

# An Inversion Approach Towards a Reduction of Fluid Flow Model Complexity

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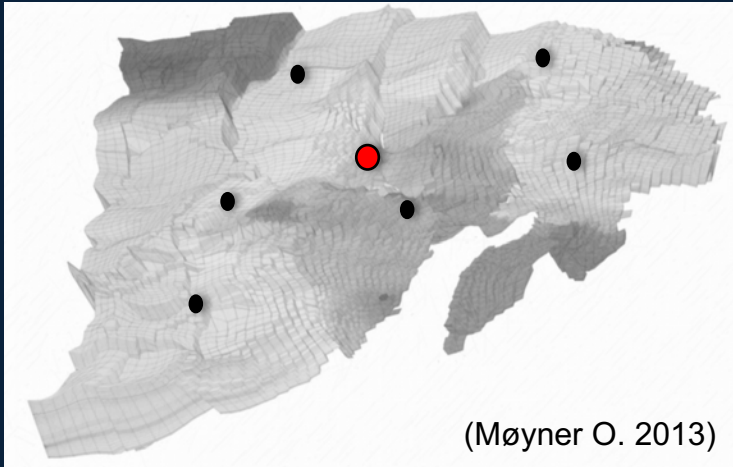
STUDENT [ EARTH, ATMOSPHERIC AND PLANETARY SCIENCES ]

*In collaboration with Prof Dale Morgan*

أرامكو السعودية  
saudi aramco



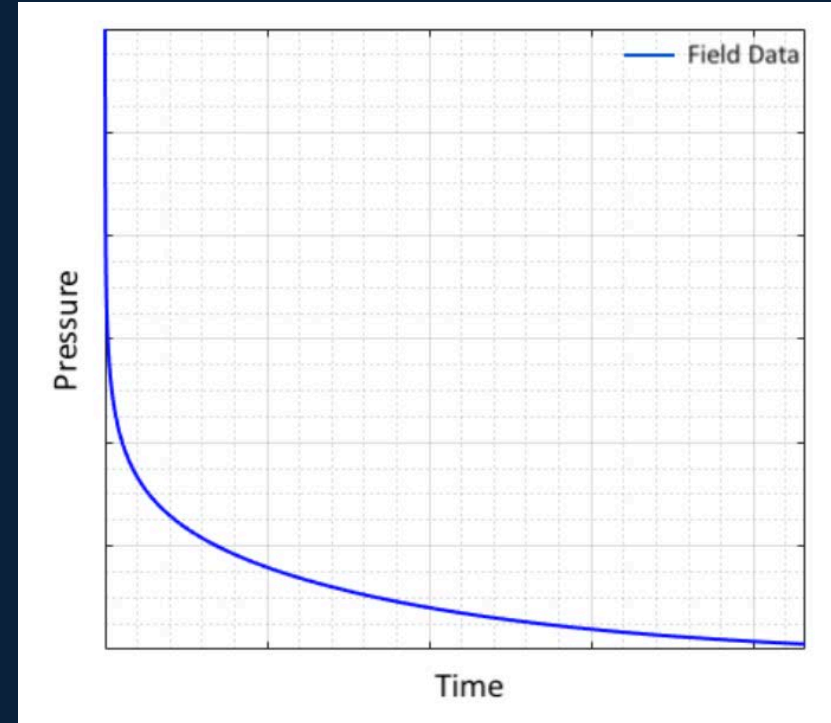
# Motivation



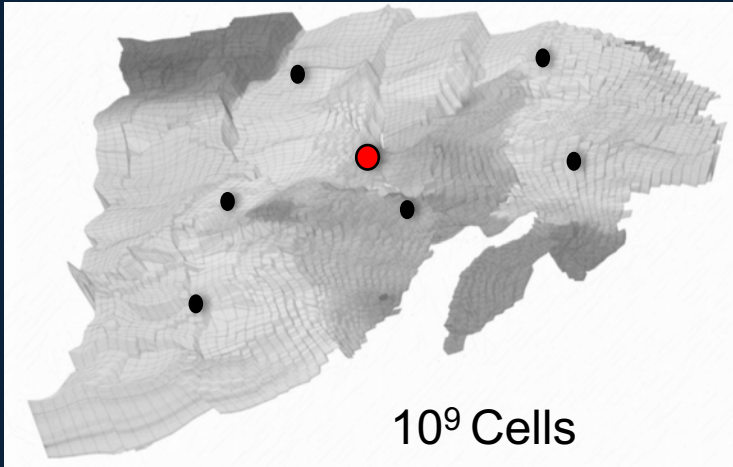
(Møyner O. 2013)

Simulation  
 $10^9$  Cells

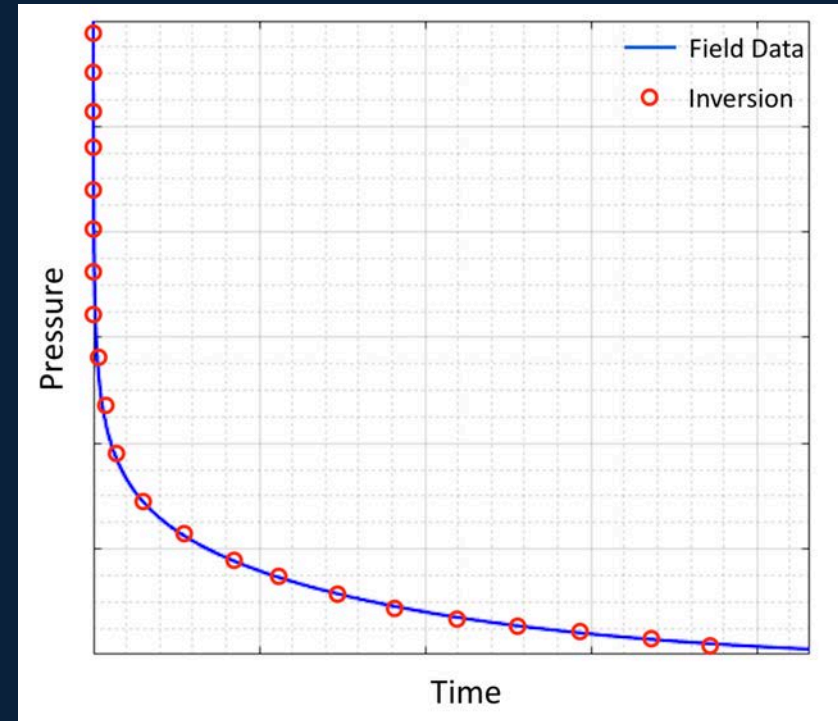
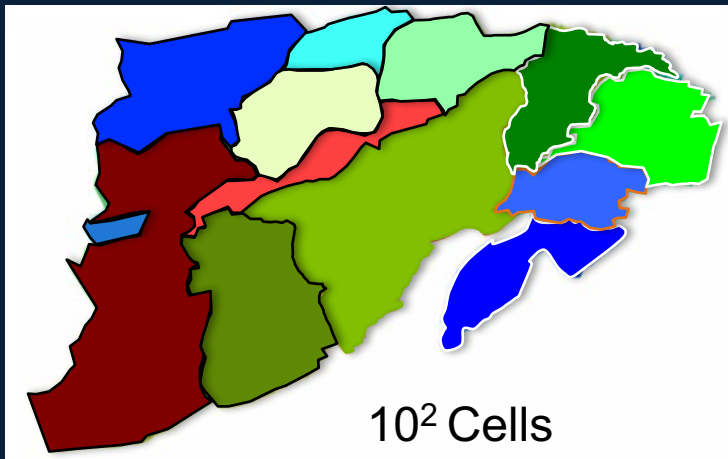
- Observation well
- Production well



# Motivation

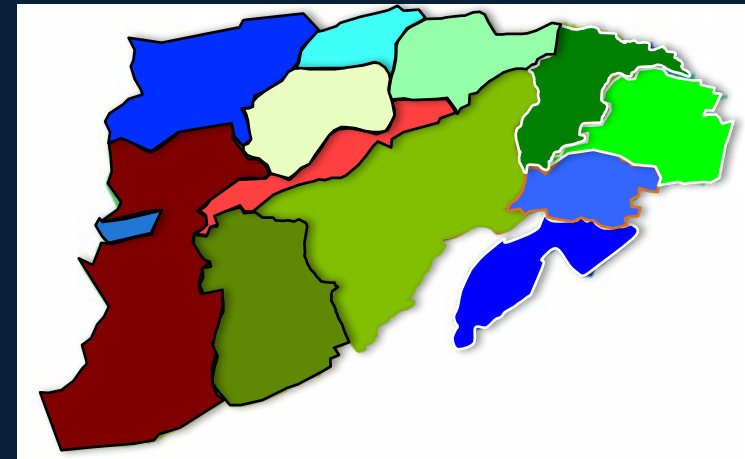
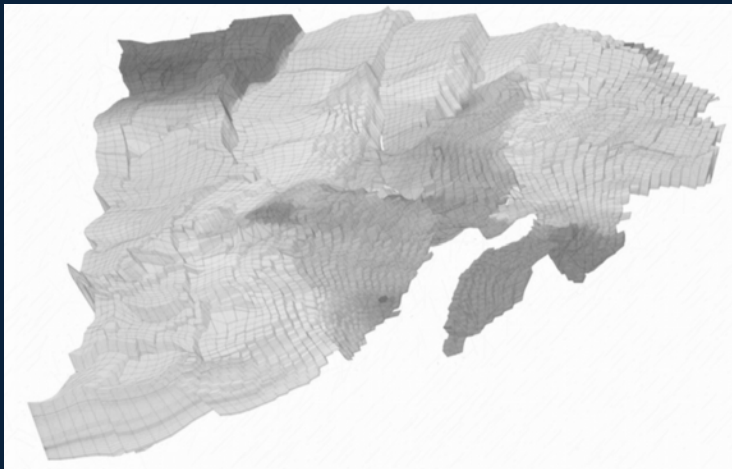


- Observation well
- Production well



# Objective

INVERTING FOR THE HYDRAULIC PARAMETERS USING A REDUCED HYDRAULIC CONDUCTIVITY STRUCTURE.



