Fast solvers for seismic imaging

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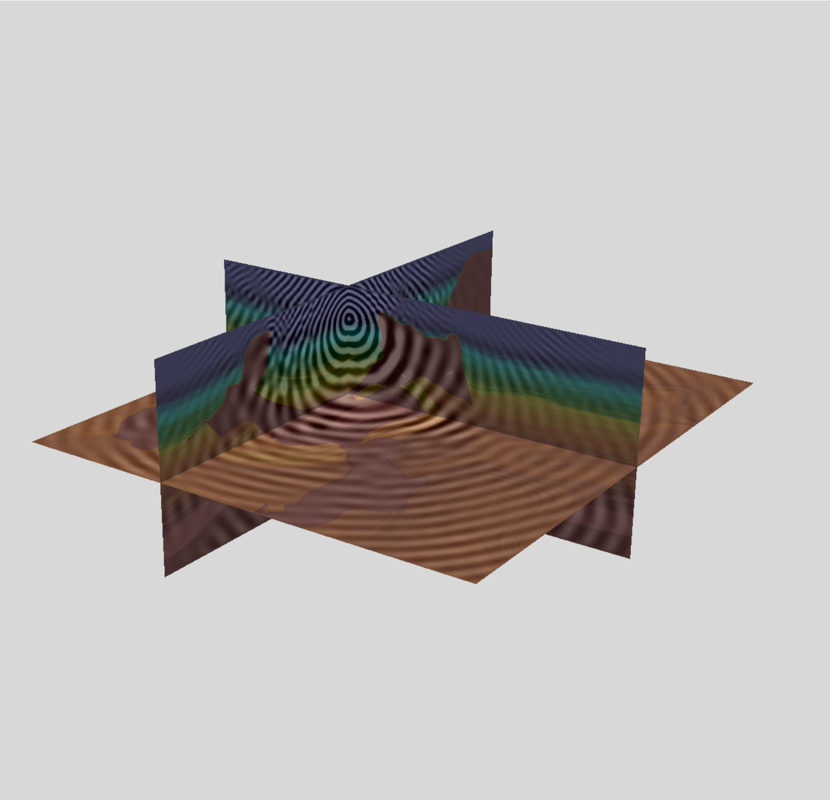
Motivation



- Inhomogeneous media
- Point sources
- High frequency

Model problem

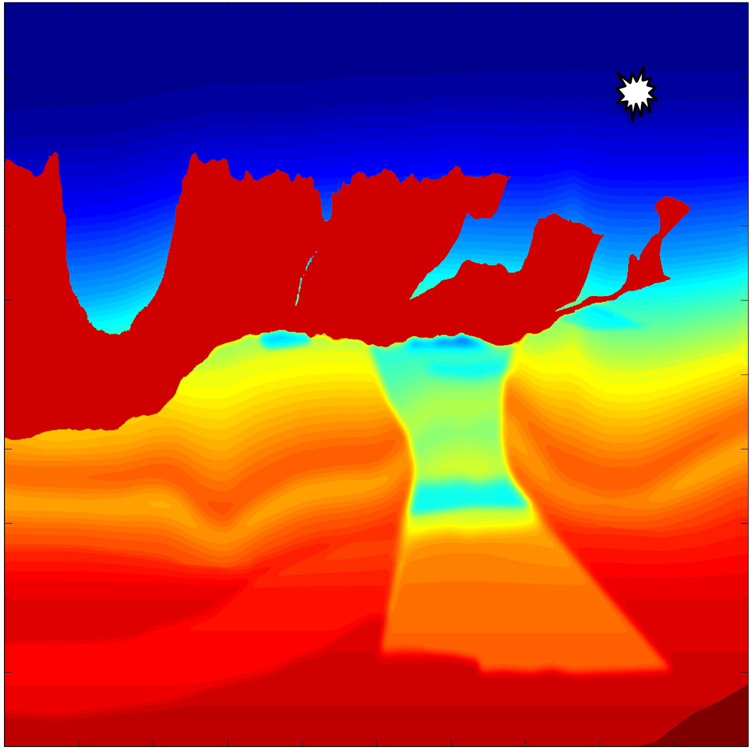
$$-\Delta u - \omega^2 m u = f$$

$$+\text{A.B.C.}$$

ω ... Frequency

m ... Squared Slowness

f ... Source term



Discretization – existing techniques

Finite Difference Methods:

- Discontinuous wave speed

$$\|u - u_h\|_{L^2} \leq C(\omega)(\omega h)$$

- Point sources

$$\|u - u_h\|_{L^2} = ?$$

- Pollution error

$$C(\omega) \to \infty \quad \text{as} \quad \omega \to \infty$$

Finite Element Methods (FEM):

