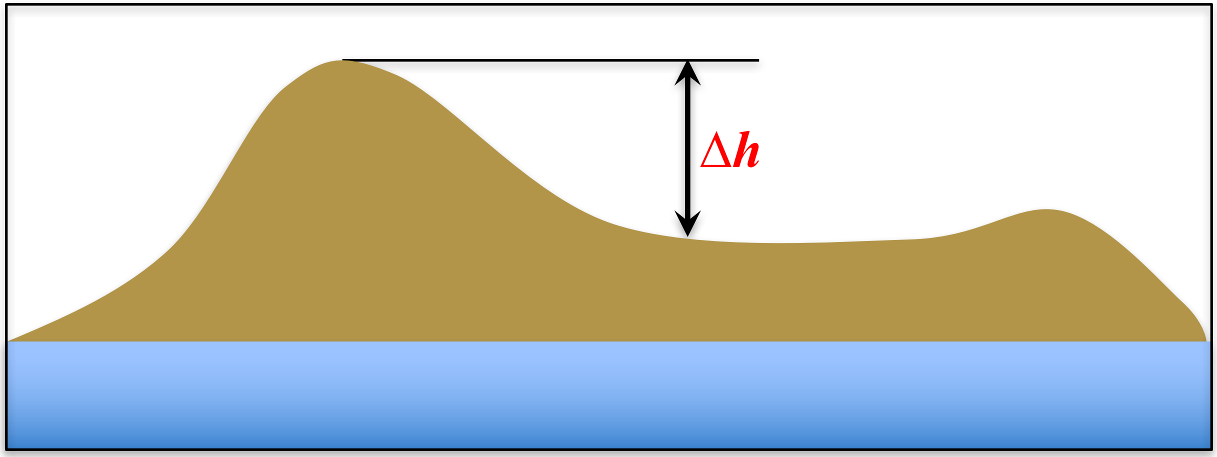
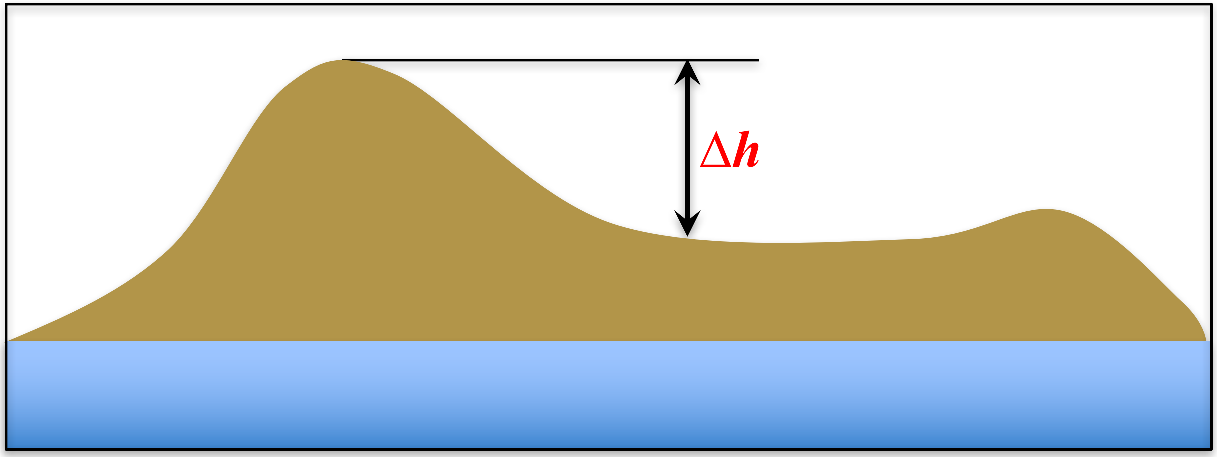
- It appears that under their own units the gravity gradient is a million times worse than the gravity.
- In reality this is not the case.
- In particular, the opposite is true in terms of dealing with the nuisance of elevation corrections.



$R_E = 6.371 \times 10^6$ meter, The average radius of the Earth

$$\Delta g = -1.9\times10^9 \mu gal \left(\frac{\Delta h}{R_E}\right)$$

$$\Delta\left(\frac{\partial g}{\partial z}\right) = -9.0\times10^3 E \left(\frac{\Delta h}{R_E}\right)$$



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The gradient has ~ Million times smaller elevation corrections than the gravity (under their own units)

Contents

1. ***S*tatic mass anomaly with gravity gradient (**a brief review**)**
2. ***D*eformation related gravity gradients (**Preliminary modeling**)**
   - Traditional gas deposit and production
   - Hydraulic cracking for shale gas

Gravity gradient tensor

$$\mathcal{G} = \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} \\ \frac{\partial g_3}{\partial x_1} & \frac{\partial g_3}{\partial x_2} & \frac{\partial g_3}{\partial x_3} \end{pmatrix}$$

Symmetric

$$\mathcal{G}_{i,j} = \frac{\partial^2 \Phi}{\partial x_i \partial x_j}$$

Zero trace

$$\sum_{i=1}^{3} \mathcal{G}_{i,i} = \sum_{i=1}^{3} \frac{\partial^2 \Phi}{\partial x_i^2} = 0$$

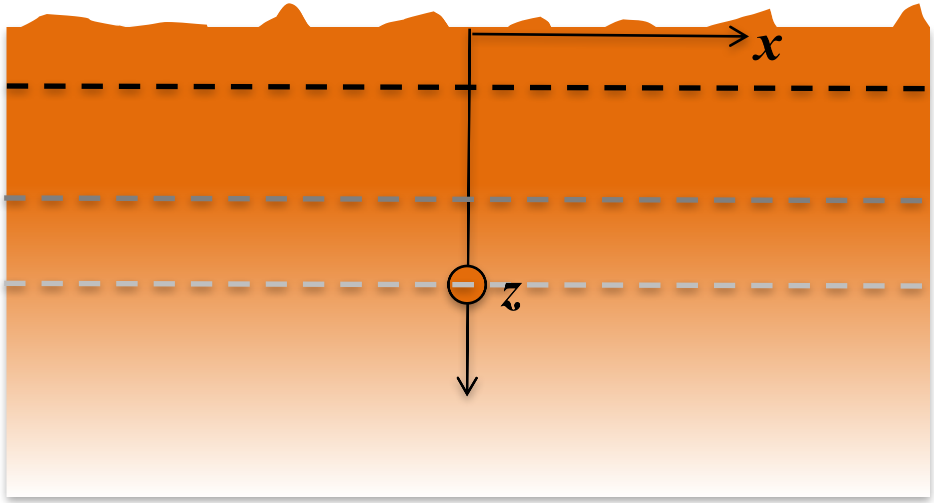
Gradient is sensitive to shallow and sharp structures

