On the Relationship between the Compressional Wave Velocity and Density of Saturated Porous Rocks: Theory and Application

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ERL Consortium Meeting

The Problem and Objective

The Problem

From theory, the wave velocity is defined by The Newton-Laplace Equation:

$$V = \sqrt{\frac{M}{\rho}}$$

$V \uparrow \longrightarrow \rho \downarrow$

V: Velocity through the rock
M: Elastic modulus   ρ: density

(Bourbie, et al 1987). *Acoustics of Porous Media.*

extent of fracture porosity by the increase in velocity with pressure. The ordinate in the diagram represents the normalized difference between the compressional velocity without pressure and that under a pressure of 1 GPa for dry samples: the greater the number of cracks, the more the velocity varies with pressure.

As already noted, the Pierre shale samples used in Fig. 5.2 display a different velocity function than Fig. 5.1. The velocity plateau as a function of pressure, which represents the closure of its "last" crack for the sample concerned, is not observed for this rock at 120 MPa. In fact, Jones and Wang (1981) observed this phenomenon continuing up to 0.4 GPa. It is possible that the increase in velocity as a function of pressure shows, in addition to the continuous closure of the microcracks, an alignment of the clay crystals in the minimum shear strength plane (Tosaya, 1982). Chalk, the second example in Fig. 5.2, is very complicated because the structure of the material varies according to the pressure applied (creep). We shall not go into further detail for either Pierre shale or chalk, assuming that the variation in velocity versus pressure are only important if the structure of the material investigated is not changed irreversibly by the experiment. For clays as well as chalk, there is good reason to suspect hysteresis in the curve of velocity vs. effective pressure.

We have shown that the increase in velocity with pressure results from the closure of the cracks, and that this closure is reflected by a greater rigidity of the material under pressure (i.e. an increase in the corresponding elastic modulus). In fact, it must be remembered that any velocity V can be expressed in the form:

$$V = \sqrt{\frac{M}{\rho}} \tag{5.1}$$

where M is the elastic modulus, and ρ the density, and that, consequently, at constant ρ, an increase in elastic modulus implies a rise in velocity. The behavior of cracks and pores under confining pressure was modeled by Walsh (1965, 1969) and Wu (1966) for a pore or crack included in a matrix. The equations satisfied by K, the bulk modulus, and μ, the shear modulus of the rock for the dry sample are the following:

$$\frac{1}{K} = \frac{1}{K_1}\left(1 + A\frac{\phi}{e}\right) \tag{5.2}$$

$$\frac{1}{\mu} = \frac{1}{\mu_1}\left(1 + B\frac{\phi}{e}\right) \tag{5.3}$$

where K_1 and μ_1 are the solid moduli, e the aspect ratio of the pore or crack ($e = 1$ for a sphere, $e \ll 1$ for a crack), ϕ the porosity, and A and B are constants depending on the characteristics of the medium and close to 1.

For the saturated sample:

$$\frac{1}{K} \simeq \frac{1}{K_1}(1 + A'\phi) \tag{5.4}$$

$$\frac{1}{\mu} \simeq \frac{1}{\mu_1}\left(1 + B'\frac{\phi}{e}\right) \tag{5.5}$$

where A' and B' are constants depending on K_1 and μ_1 (and fluid bulk modulus for A') and close to 1. It can be seen that, if $e = 1$, the effect of the pore on the moduli is negligible

Objective

- Investigate the apparent inconsistency between theory and reality of the relationship between velocity and density.
- Define a relationship between velocity and density, and between elastic modulus and density.

