Modeling nucleation and propagation of shear rupture on rough faults with a large range in wavelengths

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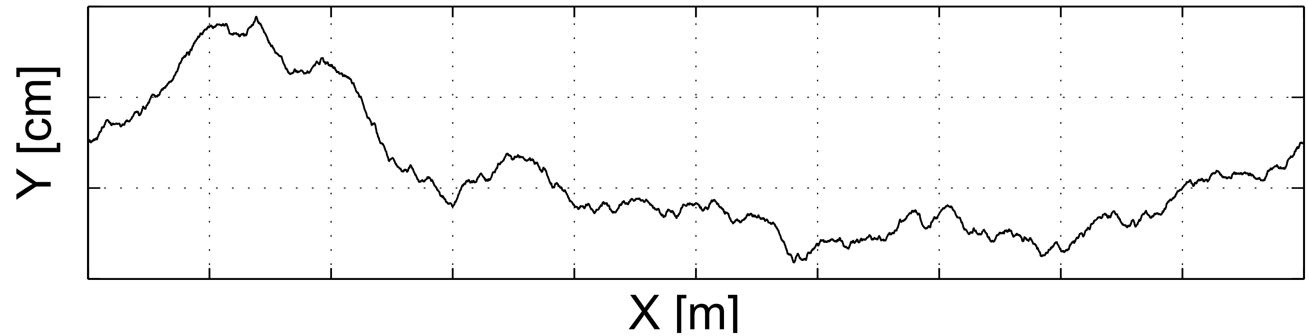
Fault roughness- self-affine fractals