$$g(x) \sim \sin \omega x, \qquad \mathcal{G}_{z,x} = \omega \cos \omega x$$

Gradient $\mathcal{G}_{z,z}(0) = \frac{1}{z^3}$

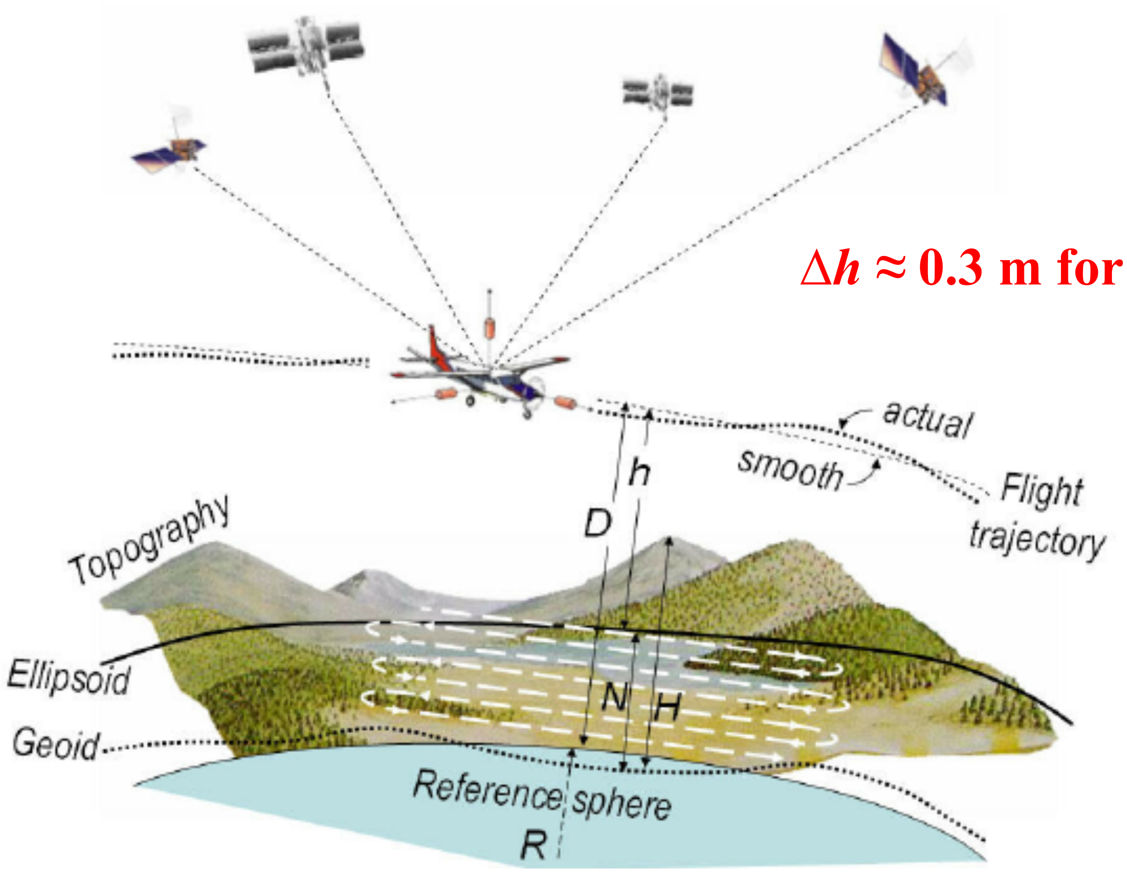
Gravity $g(0) \sim \frac{1}{z^2}$

Potential $\Phi(0) \sim \frac{1}{z}$



Depth of penetration

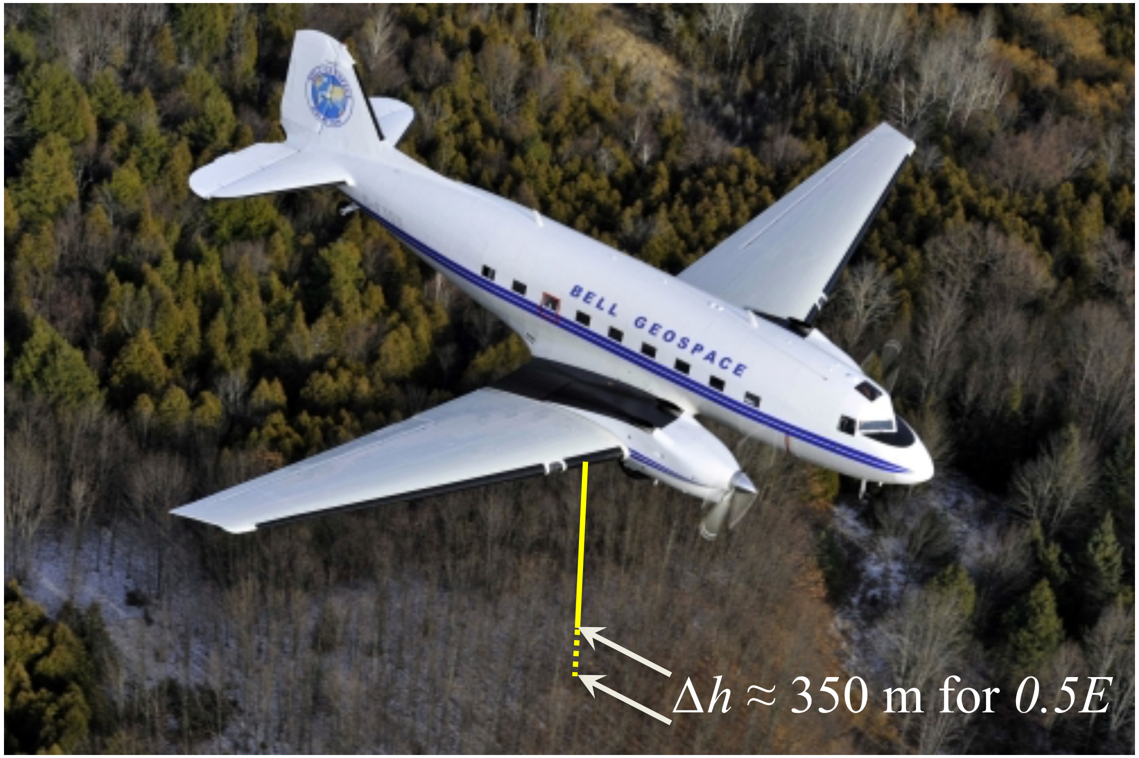
A big nuisance for airborne gravity is the height correction for repeated flyby