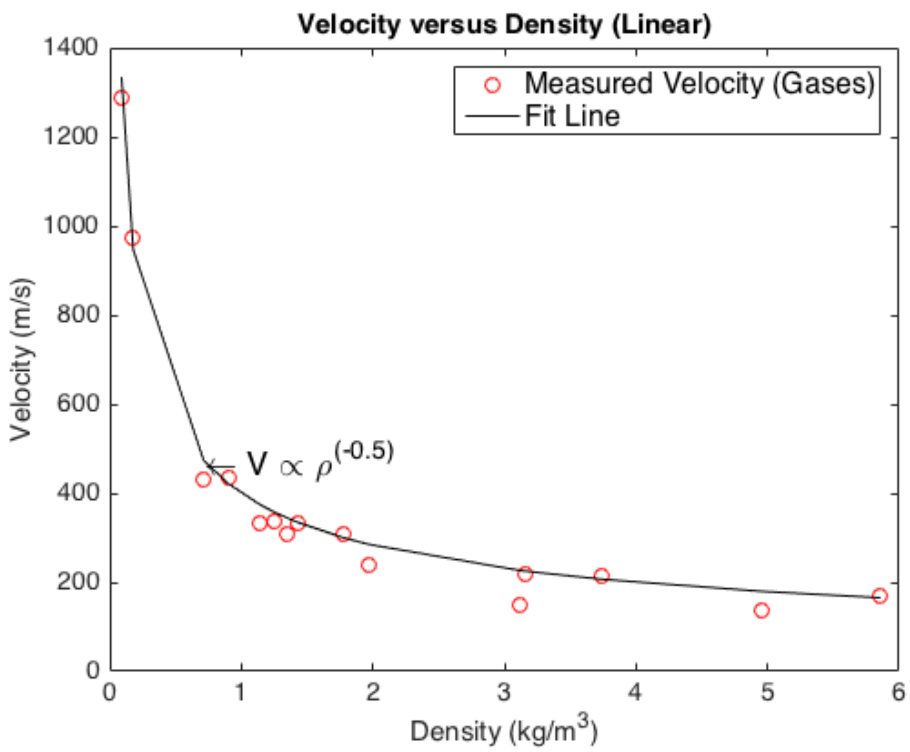
$$V = f(M, \rho)$$

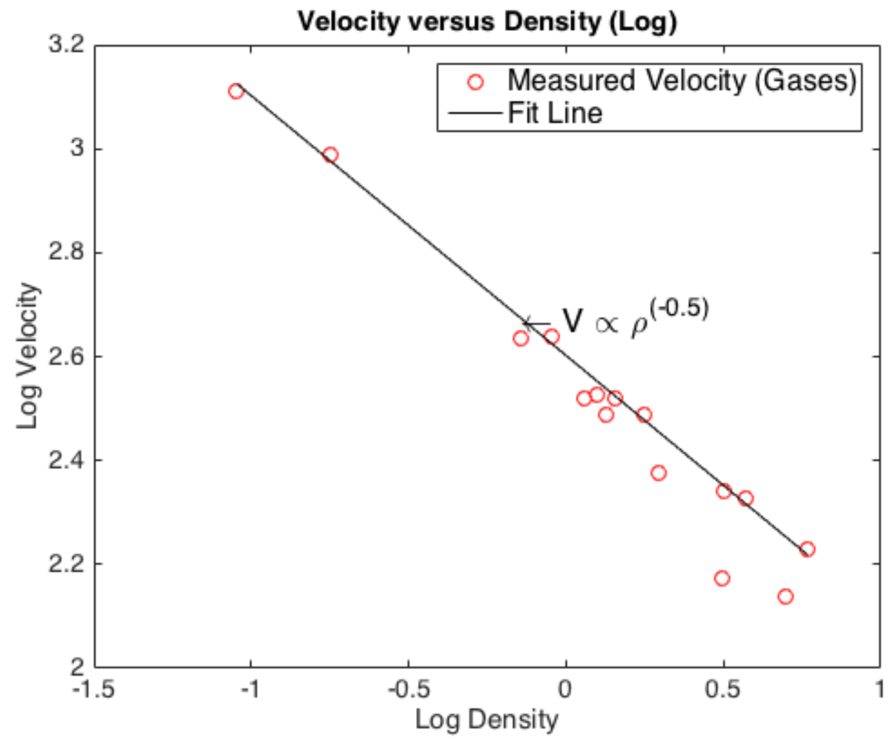
$$M = g(\rho)$$

V: Velocity through the rock M: Elastic modulus ρ: Density

Observations

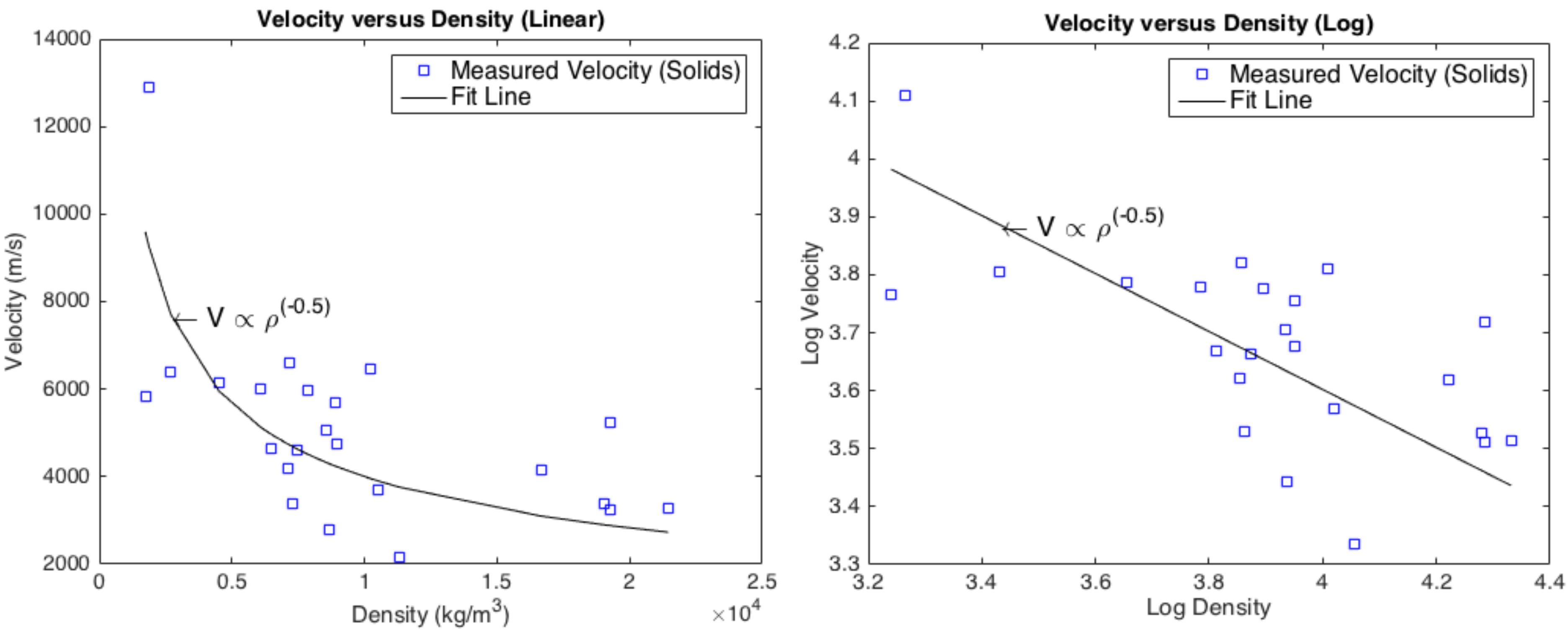
Velocity vs. Density for Gases





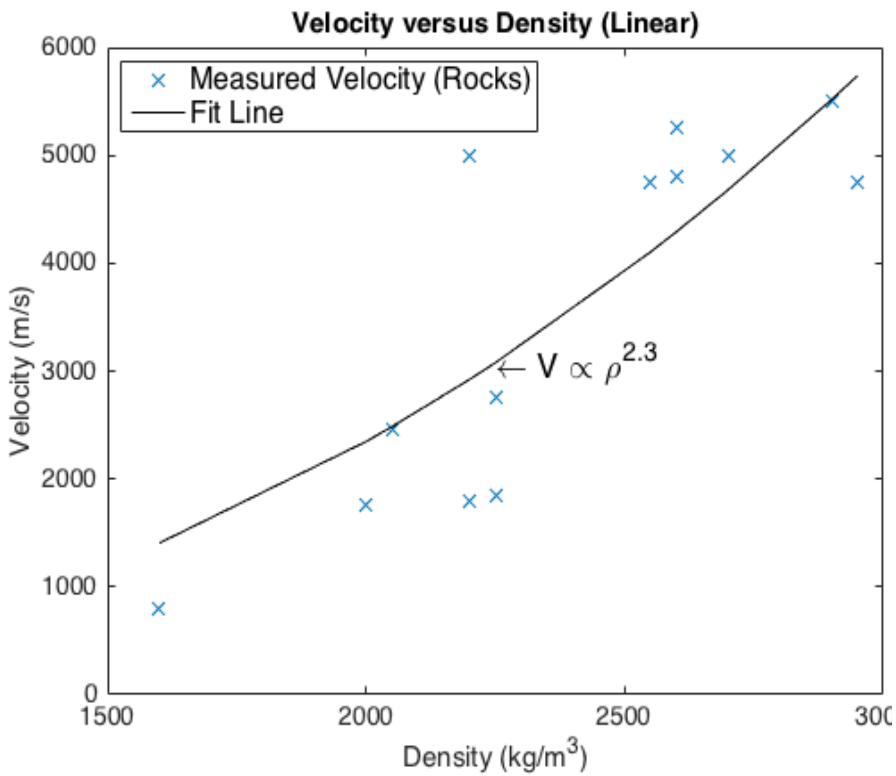
Data From: Podesta, (2002). *Understanding the Properties of Matter.*

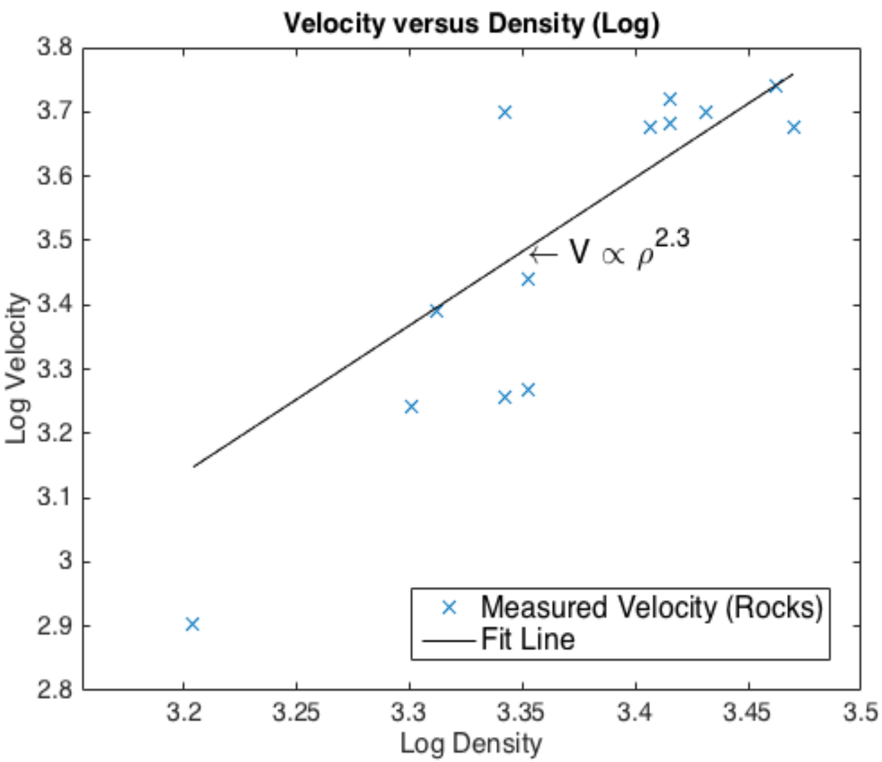
Velocity vs. Density for (non-porous) Solids