# Fluid Flow Equations

## CONFINED AQUIFERS

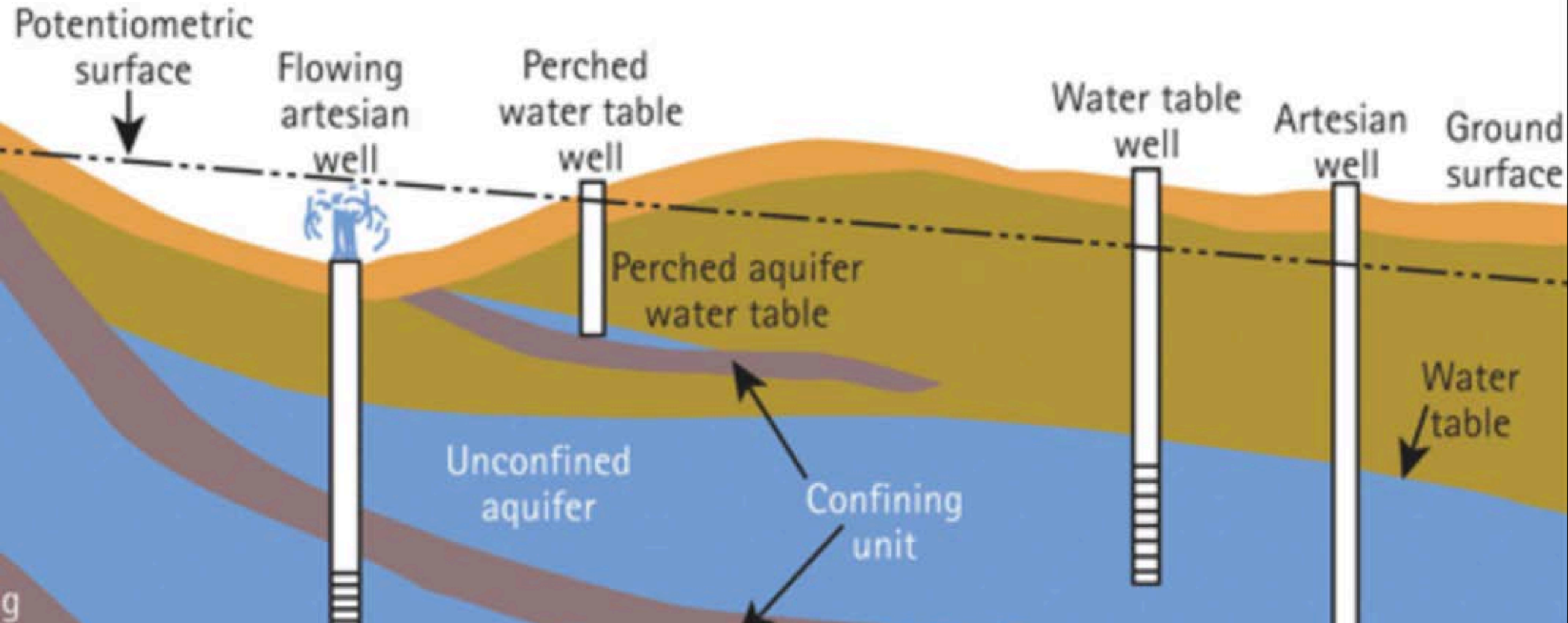
**CONFINED AQUIFERS: IS BOUNDED BY IMPERMEABLE LAYERS FROM THE TOP AND THE BOTTOM.**

$$\mathbf{T} \frac{\partial^2 h}{\partial x^2} + \mathbf{T} \frac{\partial^2 h}{\partial y^2} = S \frac{\partial h}{\partial t}.$$

## UNCONFINED AQUIFERS

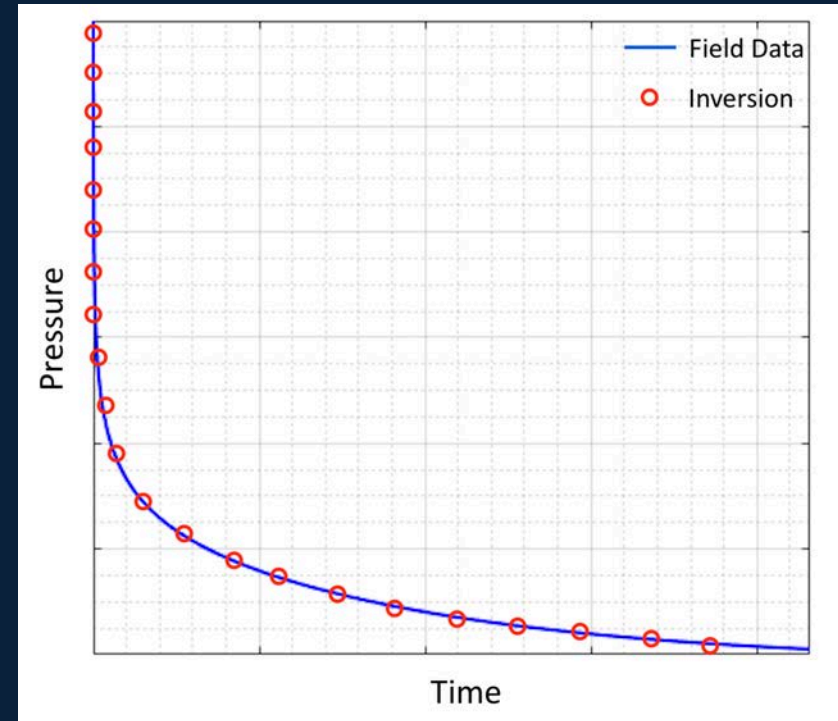
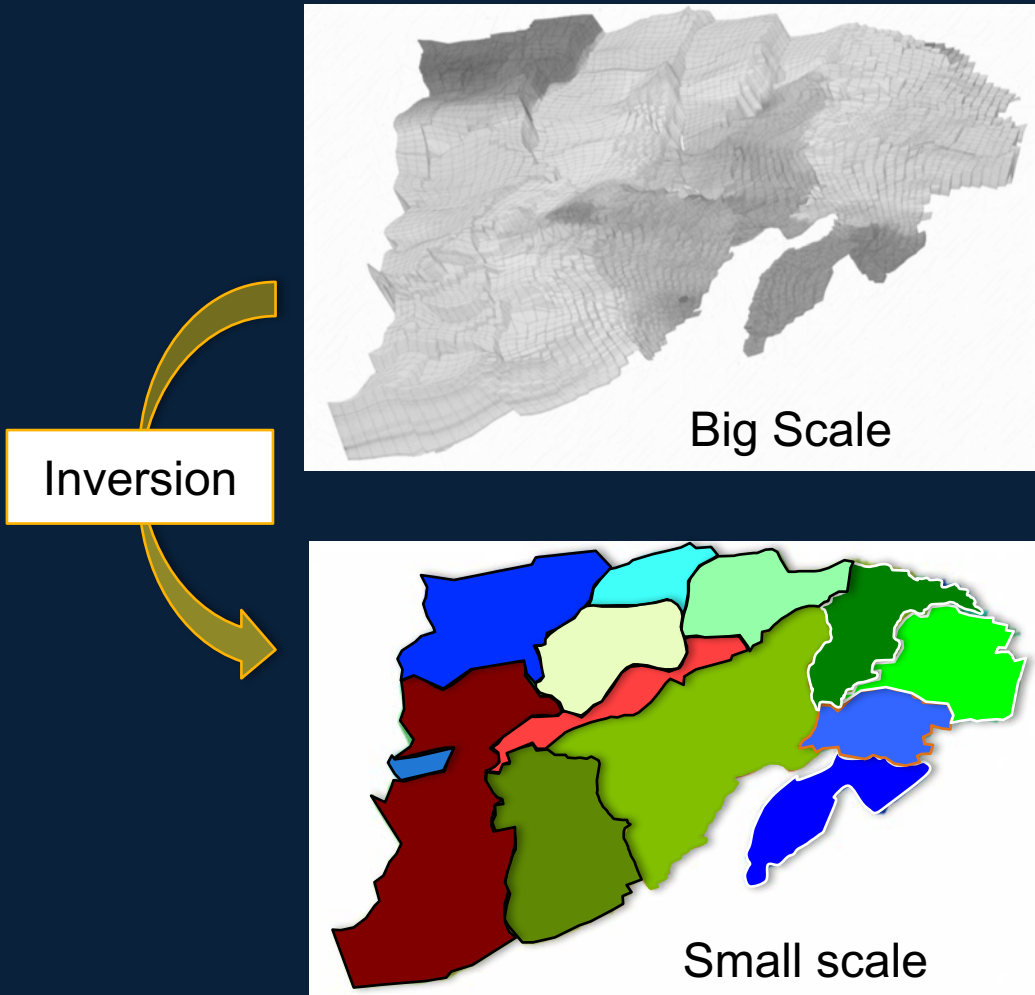
**UNCONFINED AQUIFERS: THE TOP OF THE AQUIFER IS SUBJECTED TO RECHARGE, AND ITS WATER TABLE ACTS AS THE UPPER BOUNDARY.**

$$\frac{\mathbf{K}}{2} \frac{\partial^2 h^2}{\partial x^2} + \frac{\mathbf{K}}{2} \frac{\partial^2 h^2}{\partial y^2} = S \frac{\partial h}{\partial t} - R(x, y, t).$$