$\Delta h \approx 0.3$ m for $0.1mgal$

Hajkova et al, 2010

For airborne gravity gradient the accuracy of aircraft height is much less important



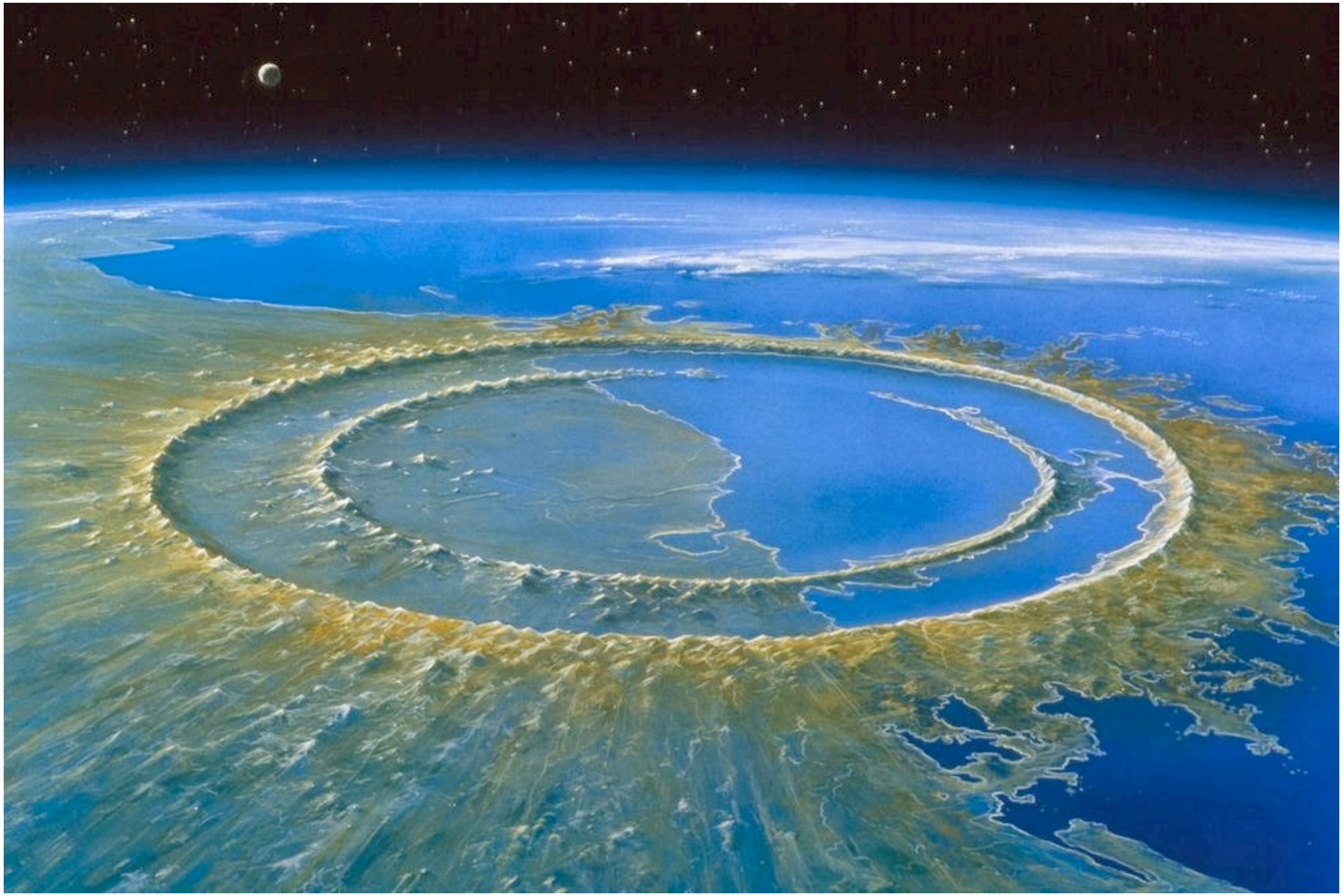
$\Delta h \approx 350$ m for $0.5E$

http://www.intrepid-geophysics.com/ig/index.php?page=gradiometry

Probe the static structure: the double rings of Vredefort Crater, South Africa, with Gravity Gradient

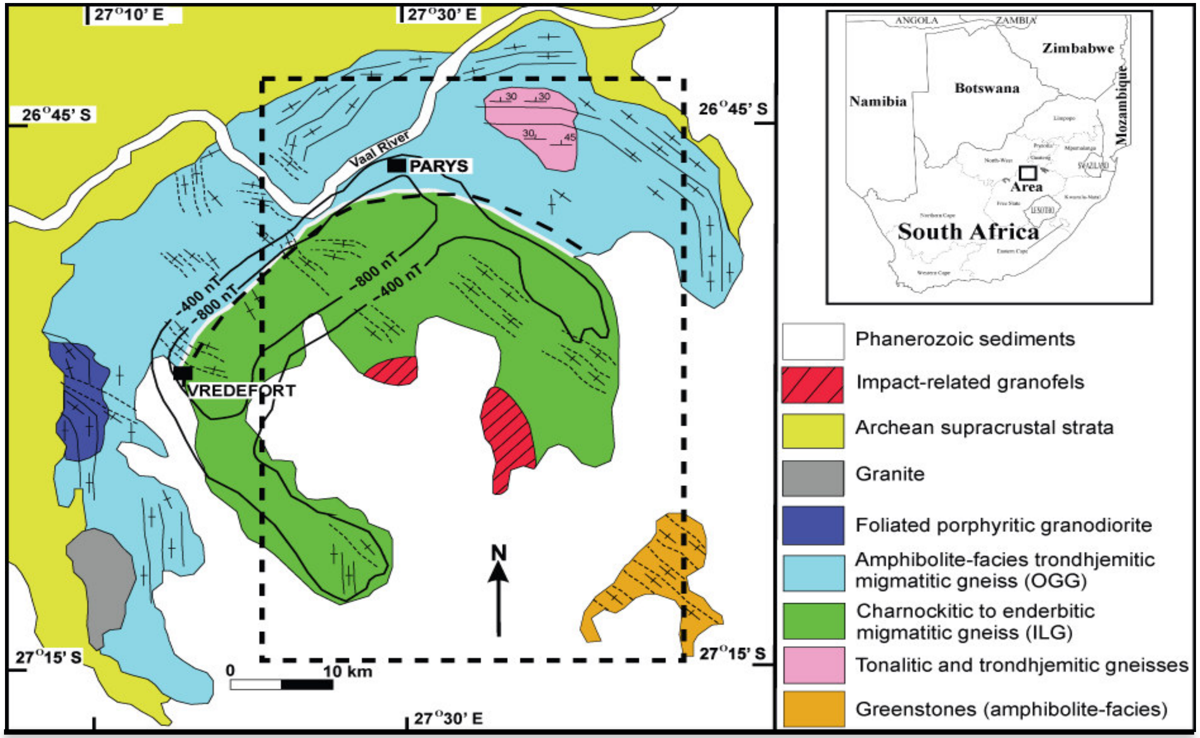
https://en.wikipedia.org/wiki/Vredefort_crater#/media/File:Vredefort_crater.jpg