Data From: Podesta, (2002). *Understanding the Properties of Matter.*

Velocity vs. Density for Rocks





Data From: Mavko, (2016). *Introduction to Rock Physics.*

Gardner's Plot

- Experimental data for rocks
  - Gardner's equation

$$V = 108.28\rho^4$$

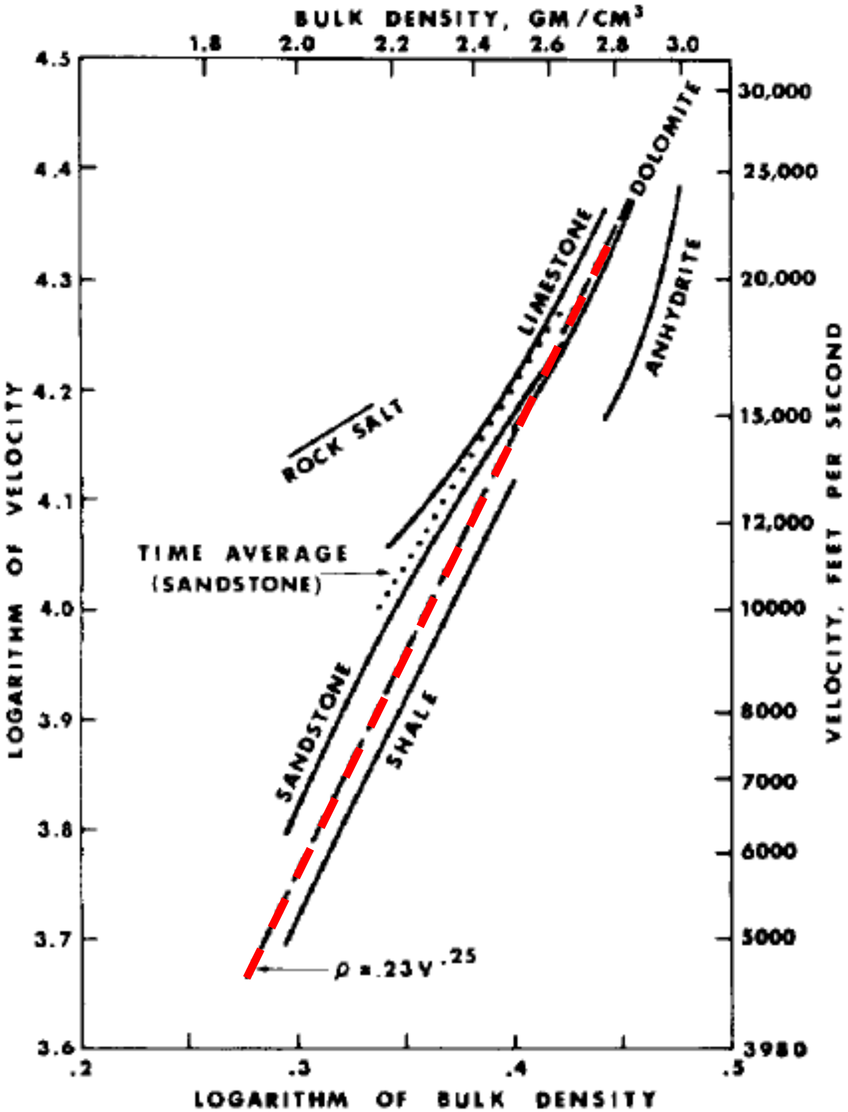


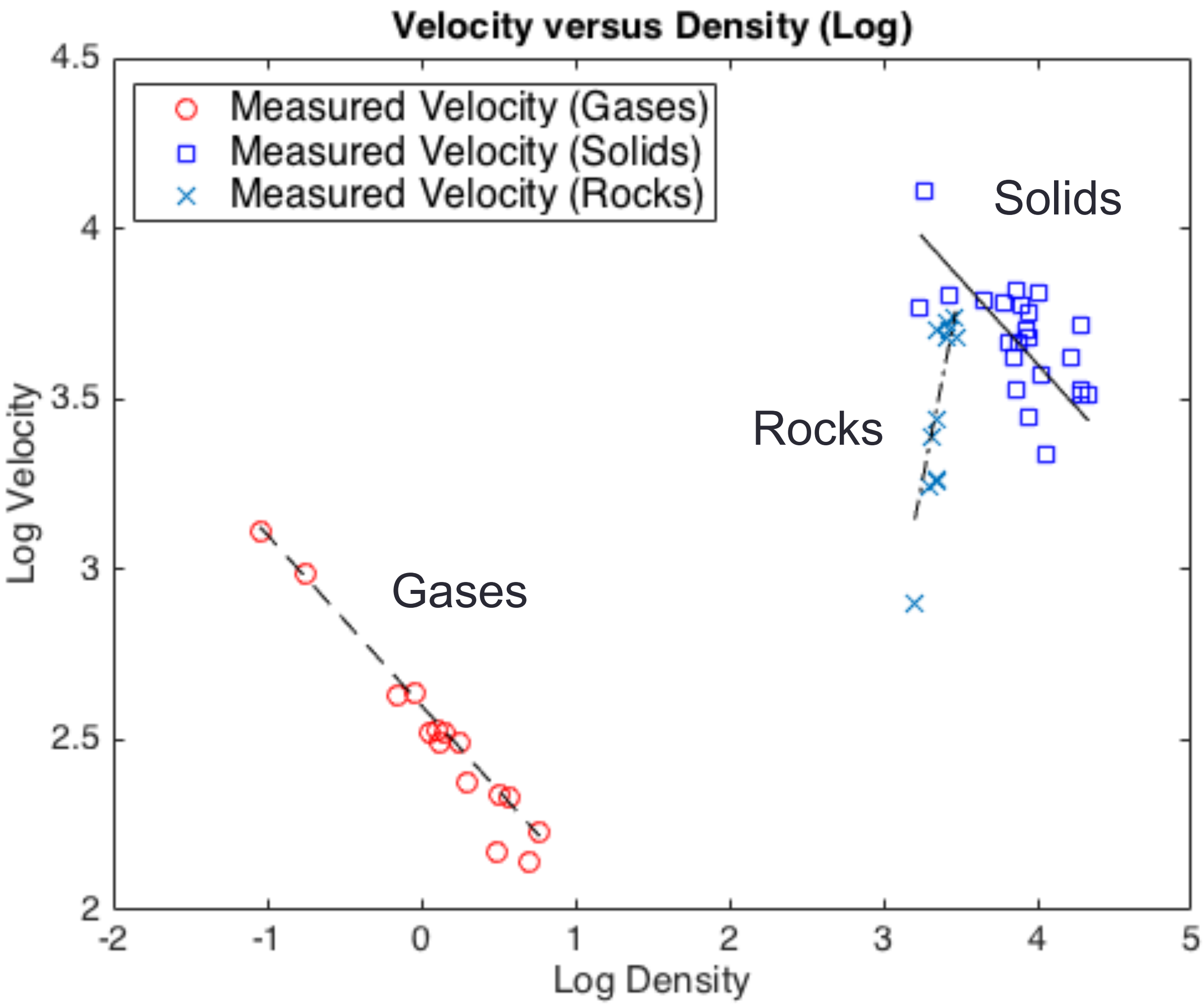
FIG. 1. Velocity-density relationships in rocks of different lithology.