## Confined & Unconfined Aquifers

# Inversion





# Inversion

## OBJECTIVE FUNCTION:

$$\min \|h_S - h_B\|_2$$

$h_S$ : data from small scale

$h_B$ : data from big scale

## DAMPED LEAST SQUARE:

A: Jacobian matrix

I: Identity matrix

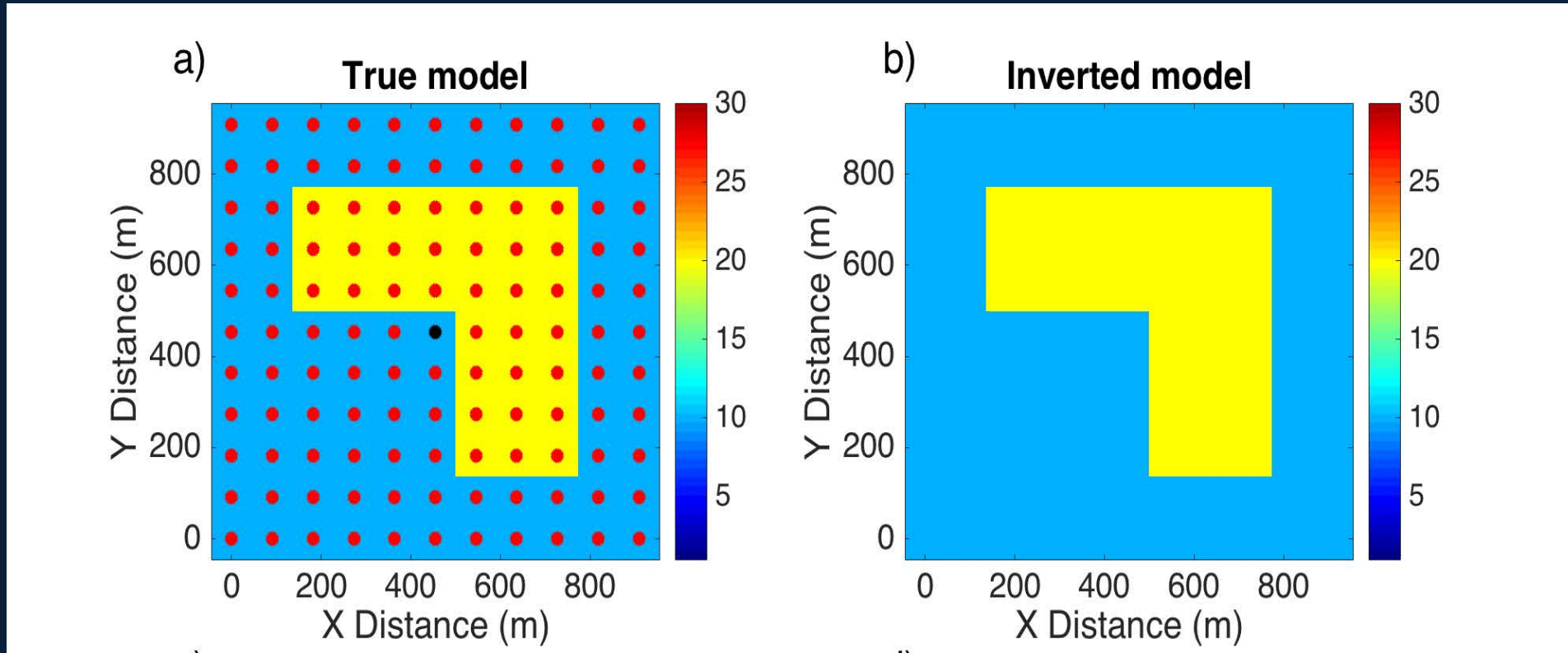
$\alpha$ : Damping factor

d: data

$$\delta h_S = (A^T A + \alpha^2 I)^{-1} A^T \delta d.$$

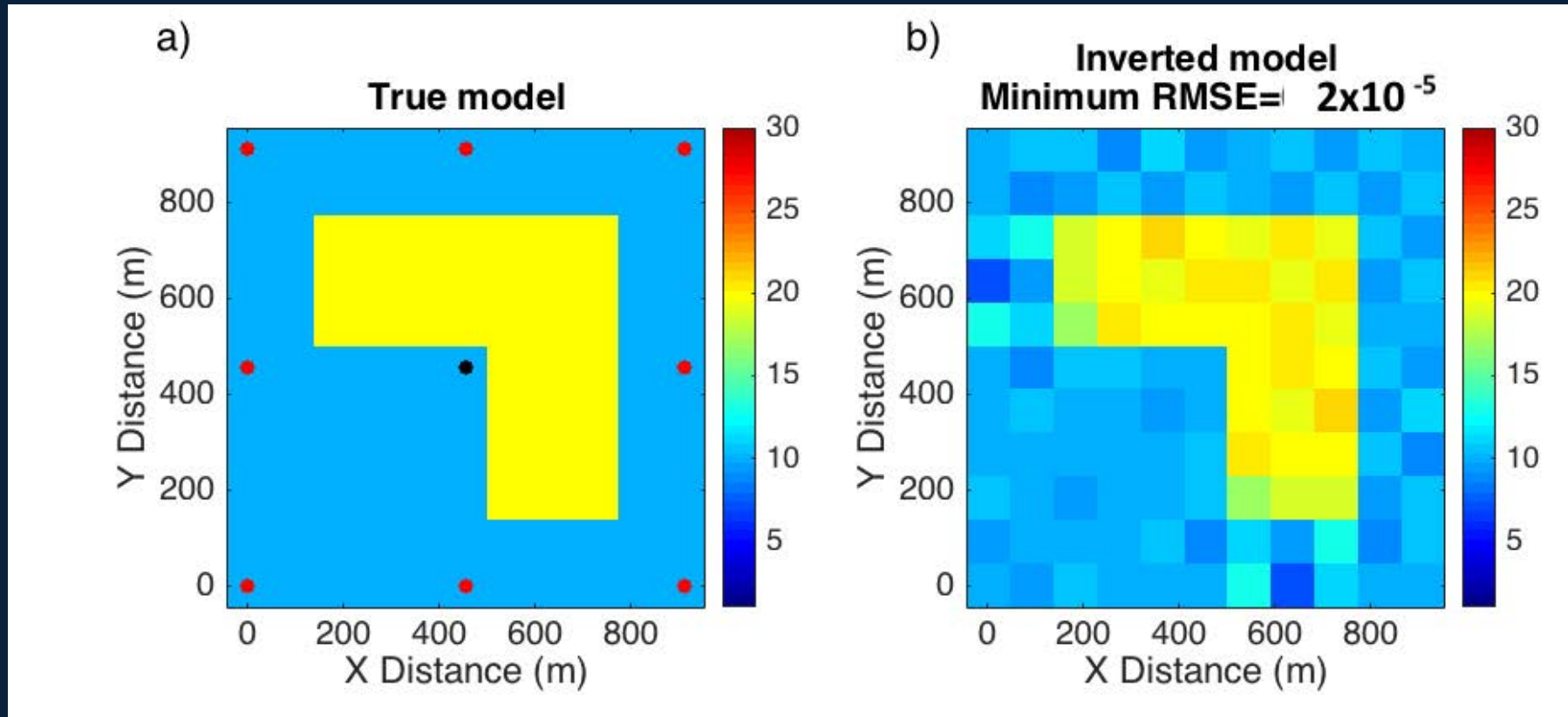


# Case 1 (x,y)



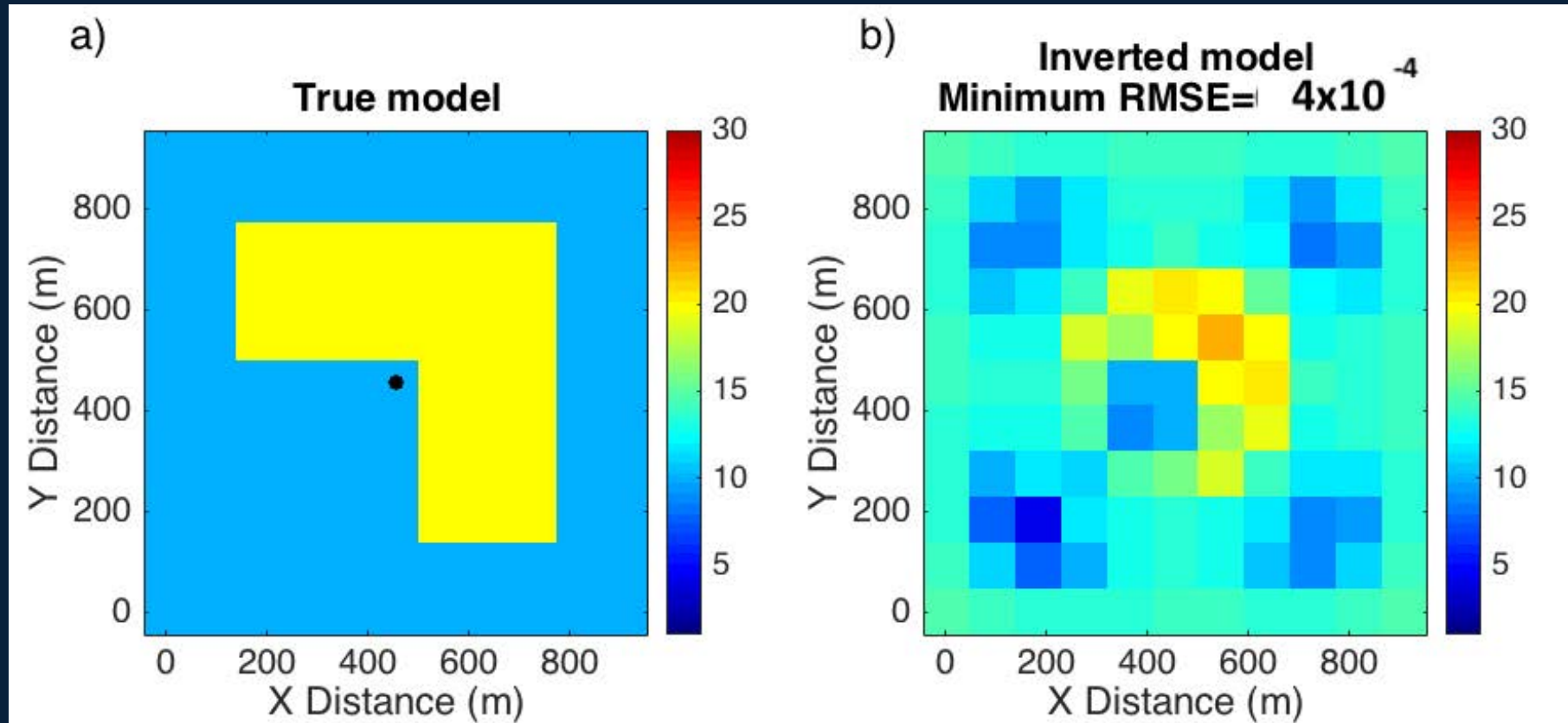
- 1 pumping well
- 1000mx1000m
- 11x11 grid
- 120 observation wells.