Artistic Reconstruction of the Crater from geological evidence



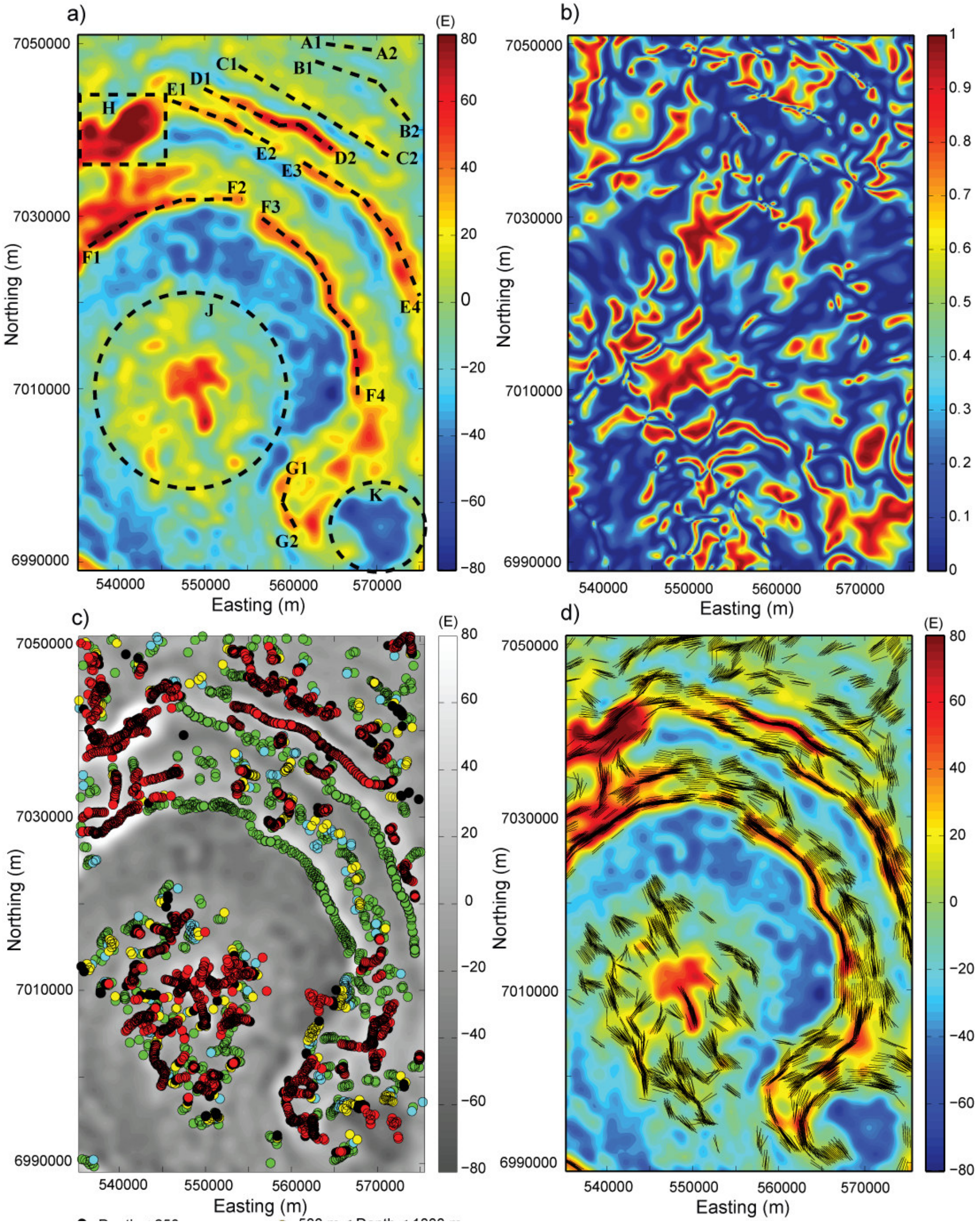
http://news.nationalgeographic.com/news/2013/13/130214-biggest-asteroid-impacts-meteorites-space-2012da14/

Investigating area by airborne gradiometer



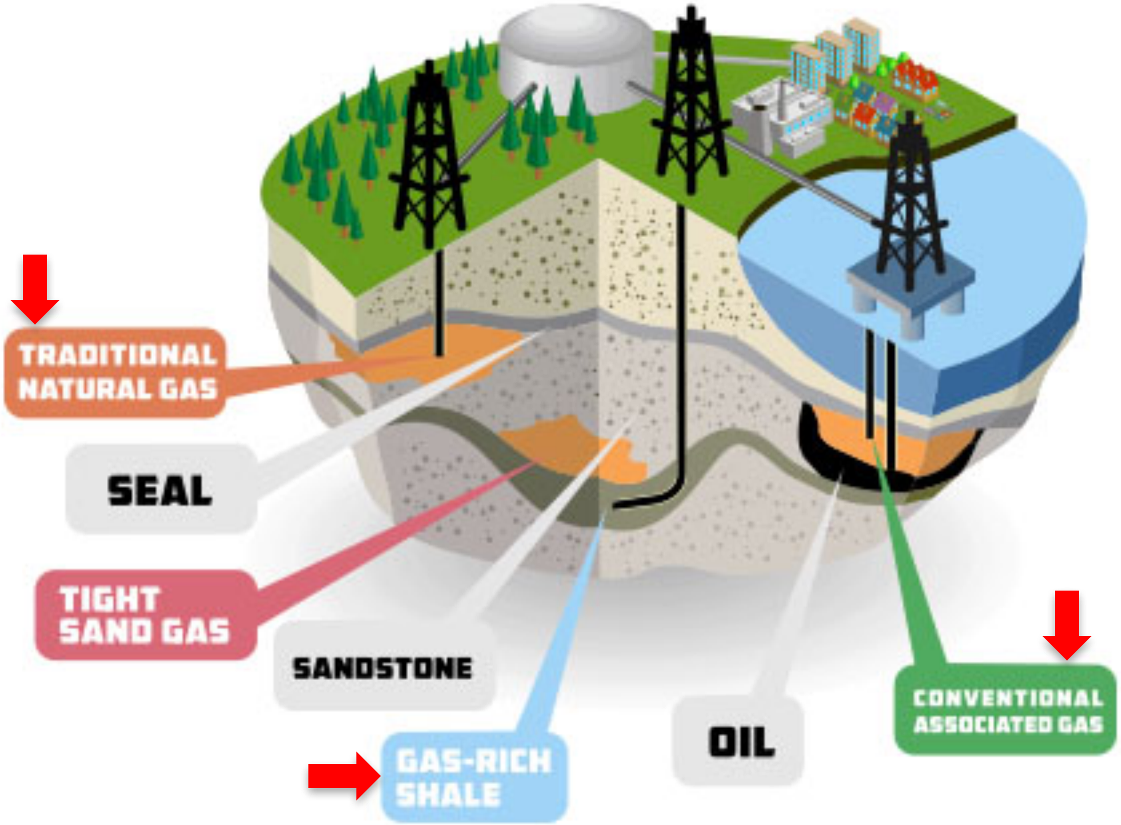
Beiki *at al*. (2010)