V: Velocity through the rock
ρ: Density

(Gardner et al, 1974)

Conclusion

Single Phase



2-Phase

Methodology

Wylie's Time Average Equation

- Wyllie's Time Average equation:

$$\frac{1}{V} = \frac{\phi}{V_f} + \frac{(1-\phi)}{V_m}$$

- Weighted average density:

$$\rho = \phi\rho_f + (1-\phi)\rho_m$$

$$\phi = \frac{\rho_m - \rho}{\rho_m - \rho_f}$$

V_f: Fluid velocity
ϕ: Porosity
ρ_m: Mineral density
V_m: Mineral velocity
ρ: Rock bulk density
ρ_f: Fluid Density

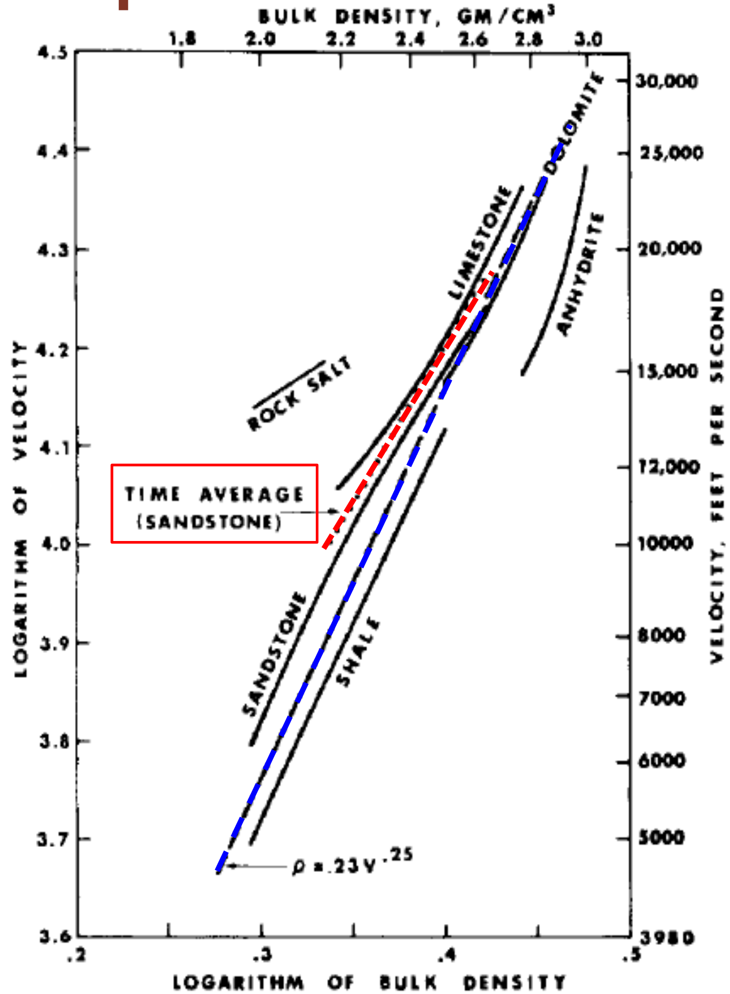


FIG. 1. Velocity-density relationships in rocks of different lithology.

(Gardner et al, 1974)

Derivation of New Expression of the Elastic Modulus, M

The Newton-Laplace equation

$$V = \sqrt{\frac{M}{\rho}}$$

Wyllie's equation

$$\frac{1}{V} = \frac{\phi}{V_f} + \frac{(1-\phi)}{V_m}$$

$$M = \frac{(\rho_m - \rho_f)^2 V_f^2 V_m^2 \rho}{[(V_f - V_m)\rho + \rho_m V_m - \rho_f V_f]^2}$$

V: Velocity through rock

V_m: Velocity through mineral

V_f: Velocity through fluid

ρ: Rock bulk density

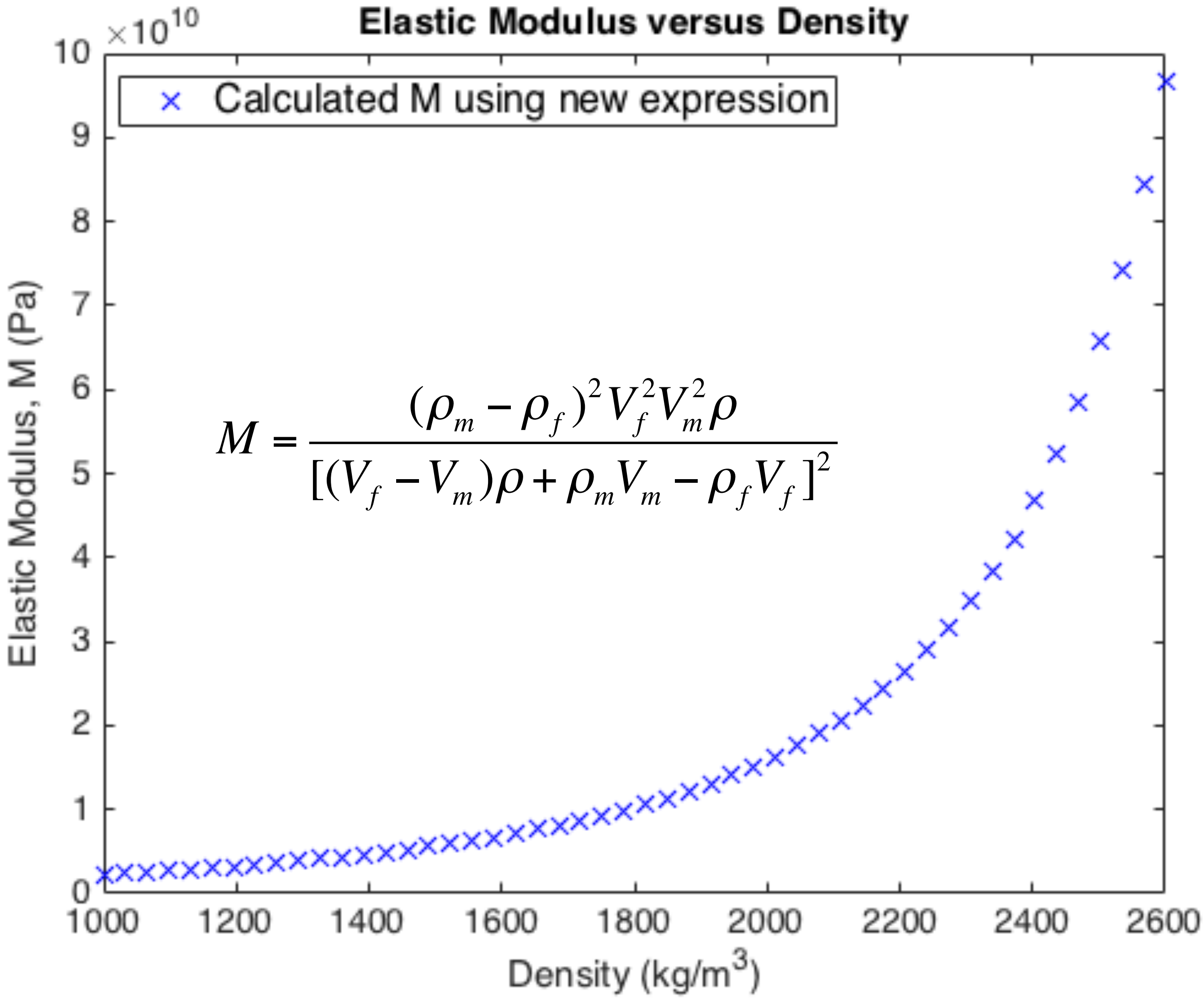
ρ_m: Mineral density

ρ_f: Fluid Density

M: Rock Elastic parameter

Φ: Porosity

Elastic Modulus vs. Density



Numerical Approximation of the Elastic Modulus

The Newton-Laplace equation

$$V = \sqrt{\frac{M}{\rho}}$$

Numerical Approx. Of Wyllie's equation

$$V = a\rho^b + c$$

$$\sqrt{\frac{M}{\rho}} = a\rho^b + c$$

$$M(\rho) \approx (a\rho^b + c)^2 \rho$$

Using b = 4:

$$M(\rho) \approx A\rho^9 + B\rho^5 + C\rho$$

V: P-wave velocity M: Elastic modulus ρ: Density a, b, c, A, B, and C: Constants

Generalized Gardner's Equation

- Gardner's equation:

$$V = a\rho^4$$

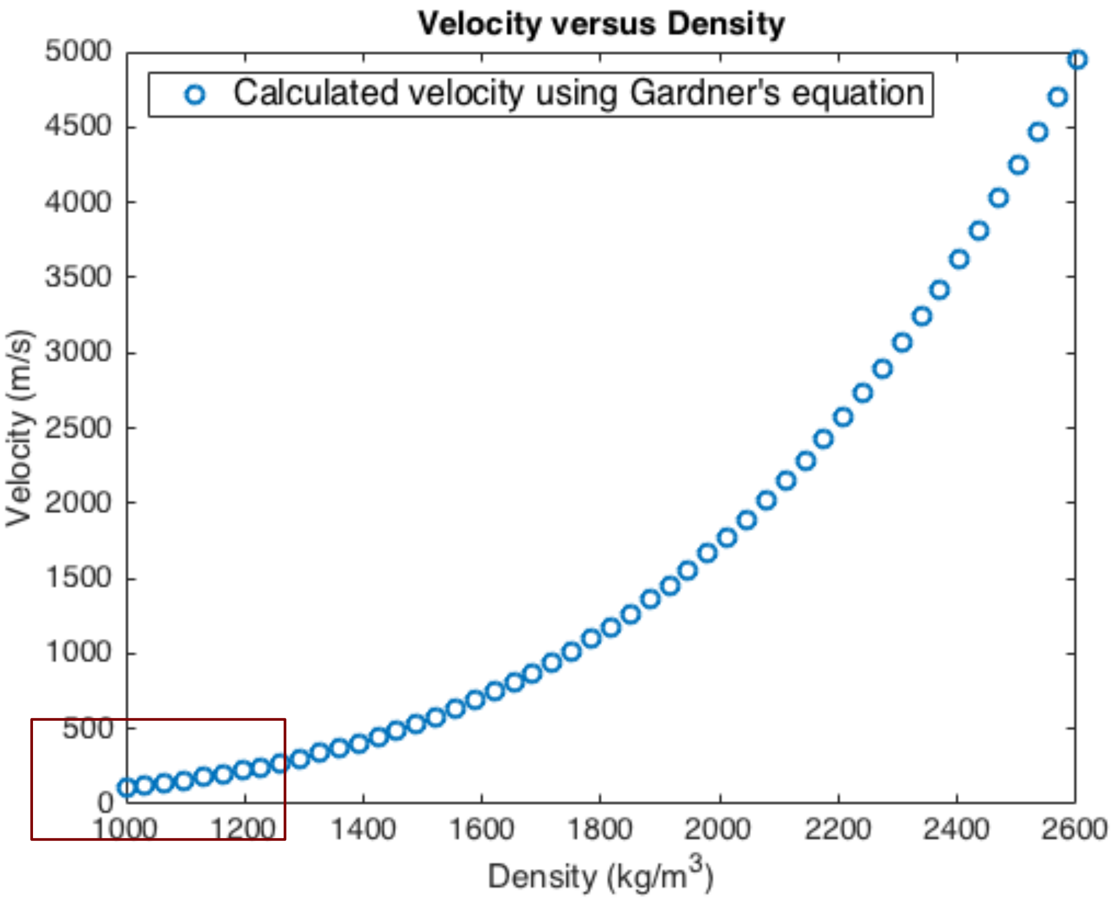
- Wyllie's approx. relationship:

$$V = a\rho^4 + c$$

- Proposed generalized form of Gardner's equation:

$$V = a\rho^b + c$$

- Where:
  - c = 1500 – a
  - b = 4



Applying Findings to Field Data

Field Data

- Well-log data that includes:
  - P-wave Velocity
  - Density
  - Porosity
  - Mineralogy

- Source:
  - 3 wells (Carbonate rocks): from Saudi Aramco
  - 1 well (Clastic rocks): from the *Quantitative Seismic Interpretation* book *(Avseth et al., 2005)*

Applying the New Expression of the Elastic Modulus, M

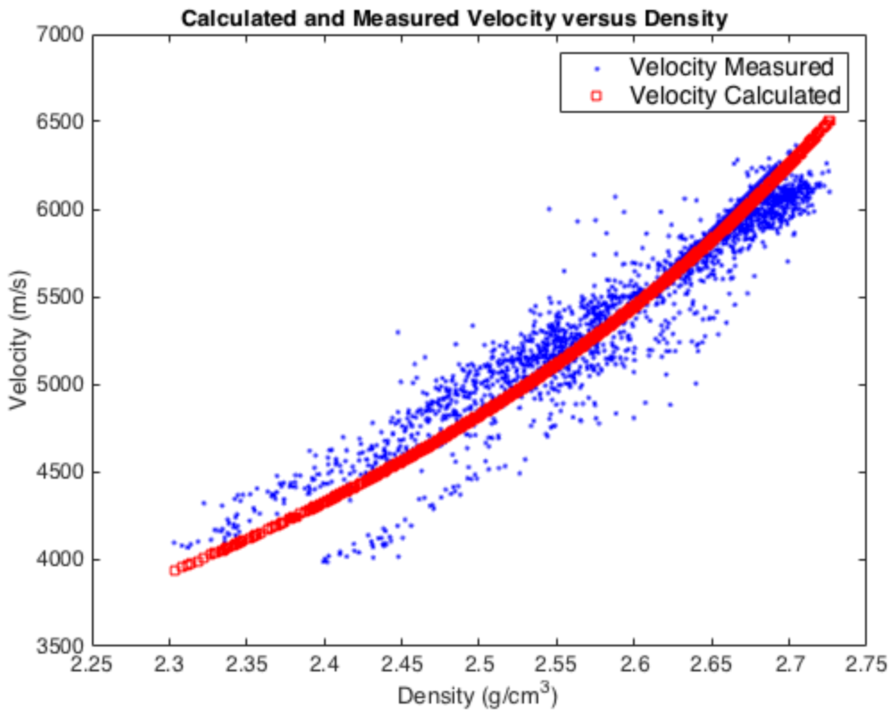
Applying the New Expression of the Elastic Modulus, M, to Field Data

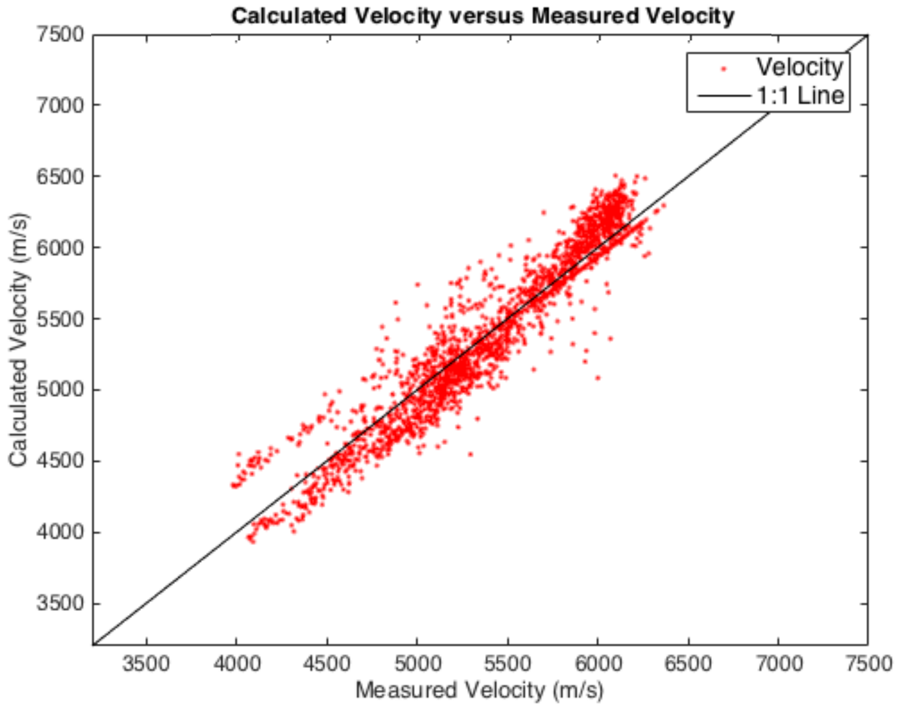
$$M = \frac{(\rho_m - \rho_f)^2 V_f^2 V_m^2 \rho}{[(V_f - V_m)\rho + \rho_m V_m - \rho_f V_f]^2}$$