# Case 1 (Reduced Number of Observation wells)



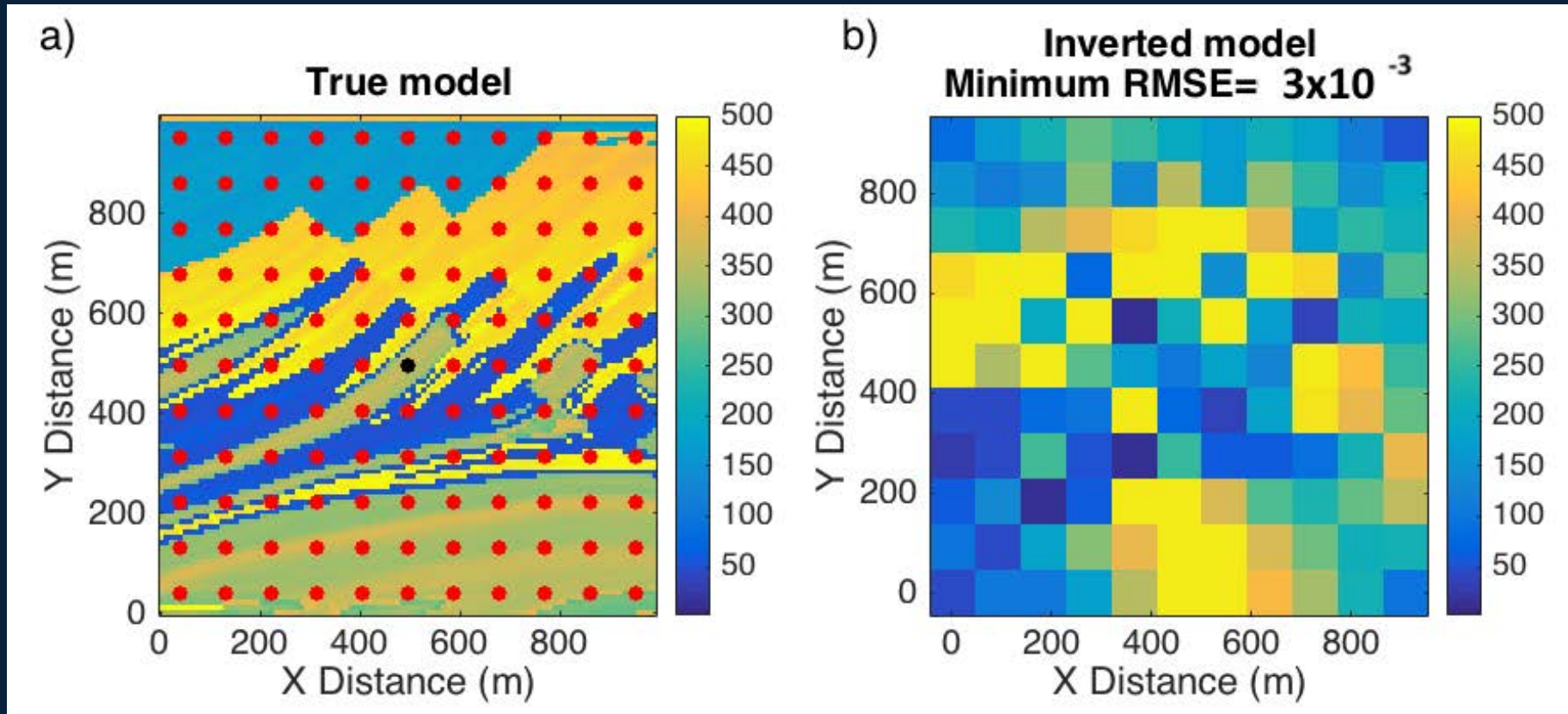
- 1 pumping well
- 1000m x 1000m
- 11x11 grid
- 8 observation wells.

# Case 1 (One Observation well)



- 1 pumping well
- 1000m x 1000m
- 11x11 grid
- 1 observation wells.

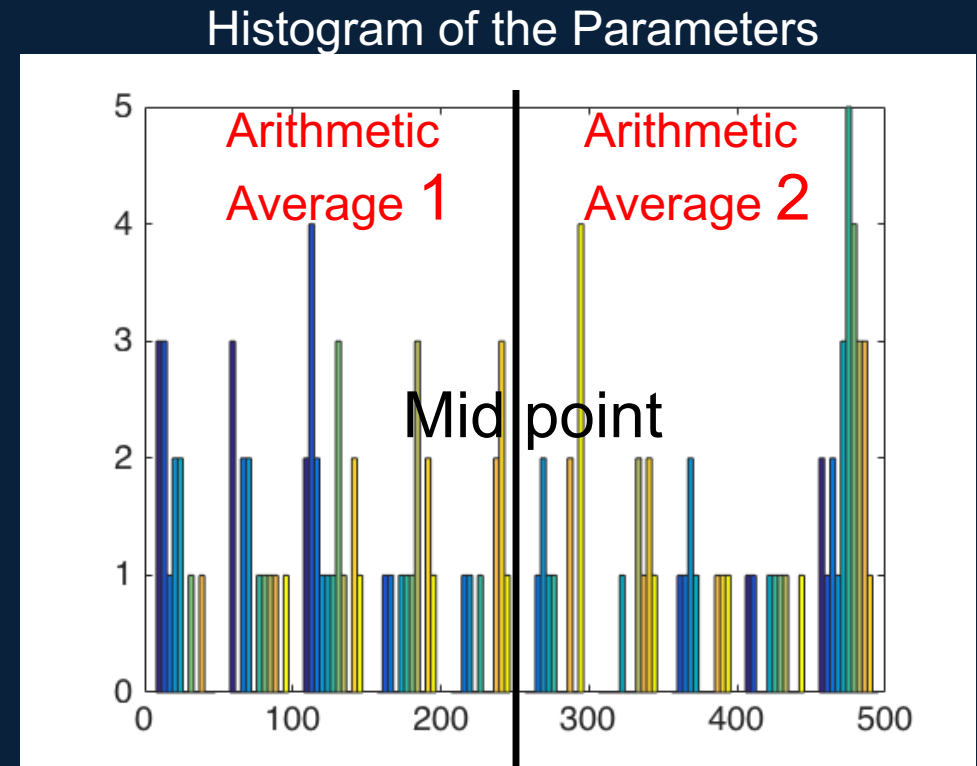
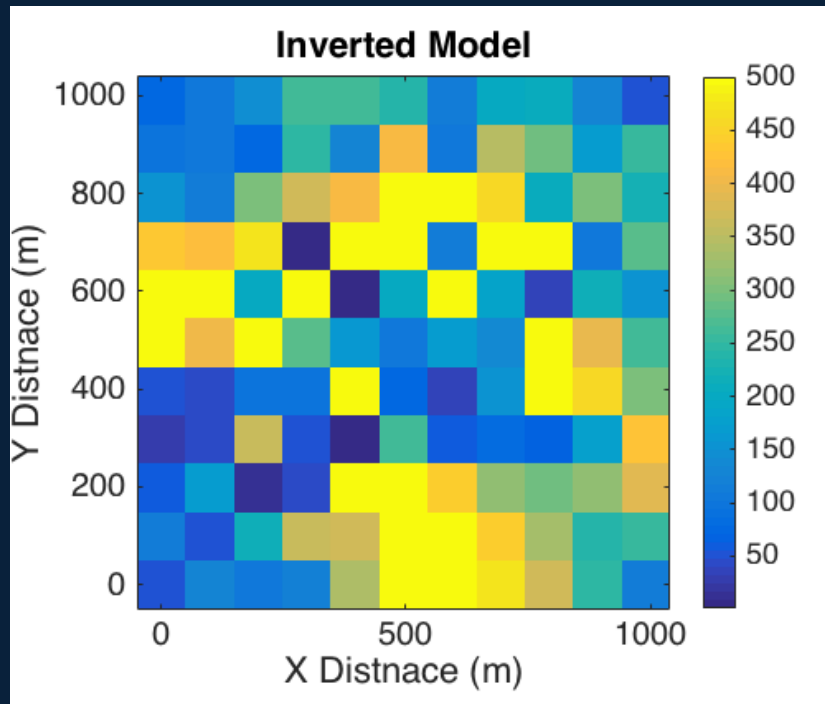
# Case 2: Big to Small