- Discontinuous wave speed

$$\|u - u_h\|_{L^2} \leq C(\omega)(\omega h)^2$$

- Point sources

$$\|u - u_h\|_{L^2} \to 0$$

- Pollution error

$$C(\omega) \to \infty \quad \text{as} \quad \omega \to \infty$$

Discretization – new technique

Hybridizable Discontinuous Galerkin Method (HDG)

Standard FEM:

- Discontinuous wave speed

$$||u - u_h||_{L^2} \leq C(\omega)(\omega h)^2$$

- Complicated meshes
- Point sources

$$||u - u_h||_{L^2} \rightarrow 0$$

- Pollution error

$$C(\omega) \rightarrow \infty \quad \text{as} \quad \omega \rightarrow \infty$$

HDG:

- Discontinuous wave speed

$$||u - u_h||_{L^2} \leq C(\omega)(\omega h)^2$$

- Simple meshes
- Point sources

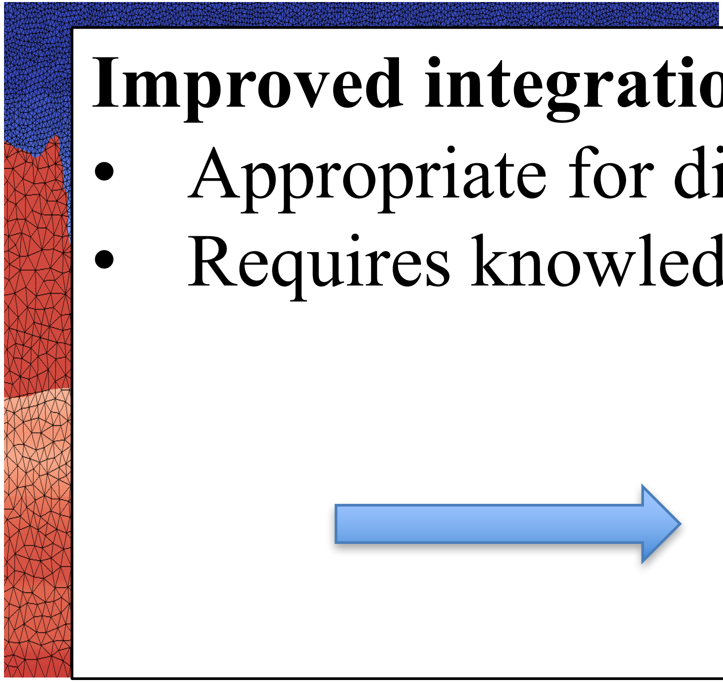
$$||u - u_h||_{L^2} \leq C(\omega)(\omega h)^2$$

- Pollution error

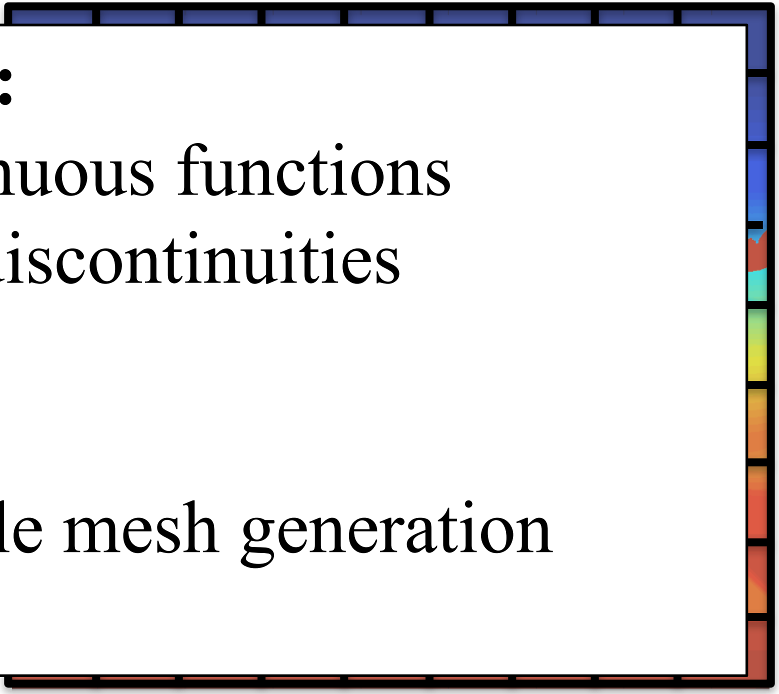
$$C(\omega) \leq C \quad \text{as} \quad \omega \rightarrow \infty$$

Discretization – Simple Meshes

Standard FEM:



HDG:



Improved integration rule:

- Appropriate for discontinuous functions
- Requires knowledge of discontinuities

Simple mesh generation

Discretization – Point sources

Singularity cancellation technique:

$$u(x) = \tilde{u}(x) - \frac{1}{2\pi}\log|x - x_S|$$

x_S ... location of singularity

$$\Longrightarrow \quad \|\tilde{u} - \tilde{u}_h\|_{L^2} \leq C(\omega)(\omega h)^2$$

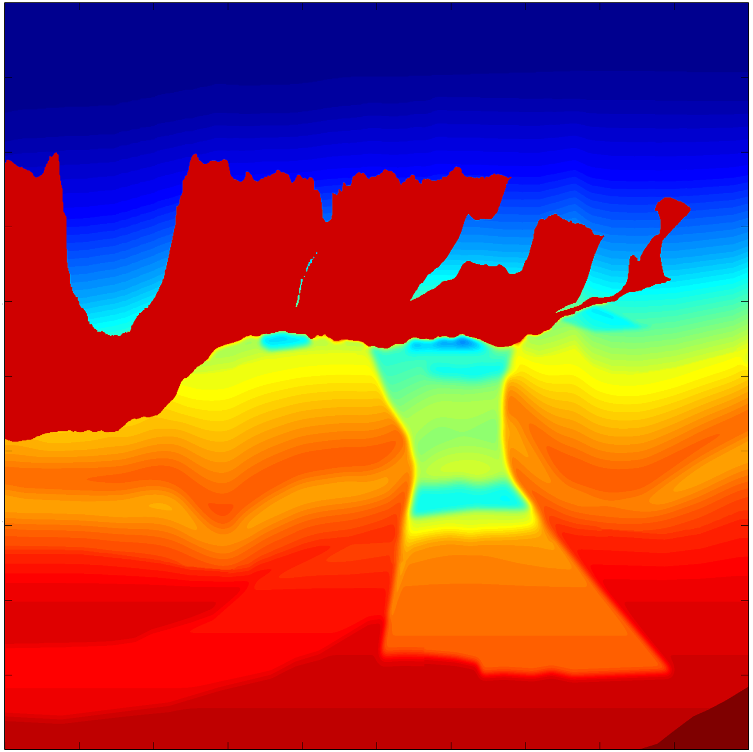
Discretization – Pollution error

Increase order of approximation:

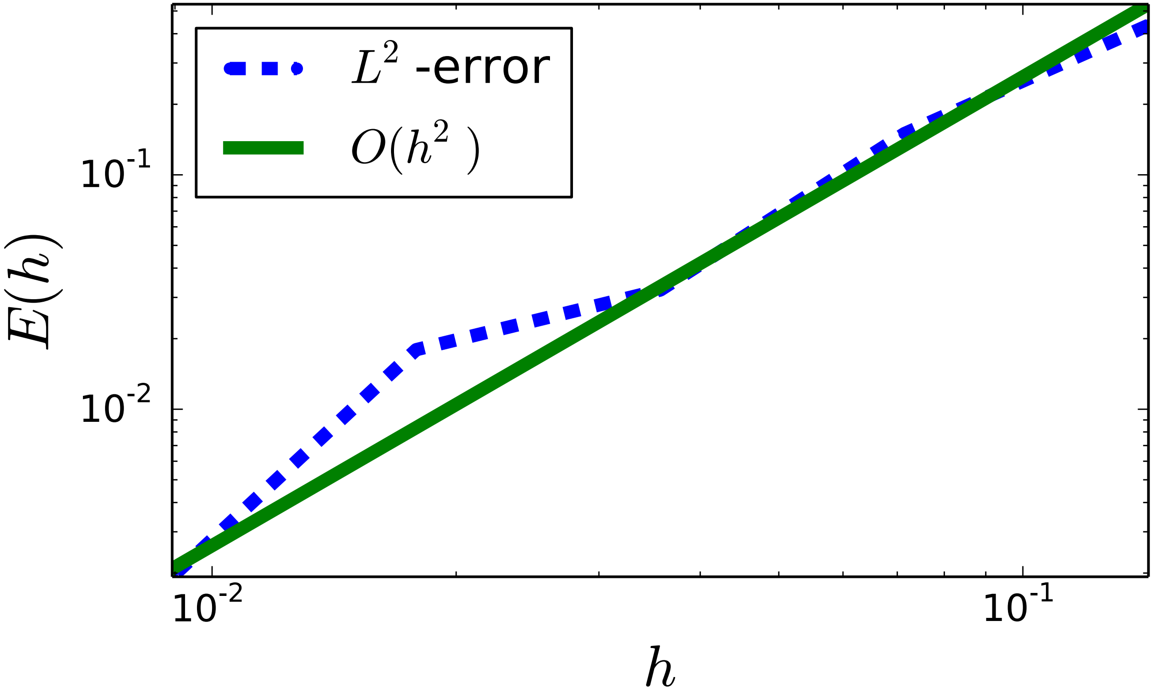
$$p = \log(\omega)$$

$$\Longrightarrow \quad C(\omega) \leq C \quad \text{as} \quad \omega \to \infty$$