Medium	Parameter	Value
Calcite	V_m	6000 m/s
	ρ_m	2.70 g/cm^3
Dolomite	V_m	7000 m/s
	ρ_m	2.70 g/cm^3
Quartz	V_m	5000 m/s
	ρ_m	2.65 g/cm^3
Water	V_f	1500 m/s
	ρ_f	1.00 g/cm^3

Data from: Gue'guen and Placiauskas, (1974). *Introduction to the Physics of Rocks.*

Calculated Velocity using the New Expression of M and Field Data – Limestone





rmse = 0.03

Conclusions

Clarification of the Relationship between Velocity and Density

extent of fracture porosity by the increase in velocity with pressure. The ordinate in the diagram represents the normalized difference between the compressional velocity without pressure and that under a pressure of 1 GPa for dry samples: the greater the number of cracks, the more the velocity varies with pressure.

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We have shown that the increase in velocity with pressure results from the closure of the cracks, and that this closure is reflected by a greater rigidity of the material under pressure (i.e. an increase in the corresponding elastic modulus). In fact, it must be remembered that any velocity V can be expressed in the form:

$$V = \sqrt{\frac{M}{\rho}} \qquad (5.1)$$

However $V \propto \rho^4$

and

$$M = \frac{(\rho_m - \rho_f)^2 \; V_f^2 \, V_m^2 \, \rho}{(\,(V_f - V_m)\,\rho + (\rho_m V_m - \rho_f V_f)\,)^2}$$

and $M \propto \rho^9$

(Bourbie, et al 1987). *Acoustics of Porous Media.*

Conclusions

- A new expression of the elastic modulus, M, is derived by linking the Newton-Laplace equation to Wyllie's equation:

$$M = \frac{(\rho_m - \rho_f)^2 \; V_f^2 \, V_m^2 \, \rho}{\left(\left(V_f - V_m \right) \rho + \left(\rho_m V_m - \rho_f V_f \right) \right)^2}$$

- The new expression of the elastic modulus, M, numericaly approximated by:

$$M(\rho) \approx (a\rho^4 + c)^2 \rho \qquad \text{approximately} \qquad M \propto \rho^9$$

- The generalized form of Gardner's equation provides more accurate velocities over the entire range of densities.

- The velocity calculated using the new expression of the elastic modulus, M, is indistiguishable when compared to the velocity calculated using the numerical approximation of velocity

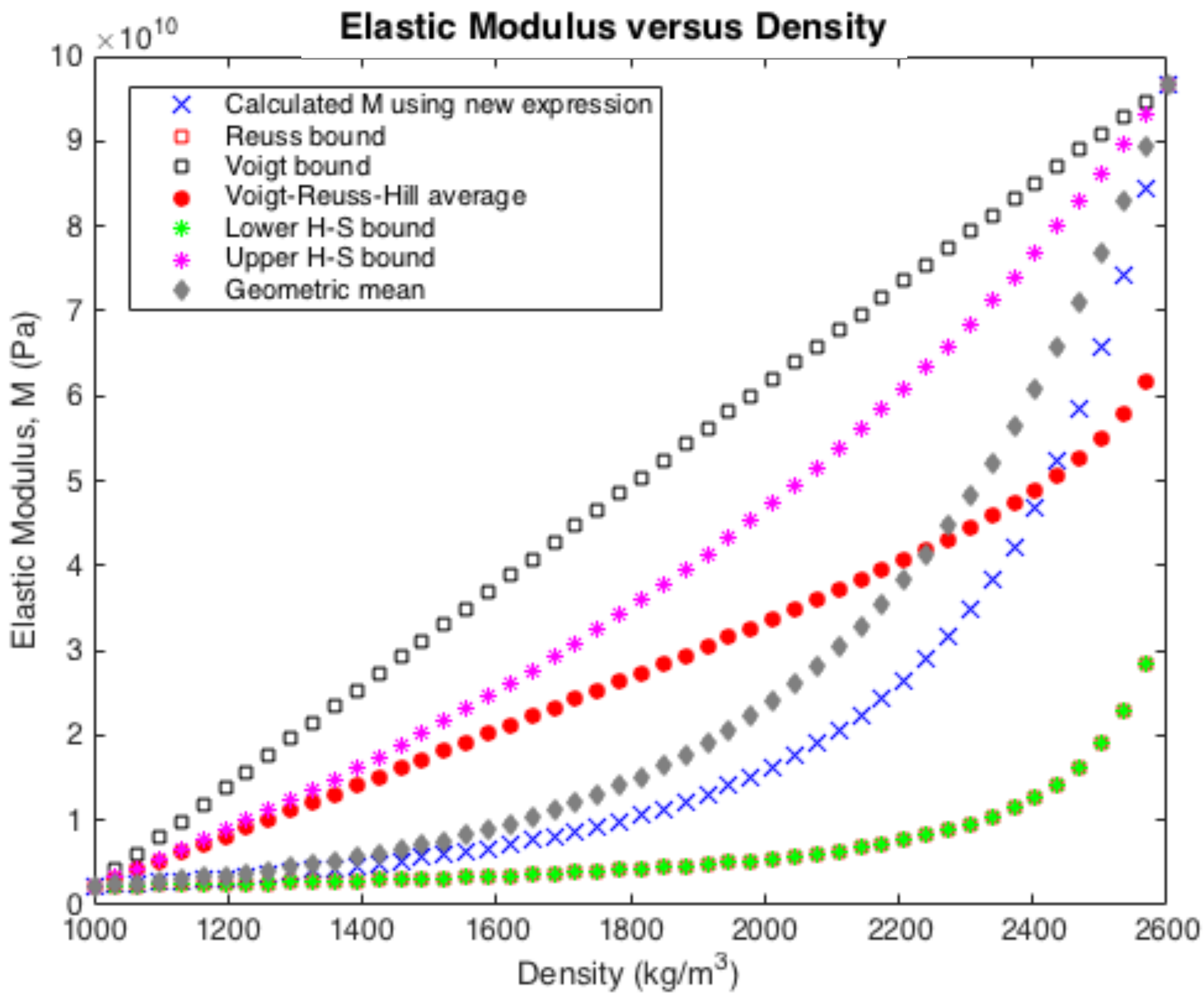
THANK YOU

Questions?!