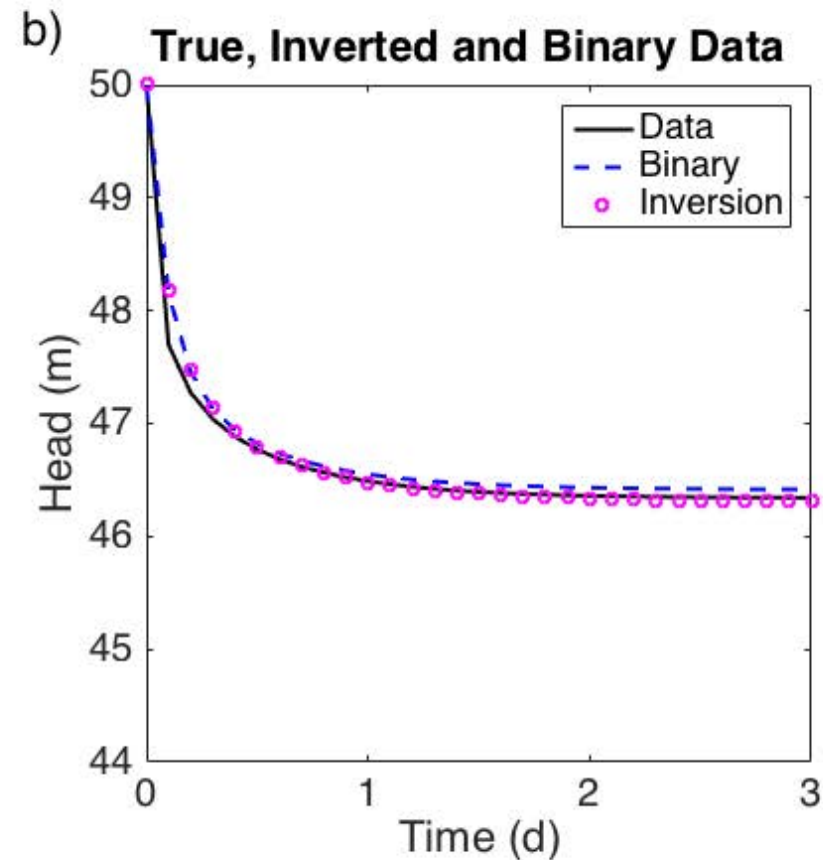
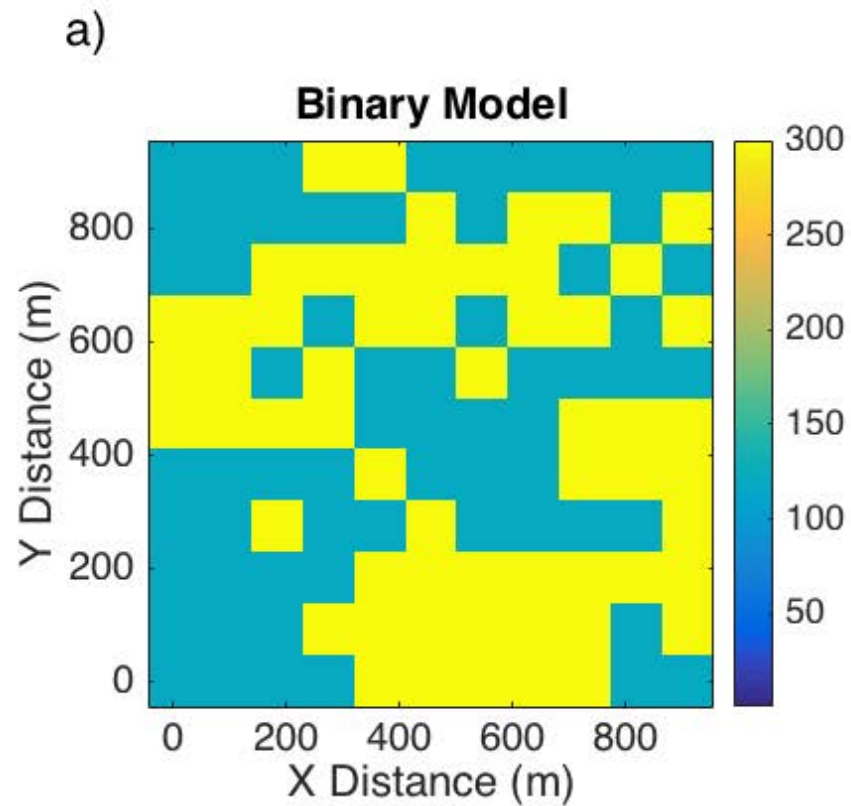
- True model: 99x99 grid
- Inverted model: 11x11 grid
- 1 pumping well
- 120 observation wells

# Binary Model (Two Conductivity Values)

THE BINARY MODEL MATCHES THE INVERTED MODEL IN TERMS OF THE GENERAL STRUCTURE, BUT IT CONTAINS TWO PARAMETERS.

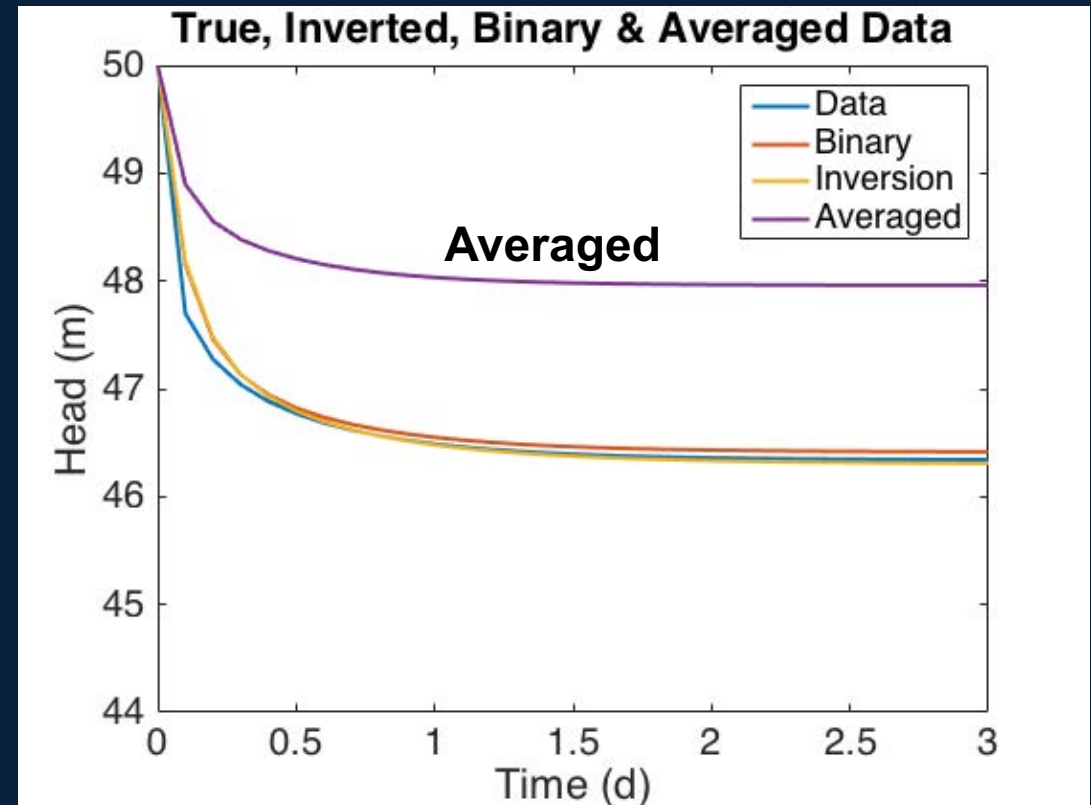
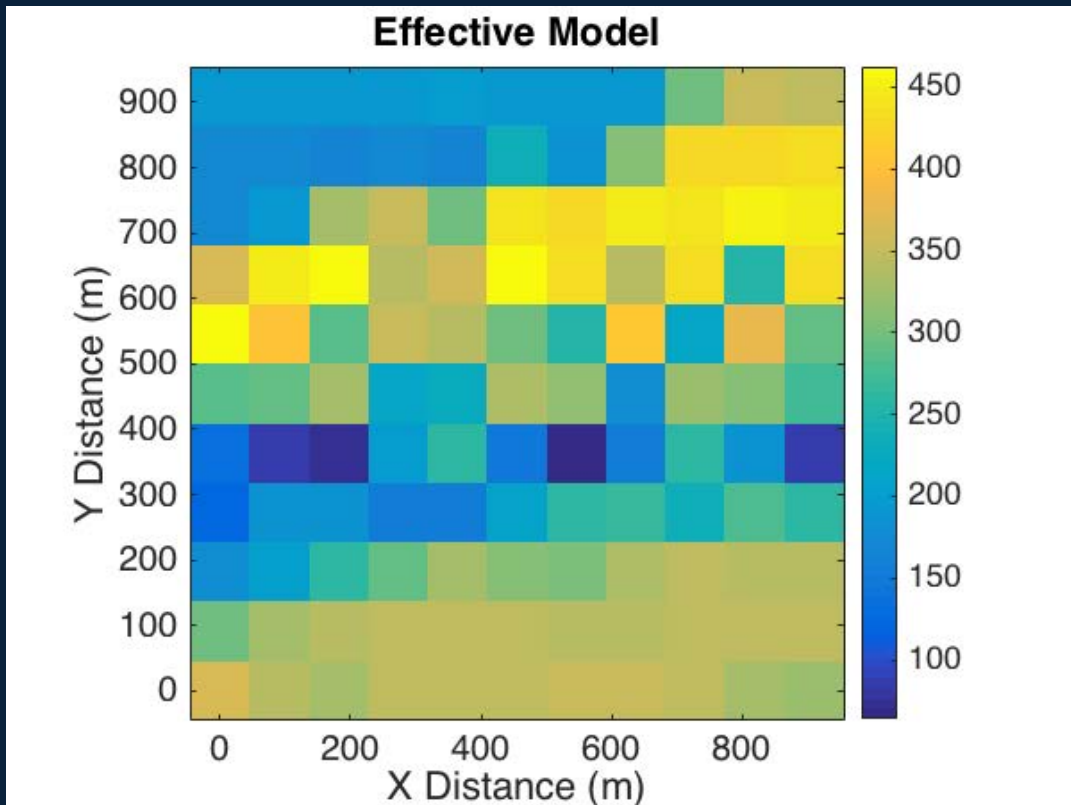