$$\frac{\partial g_z}{\partial z}$$



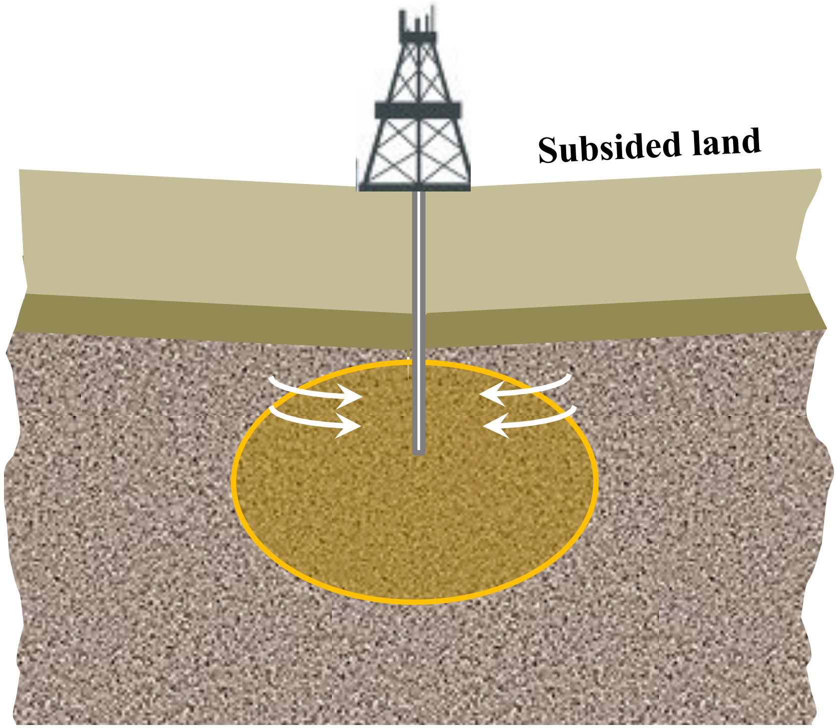
Beiki *at al*. (2010)

Probe natural gas production



https://www.waltongas.com/index.php/blog/category/44/types-of-natural-gas/

Traditional natural gas production

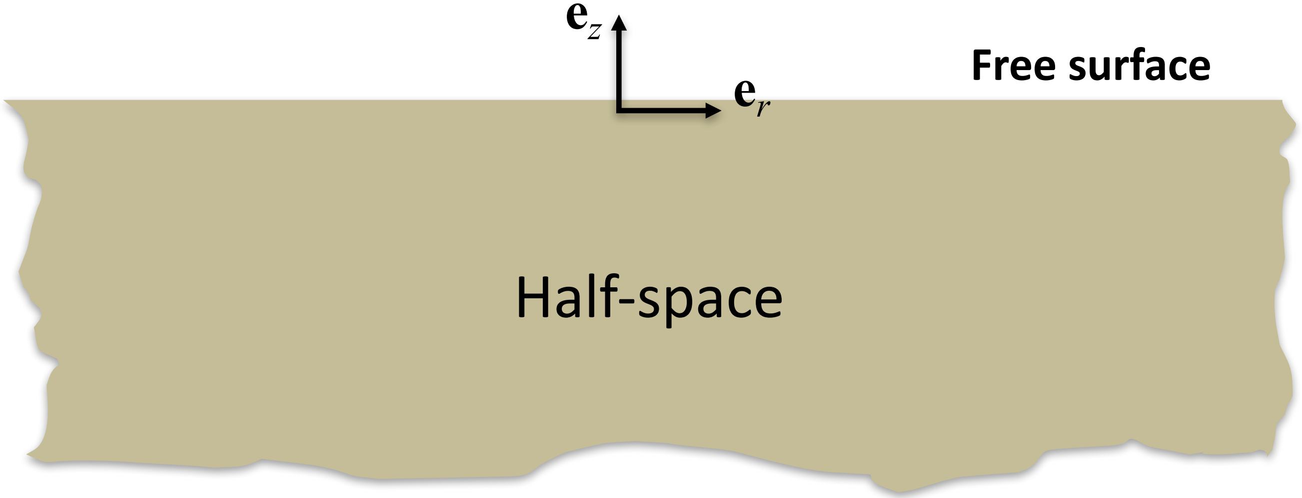


Gravity can be used to monitor the underground mass changes. We will show that the gravity signal is mostly dominated by the land subsidence /uplift, while the gravity gradient is land-subsidence/uplift proof.