Comparison

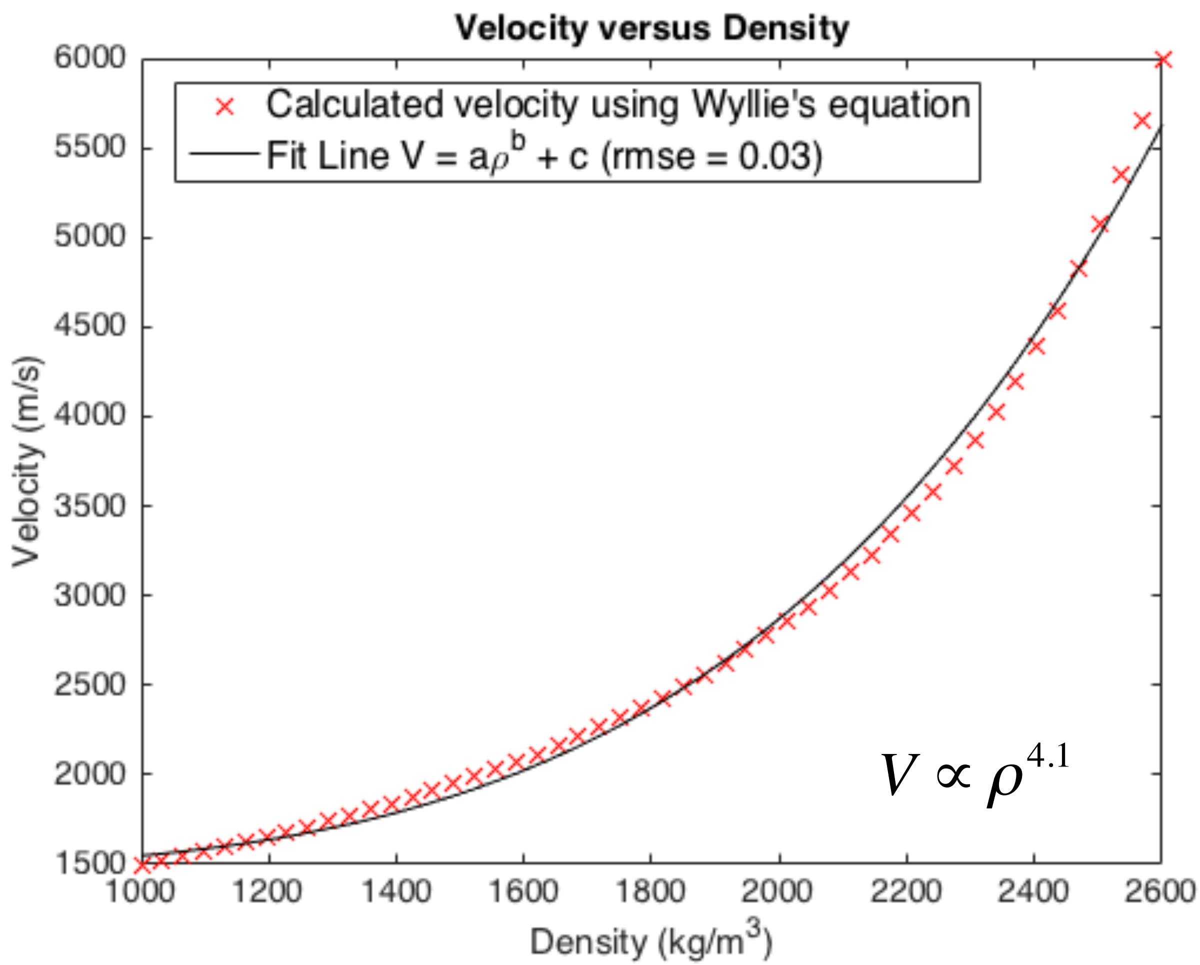
- Comparing the elastic modulus using the new expression to the elastic modulus using the effective media models:
  - Voigt upper and Reuss lower bounds
  - Hashin-Shtrikman upper and lower bounds
  - Hill average estimate
  - Geometric mean

Elastic Modulus vs. Density

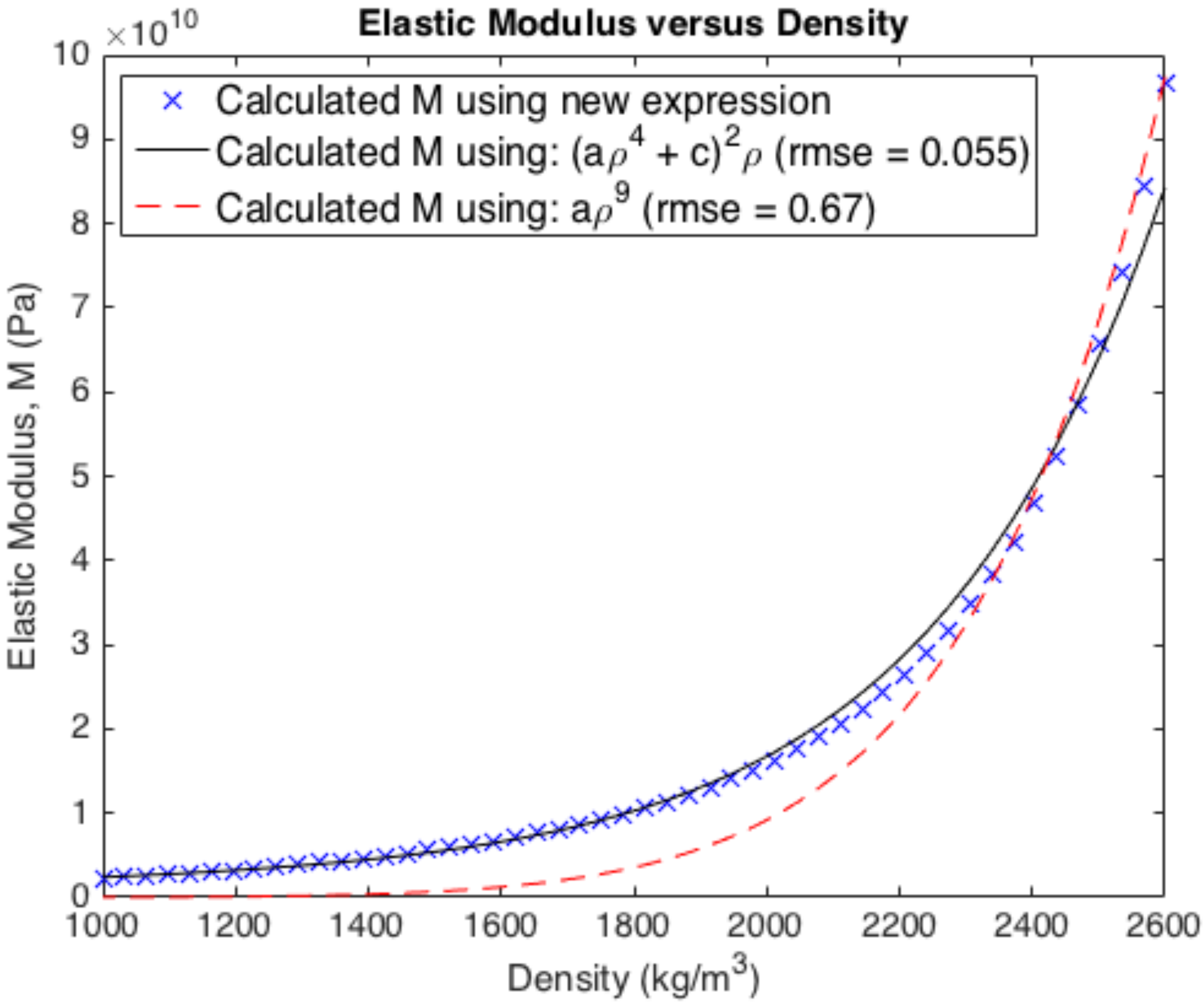


Numerical Approximation of Velocity (Wyllie's equation)

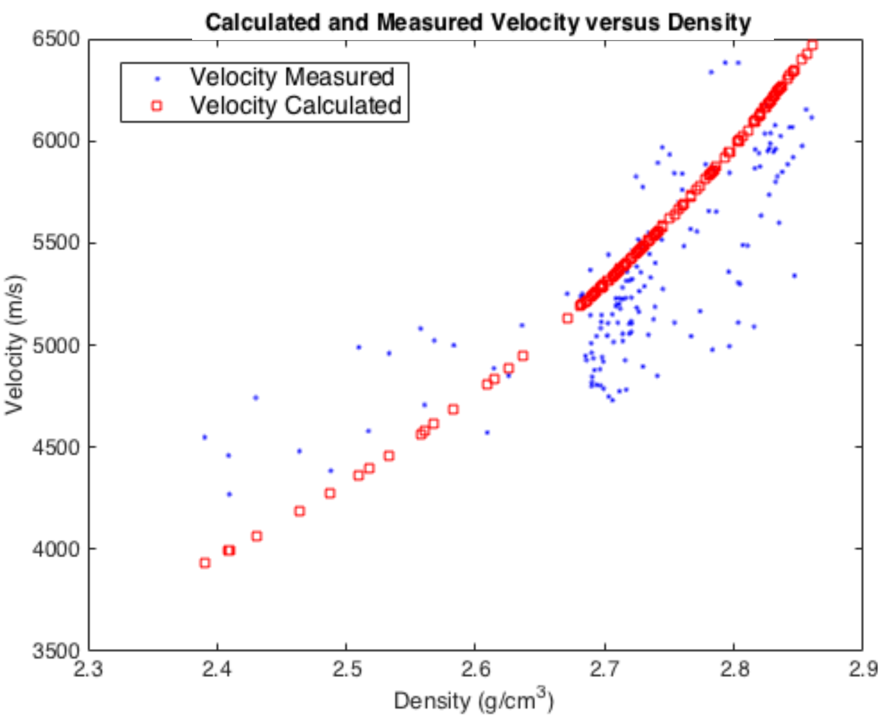
Numerical approximation form (Fit Line): $V = a\rho^{b} + c$