# Response of the Binary Model



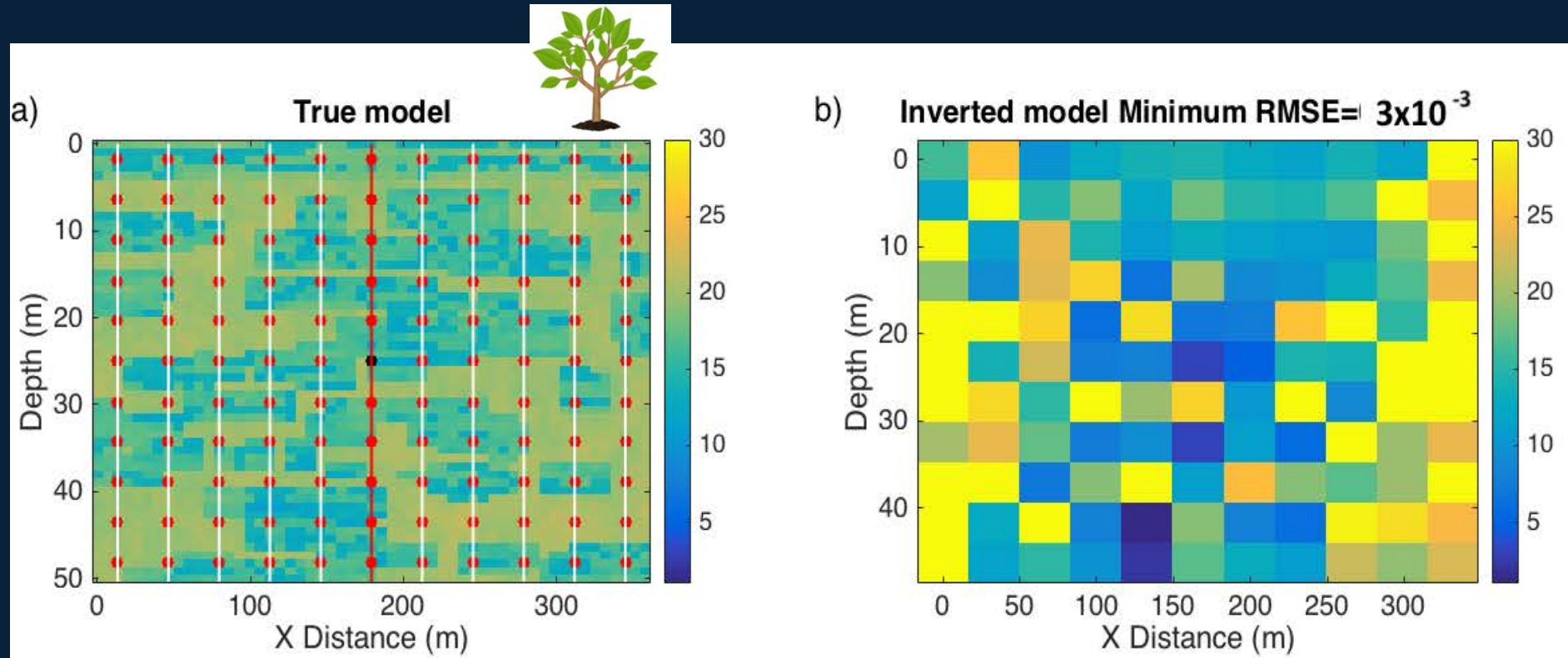


# Spatial Average of the 99x99 Model



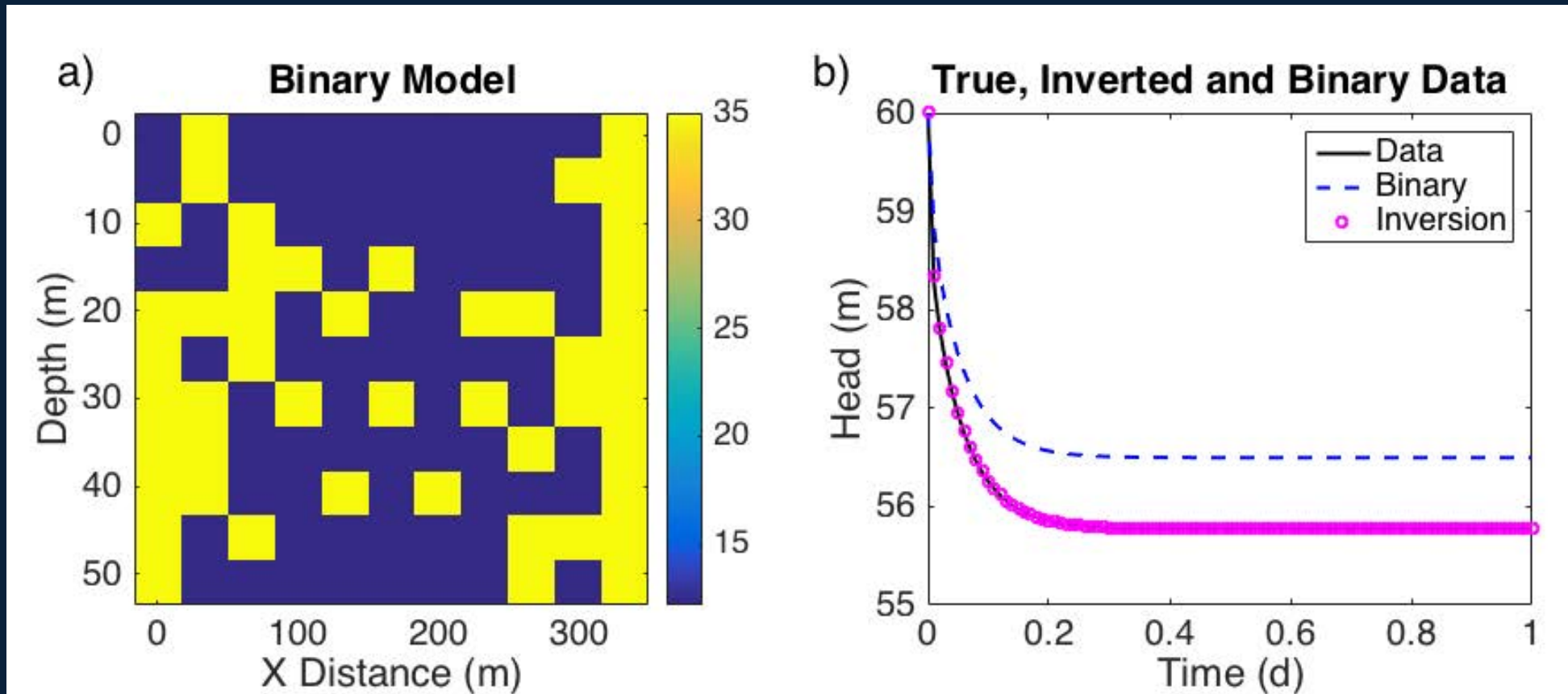


# Case 3: Big to Small (x,z)

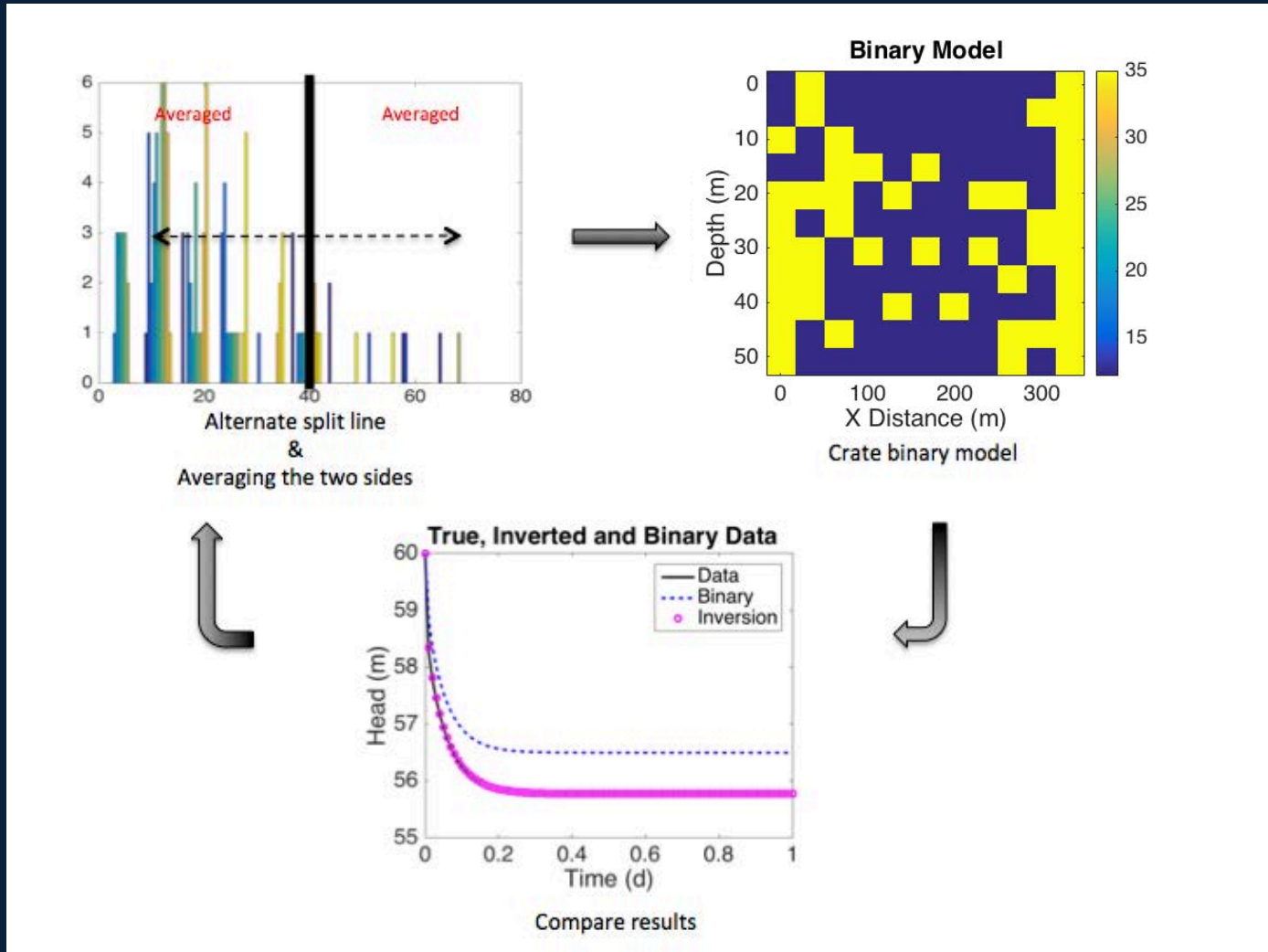


- True model: 55x55 grids
- Inverted model: 11x11 grids
- 1 pumping well
- 11 observation wells (121 data positions)