Self-gravitating elastic half space model set up

$\mathbf{u}_h$ ------ Displacement

ϕ -------- Perturbed gravitational potential



$$\nabla^2\mathbf{u}_h + \frac{1}{1-2\sigma}\nabla\nabla\cdot\mathbf{u}_h - \frac{\rho_0}{\mu}\nabla\phi + \frac{\rho_0 g}{\mu}\left(\mathbf{e}_z\cdot\nabla\mathbf{u}_h - \mathbf{e}_z\nabla\cdot\mathbf{u}_h\right) = 0$$

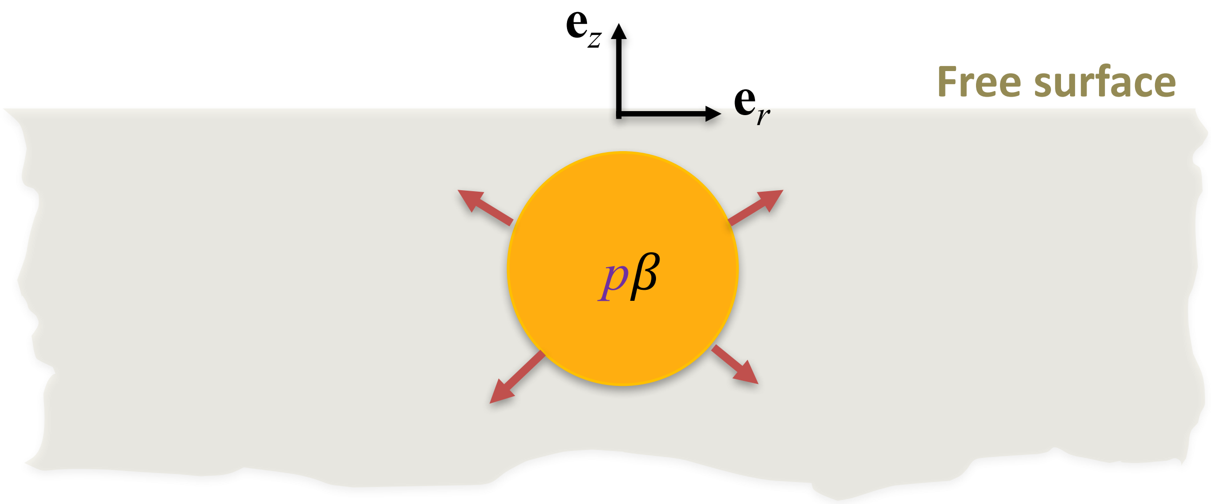
$$\nabla^2\phi = -4\pi G\rho_0\nabla\cdot\mathbf{u}_h$$

$$\lim_{\substack{R\to\infty \\ z\to+\infty}}\left(\mathbf{u}_h,\phi\right) \to 0$$

Poroelastic ball embedded in a self-gravitating half space

p ------ Incremental pressure variation

β -------- Poroelastic expansion coefficient



Decoupled pore mechanics

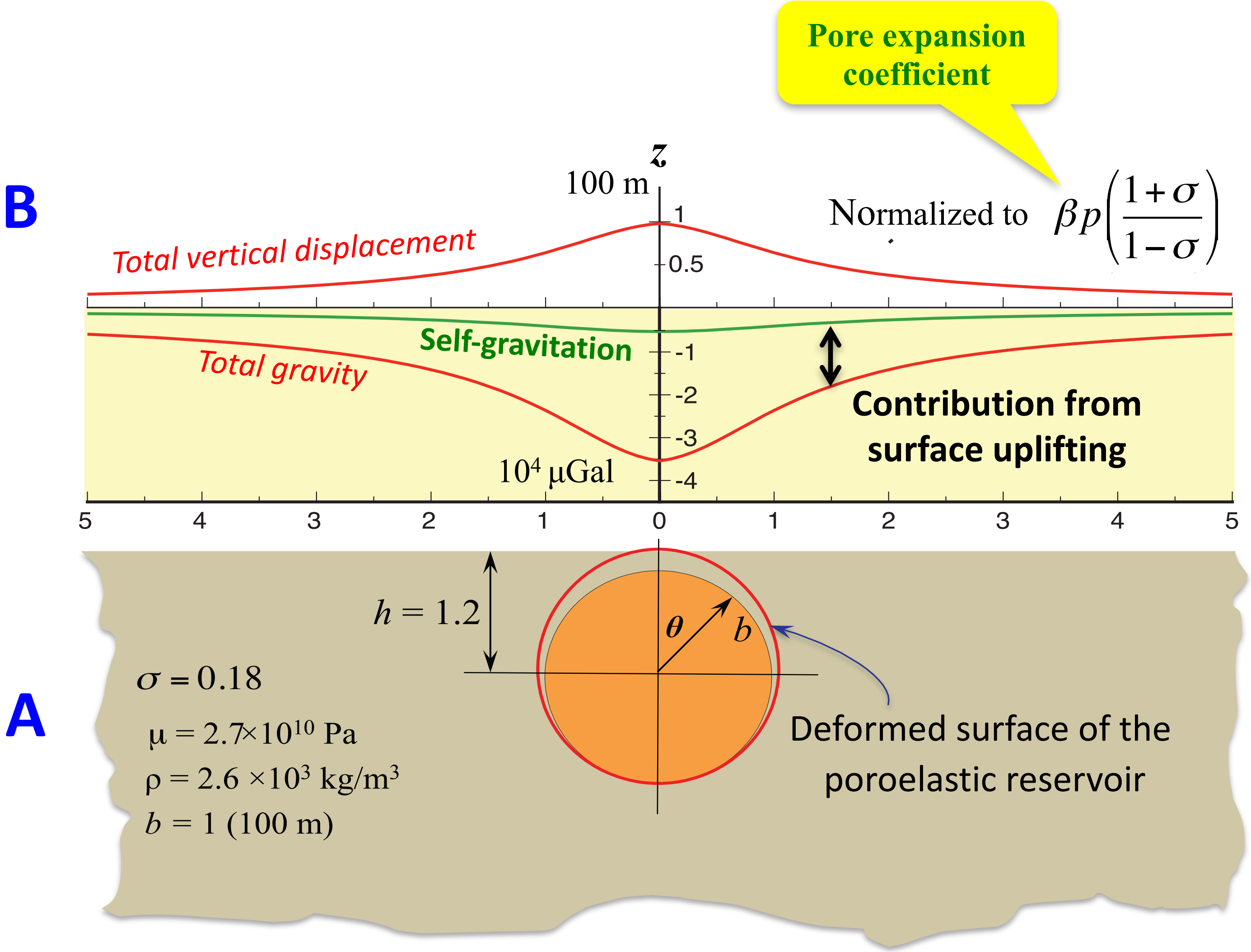
$$\lambda\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla\cdot(\nabla\mathbf{u}+\mathbf{u}\nabla) - (3\lambda+2\mu)\beta\nabla p = 0$$