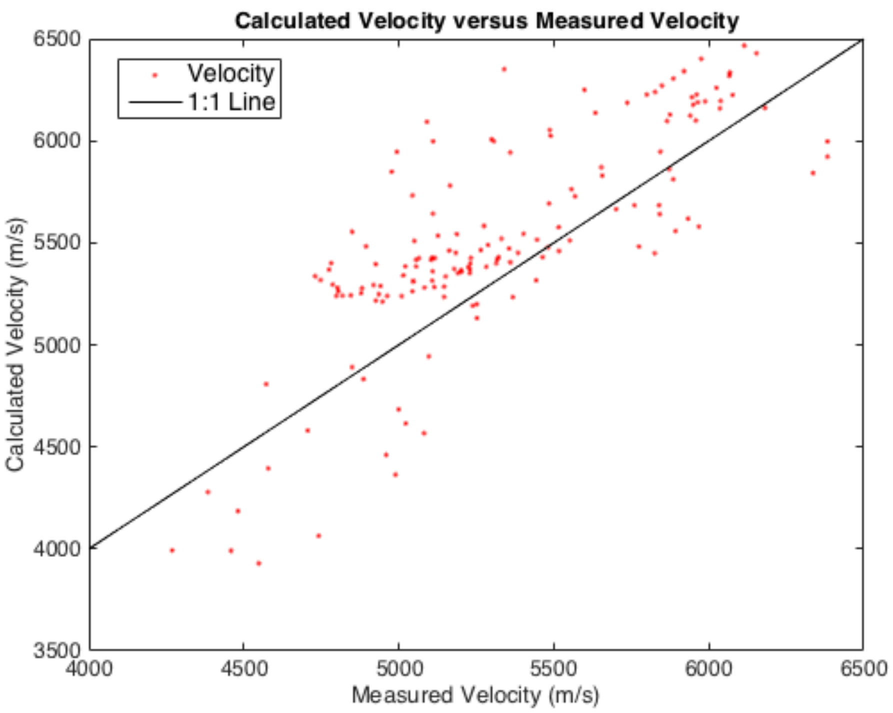


Elastic Modulus vs. Density



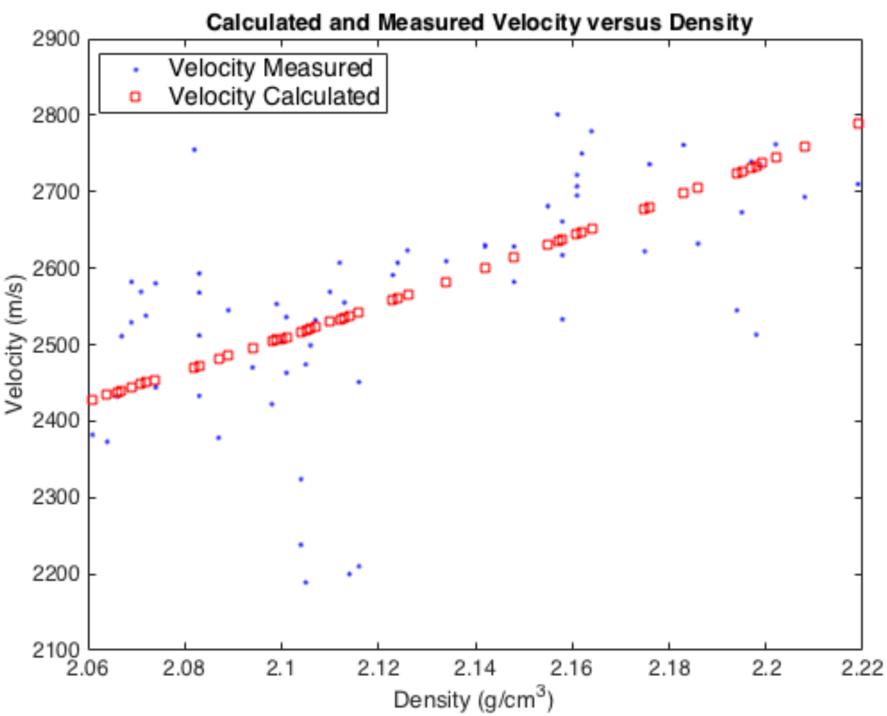
Calculated Velocity using the New Expression of M and Field Data – Dolomite

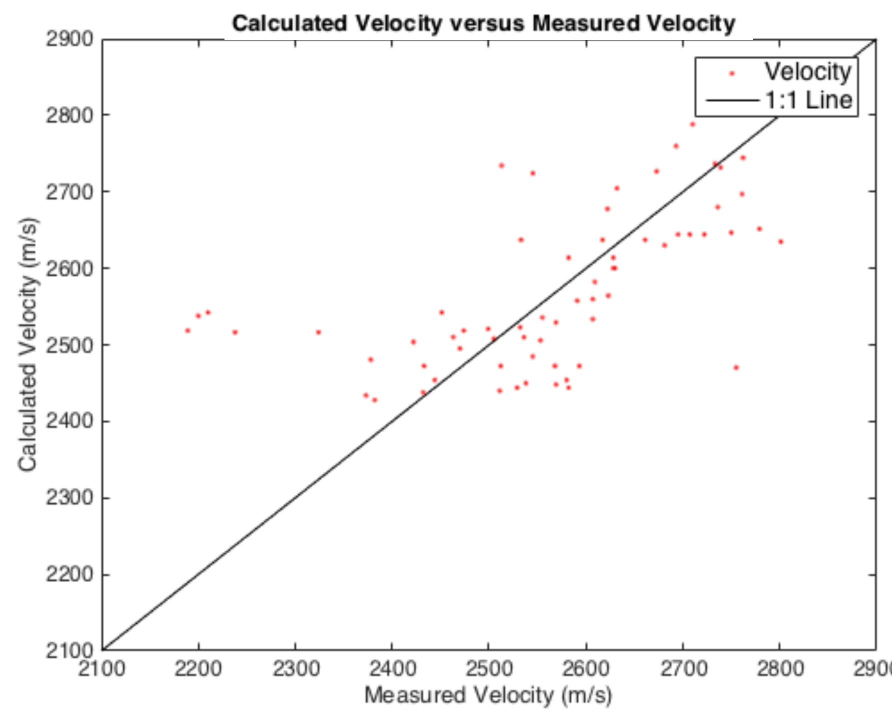




rmse = 0.05

Calculated Velocity using the New Expression of M and Field Data – Clean Sands





rmse = 0.05

Comparison

Velocity calculated using the new derived expression of M

Well No.	Lithology	rmse
Well 1	Limestone	0.03
	Dolomite	0.05
Well 2	Limestone	0.06
Well 3	Limestone	0.07
	Dolomite	0.1
Well 4	Clean Sands	0.05

Velocity calculated using the generalized form of Gardner's equation

Well No.	Lithology	rmse
Well 1	Limestone	0.03
	Dolomite	0.05
Well 2	Limestone	0.05
Well 3	Limestone	0.05
	Dolomite	0.06
Well 4	Clean Sands	0.05

Very similar rmse

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