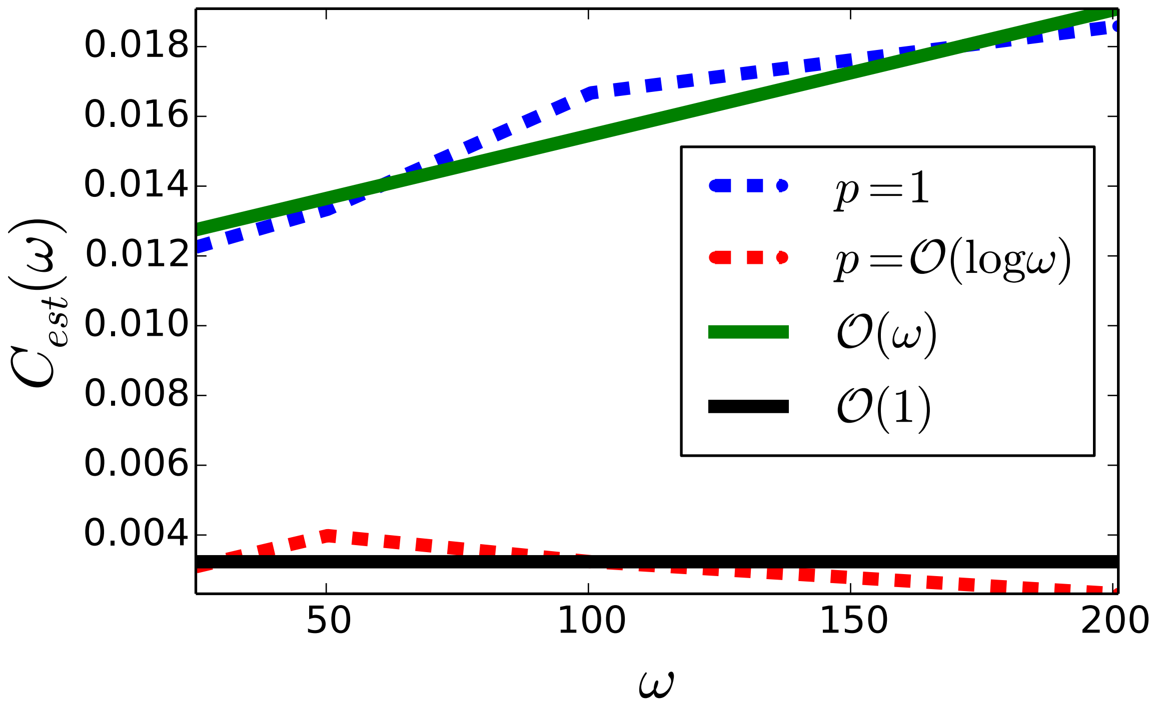
Numerical Example: BP 2004 model



Numerical Example: 2nd order accuracy



Numerical Example: Pollution error



Fast algorithms: Existing techniques

- Iterative methods: $O(\omega N)$
- Direct methods:

	1D	2D	3D
operations	$O(N)$	$O(N^{\frac{3}{2}})$	$O(N^2)$
memory	$O(N)$	$O(N \log N)$	$O(N^{\frac{4}{3}})$

- Preconditioned iterative methods: $O(N)$
  - Low-order discretizations
  - Smooth wave speeds

$O(N \log^\alpha N)$ algorithm: Method of polarized traces

Solution: Method of Polarized Traces
(PhD thesis by Leonardo Zepeda-Nunez)

- Highly inhomogeneous (discontinuous) wave speeds $O(p^\alpha N)$
- If $p = \log(\omega)$ and $\omega = O\left(\frac{1}{h}\right)$: $N = \left(\frac{p}{h}\right)^d = \omega^d \log^d \omega$

$$\longrightarrow \quad O(\omega^d \log^{\alpha+d} \omega)$$

Fast and accurate algorithm for wave propagation problems in geophysical applications

- **2nd order accuracy** and **no pollution** error in the presence of
  - Discontinuous wave speeds
  - Point sources

- $O(\omega^d \log^{\alpha+d} \omega)$ **complexity** using the method of polarized traces