Gravity and gravitational gradient from self-gravitation on the free surface

$$-\nabla\phi = -\left(\frac{\partial\phi}{\partial x}, \qquad \frac{\partial\phi}{\partial y}, \qquad \frac{\partial\phi}{\partial z}\right)$$

$$-\nabla\nabla\boldsymbol{\phi} = -\begin{pmatrix} \frac{\partial^2\phi}{\partial x^2} & \frac{\partial^2\phi}{\partial x\partial y} & \frac{\partial^2\phi}{\partial x\partial z} \\ \frac{\partial^2\phi}{\partial x\partial y} & \frac{\partial^2\phi}{\partial y^2} & \frac{\partial^2\phi}{\partial y\partial z} \\ \frac{\partial^2\phi}{\partial x\partial z} & \frac{\partial^2\phi}{\partial y\partial z} & \frac{\partial^2\phi}{\partial z^2} \end{pmatrix}$$

Perturbed gravity



Pore expansion coefficient

Normalized to $\beta p\left(\frac{1+\sigma}{1-\sigma}\right)$

Contribution from surface uplifting

Deformed surface of the poroelastic reservoir

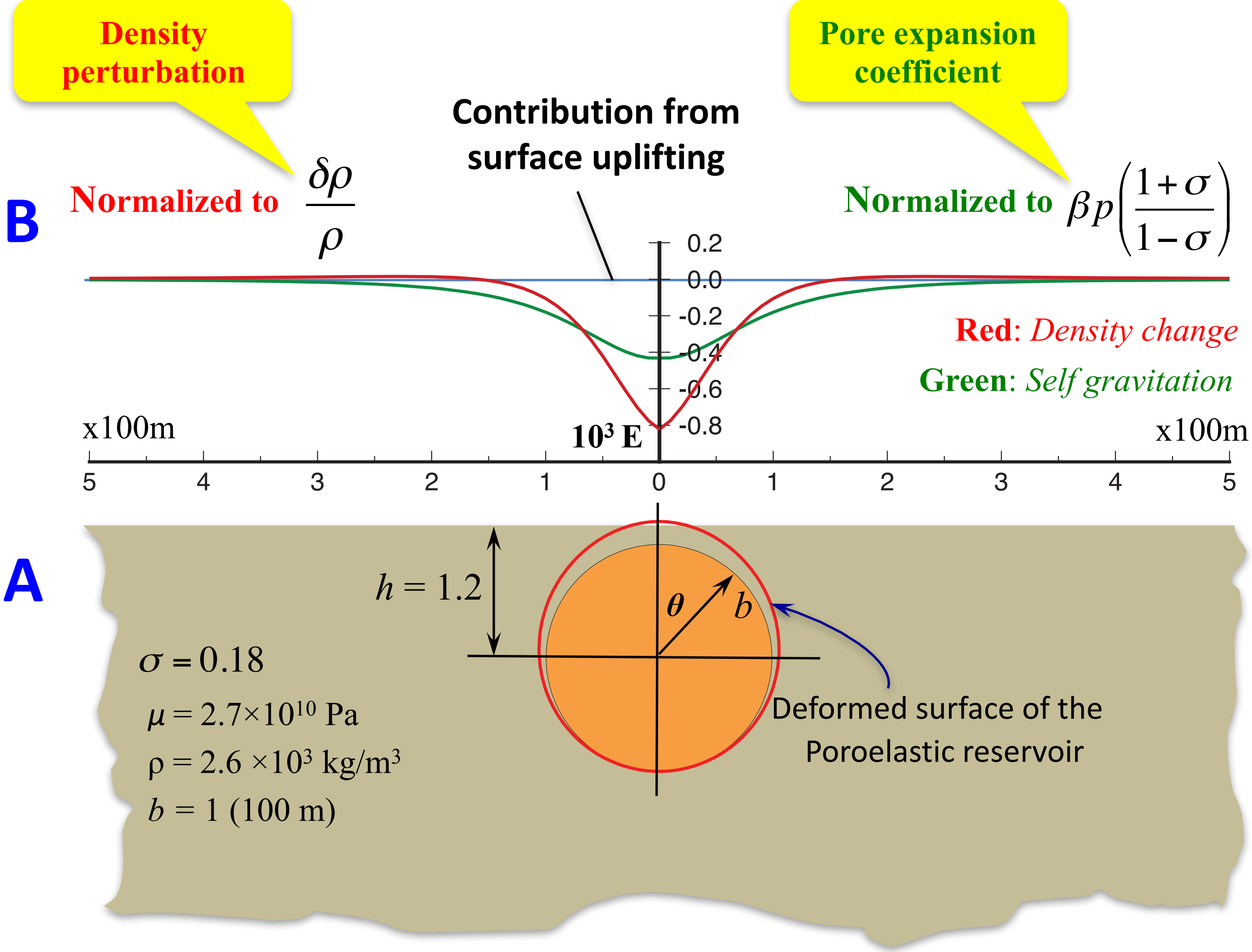
$\sigma = 0.18$

$\mu = 2.7\times10^{10}$ Pa

$\rho = 2.6\times10^{3}$ kg/m^3

$b = 1$ (100 m)