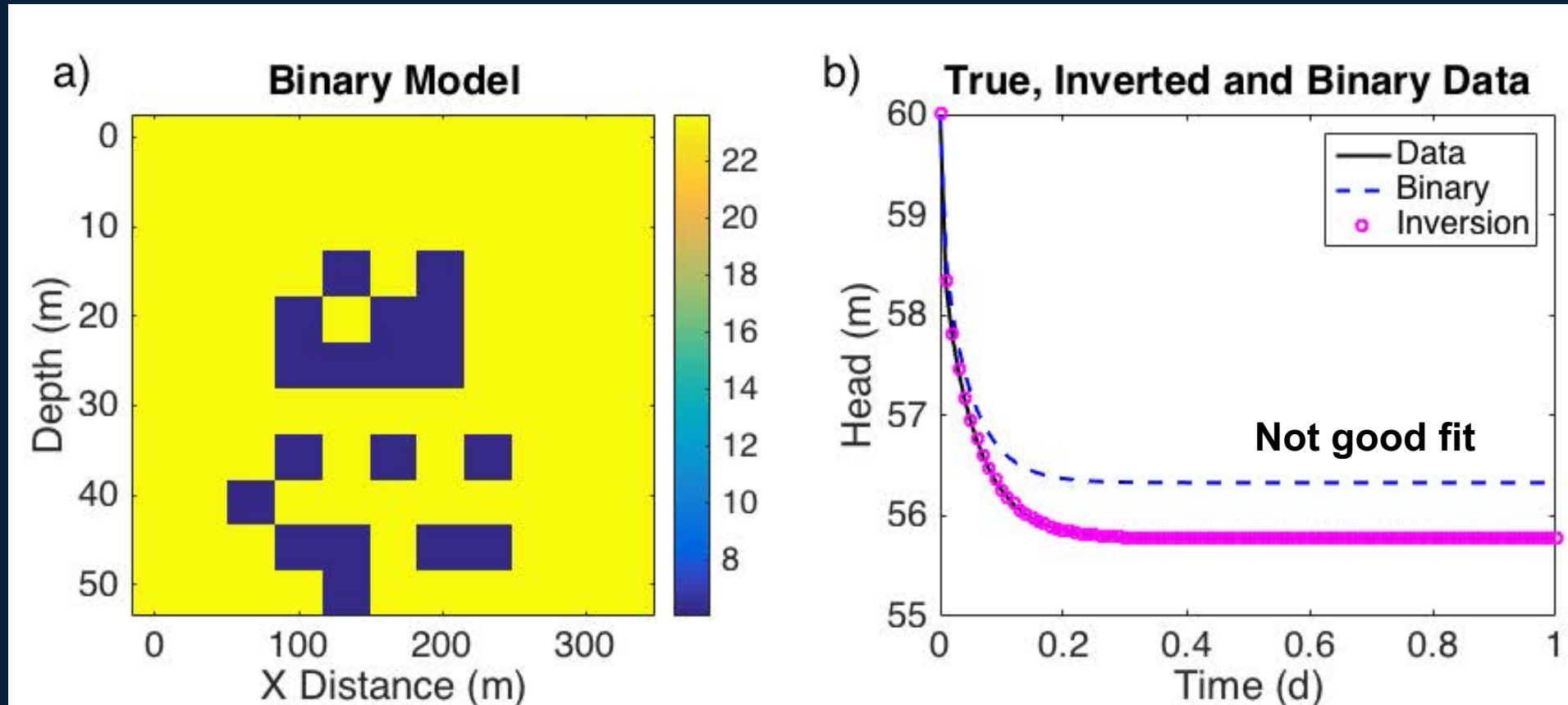
# Case 3: (Binary Model)



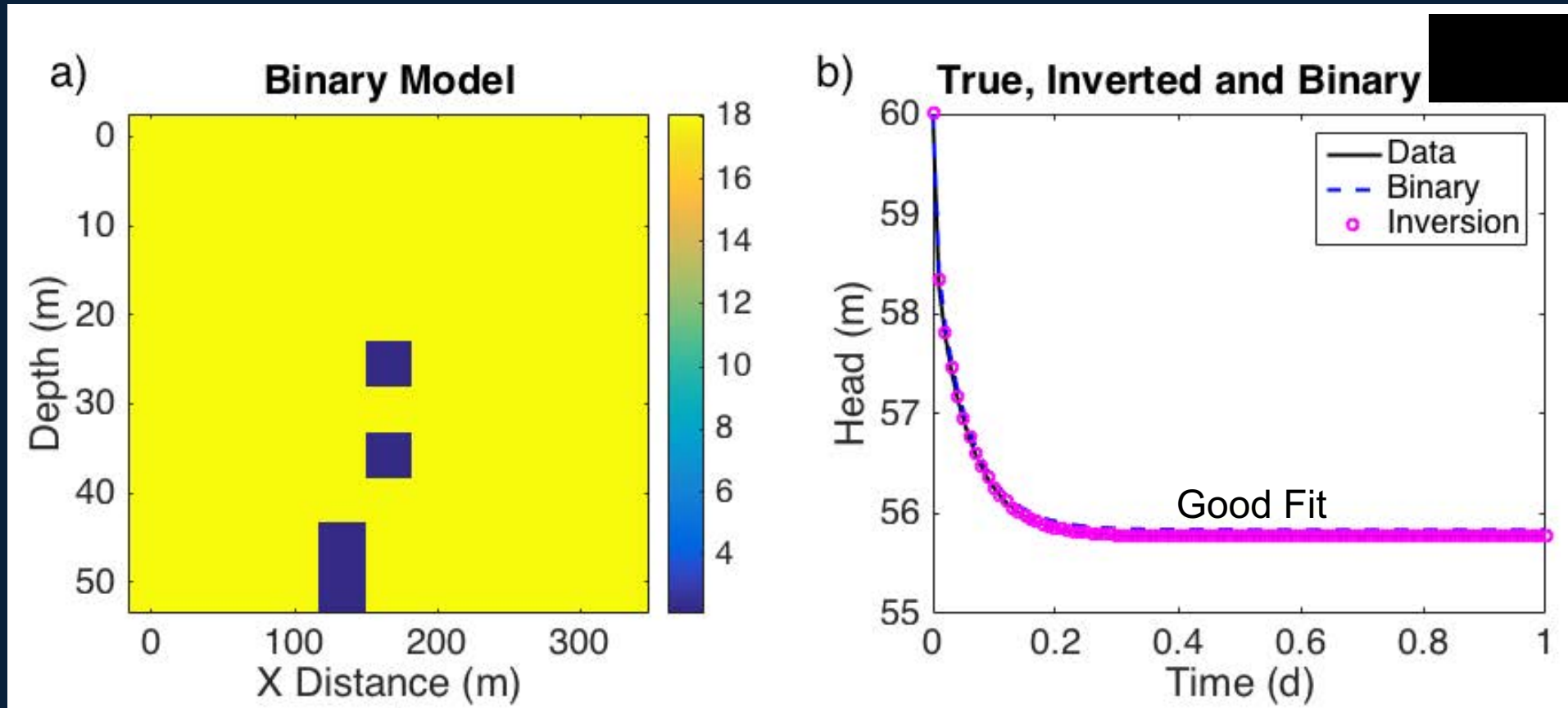
# Optimum Binary Model



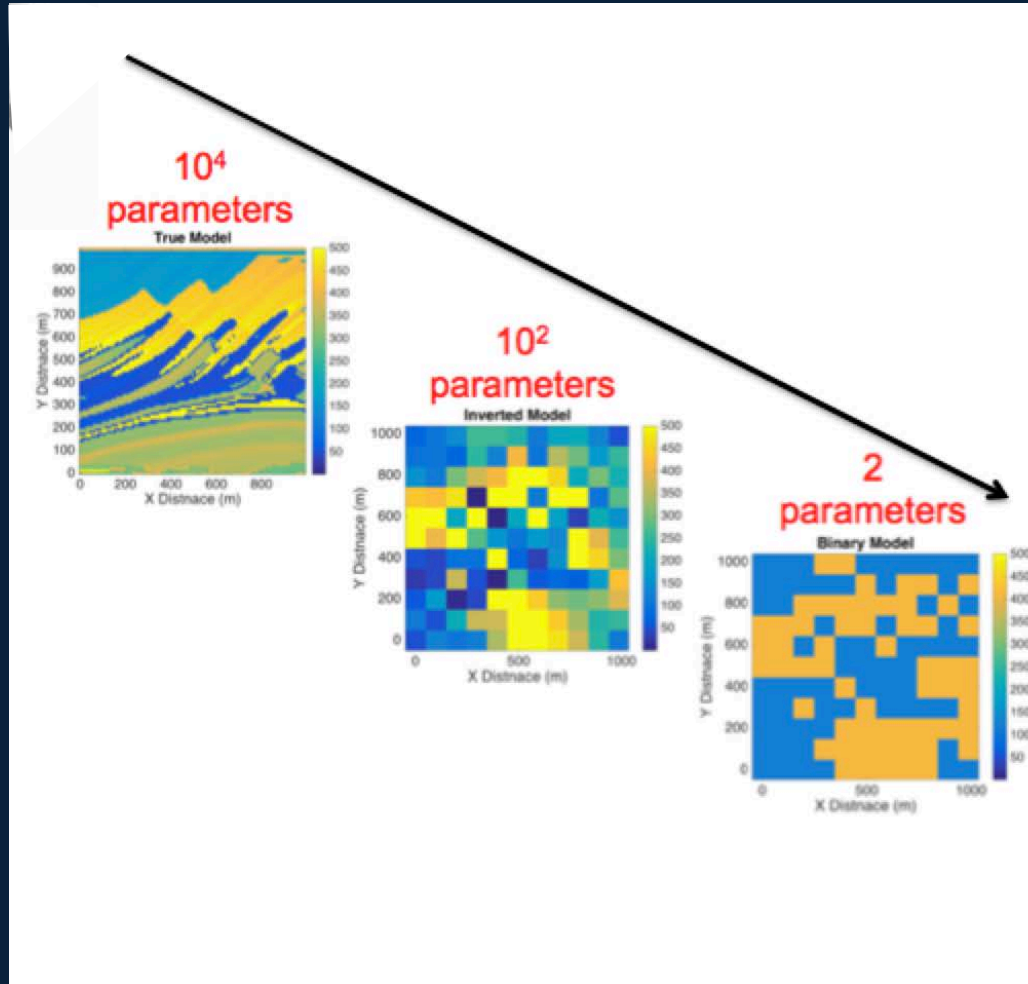
# Optimum Binary Model (Arithmetic Averaging)