Perturbed gravity gradient $\dfrac{\partial g_z}{\partial z}$



Density perturbation

Pore expansion coefficient

Contribution from surface uplifting

B

Normalized to $\dfrac{\delta\rho}{\rho}$

Normalized to $\beta p\left(\dfrac{1+\sigma}{1-\sigma}\right)$

Red: *Density change*

Green: *Self gravitation*

A

$h = 1.2$

$\sigma = 0.18$

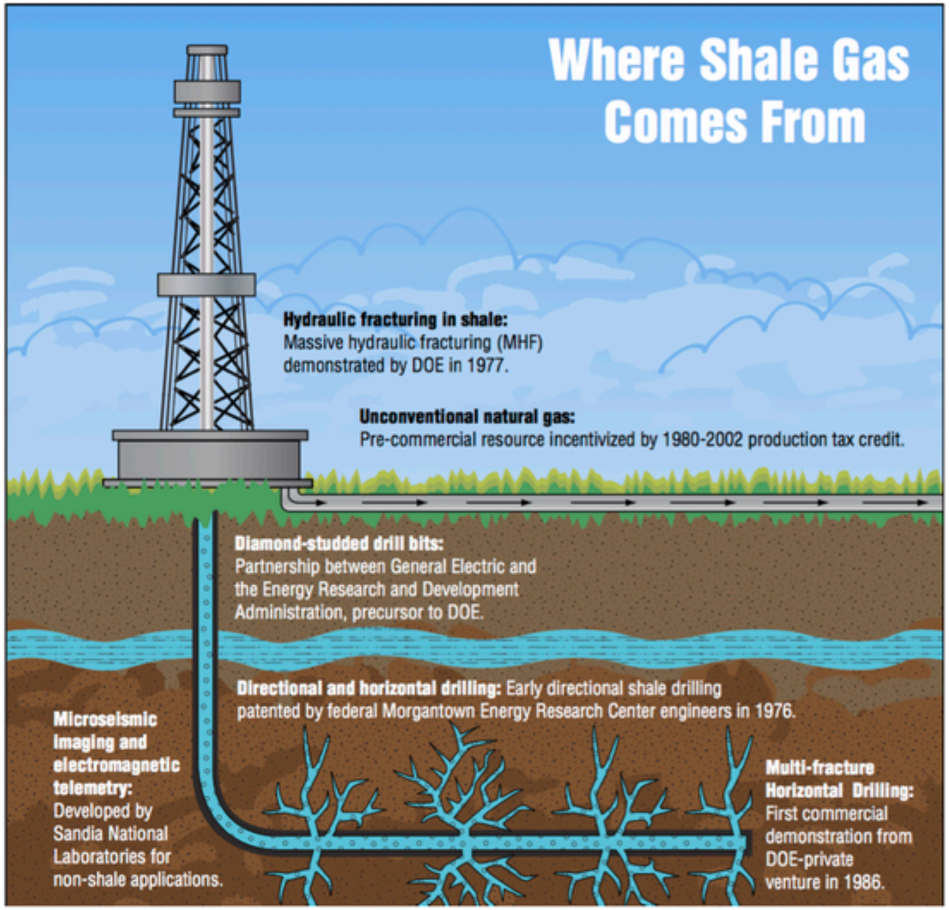
$\mu = 2.7\times10^{10}$ Pa

$\rho = 2.6\times10^3$ kg/m^3

$b = 1$ (100 m)

Deformed surface of the Poroelastic reservoir

Hydraulic fracturing for shale gas



Where Shale Gas Comes From

Hydraulic fracturing in shale: Massive hydraulic fracturing (MHF) demonstrated by DOE in 1977.

Unconventional natural gas: Pre-commercial resource incentivized by 1980-2002 production tax credit.

Diamond-studded drill bits: Partnership between General Electric and the Energy Research and Development Administration, precursor to DOE.

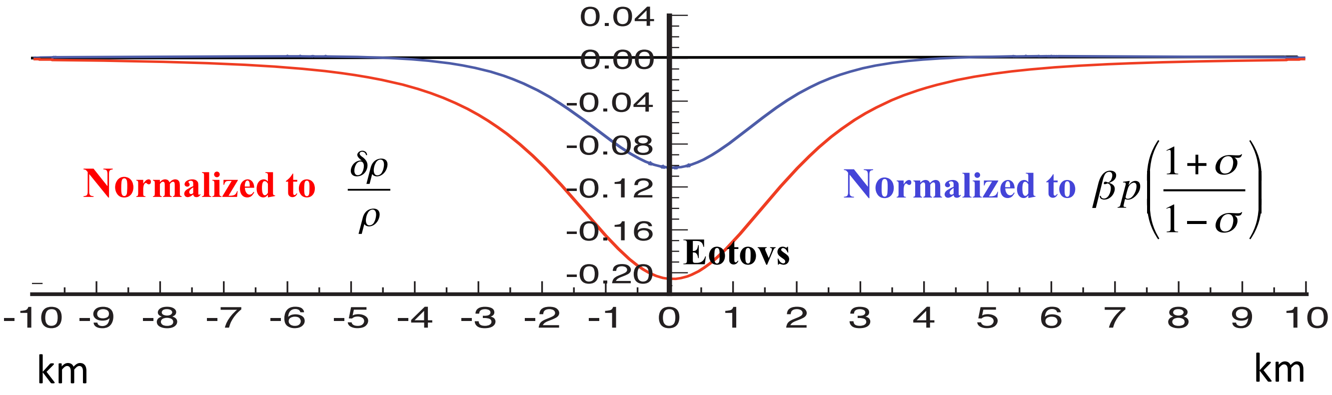
Directional and horizontal drilling: Early directional shale drilling patented by federal Morgantown Energy Research Center engineers in 1976.

Microseismic imaging and electromagnetic telemetry: Developed by Sandia National Laboratories for non-shale applications.

Multi-fracture Horizontal Drilling: First commercial demonstration from DOE-private venture in 1986.

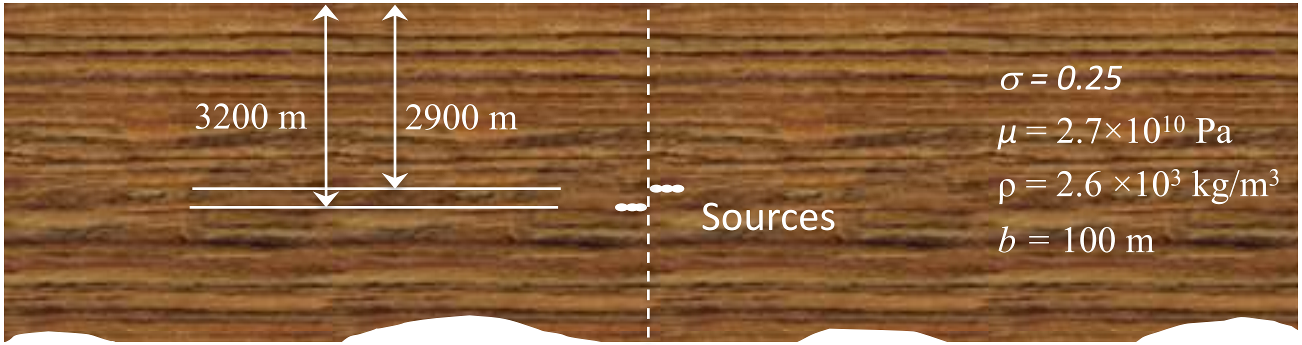
https://thebreakthrough.org/index.php/programs/energy-and-climate/where-the-shale-gas-revolution-came-from

Perturbed gravity gradient from density (red) self-gravitation (blue) and surface uplift (black) $\dfrac{\partial g_z}{\partial z}$



Normalized to $\dfrac{\delta\rho}{\rho}$

Normalized to $\beta p\left(\dfrac{1+\sigma}{1-\sigma}\right)$



$\sigma = 0.25$
$\mu = 2.7 \times 10^{10}$ Pa
$\rho = 2.6 \times 10^3$ kg/m^3
$b = 100$ m

Conclusions

The gravity gradient can effectively denuisance the ground subsidence/uplift effect in deformation-related time-varying gravity variations.