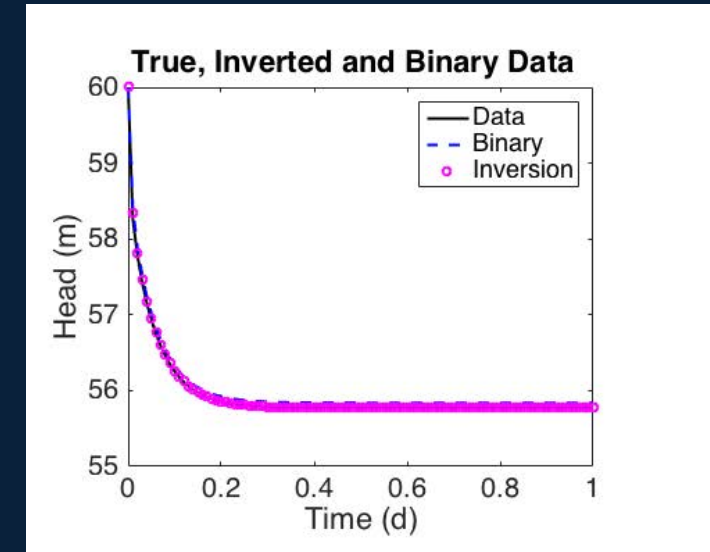
# Optimum Binary Model (Geometric Averaging)



# Summary

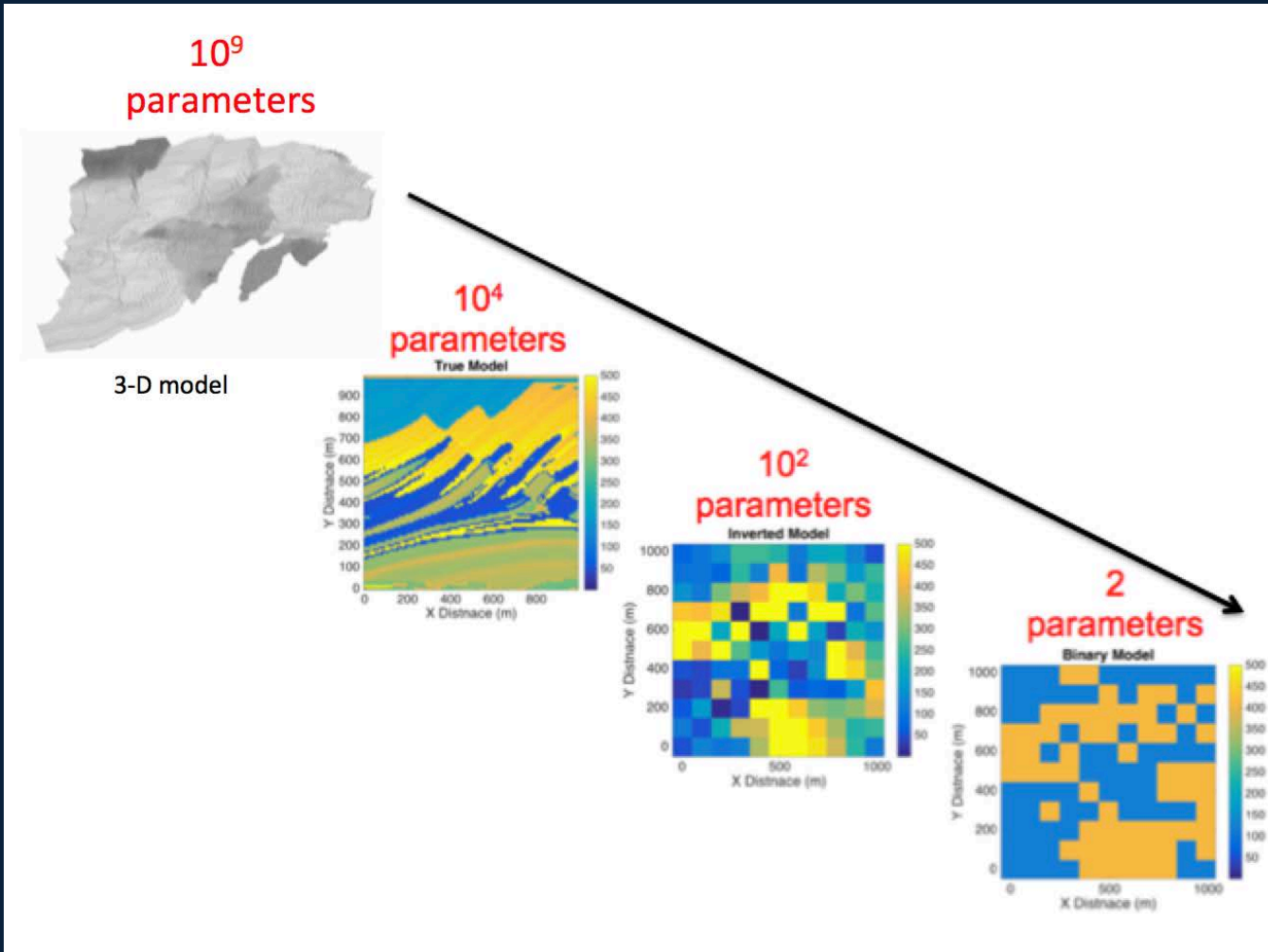


“Perfect Fit”

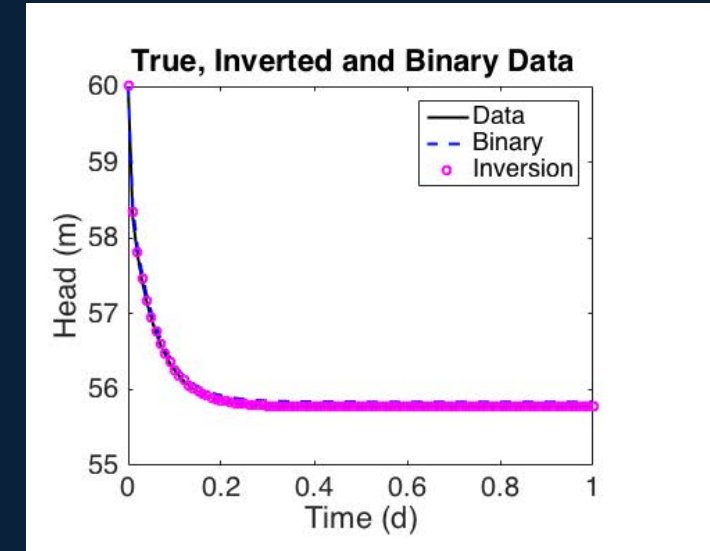




# Summary



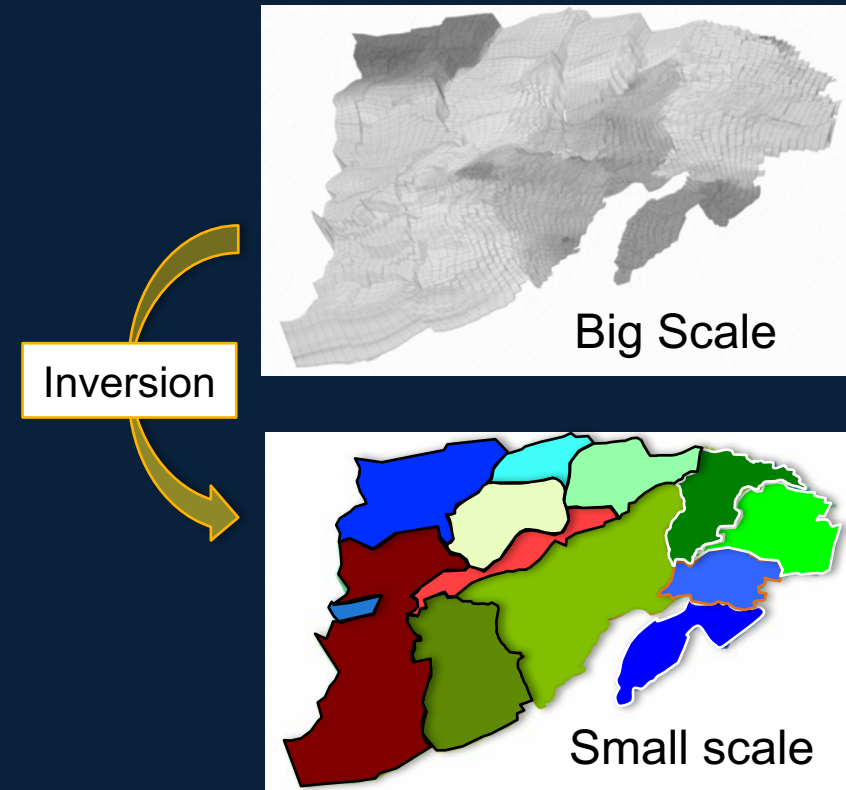
“Perfect Fit”





# Conclusions

- A reduced model could fit the production data.
- The model can be reduced to binary model and also fits the production data.
- We had a better success using the geometric mean in creating the optimum binary model.



# Thanks