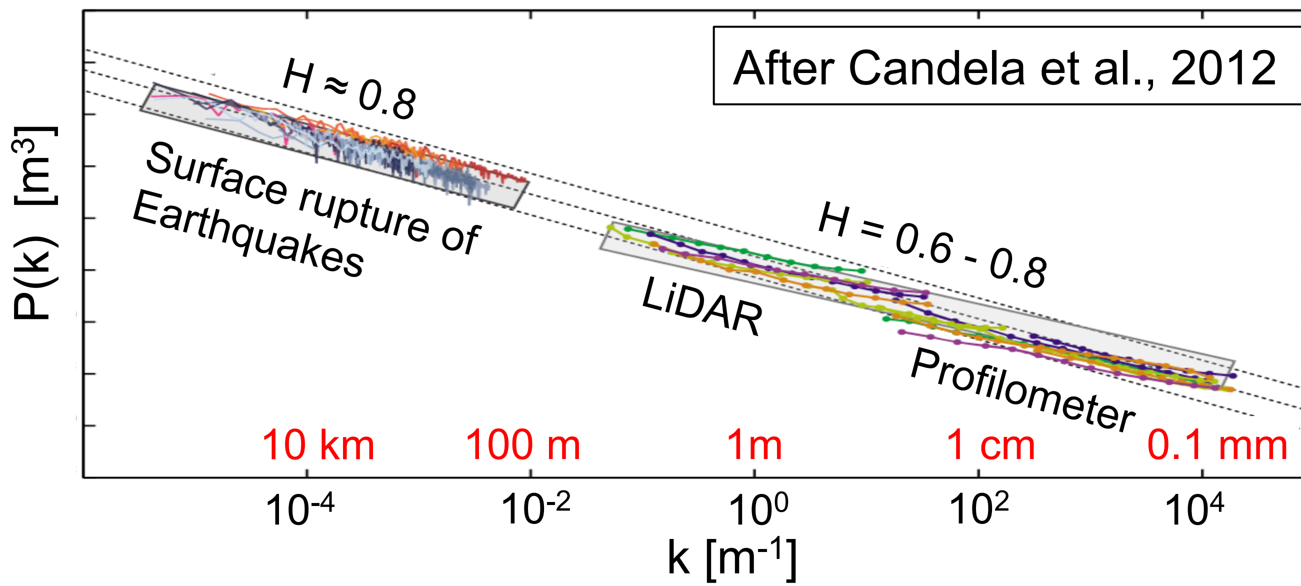
Spectral density

$$P(k) = Ck^{-(2H+1)}$$

H - Hurst exponent

C = $1\text{x}10^{-6} - 2\text{x}10^{-4}$



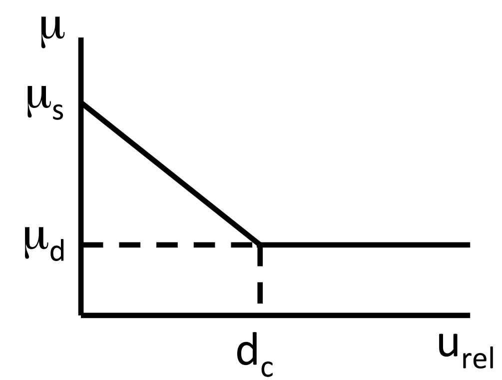


Study numerically the effect of roughness on the nucleation and propagation of shear rupture

Friction laws

Slip weakening- $\mu\ (u_{rel})$

$$\mu = \begin{cases} \mu_s + \dfrac{\mu_d - \mu_s}{d_c} u_{\mathrm{rel}}, & u_{\mathrm{rel}} \le d_c \\ \mu_d, & u_{\mathrm{rel}} > d_c \end{cases}$$



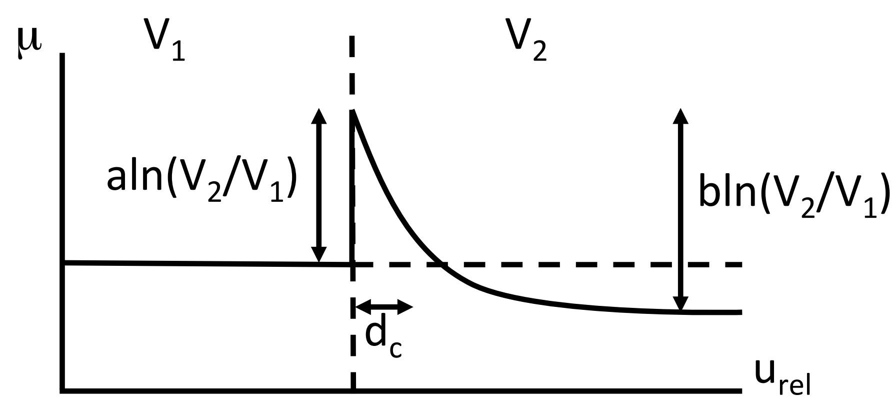
Rate and state friction (Dieterich, 1979)- $\mu\ (V_{rel}, \theta)$

$$\mu = \mu^* + a\ln\left(\frac{v_{\mathrm{rel}}}{v^*}\right) + b\ln\left(\frac{v^*\theta}{d_c}\right)$$

V_{rel}- slip velocity, V^*- reference velocity
μ^*- steady-state friction at V = V*
a and b- material dependent empirical constants
d_c- critical slip distance, θ- state variable:

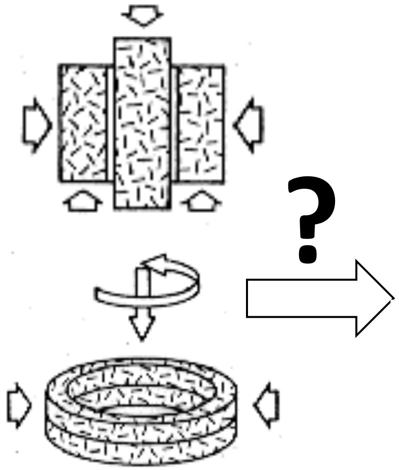
Aging law

$$\dot{\theta} = 1 - \frac{\theta v_{\mathrm{rel}}}{d_c}$$



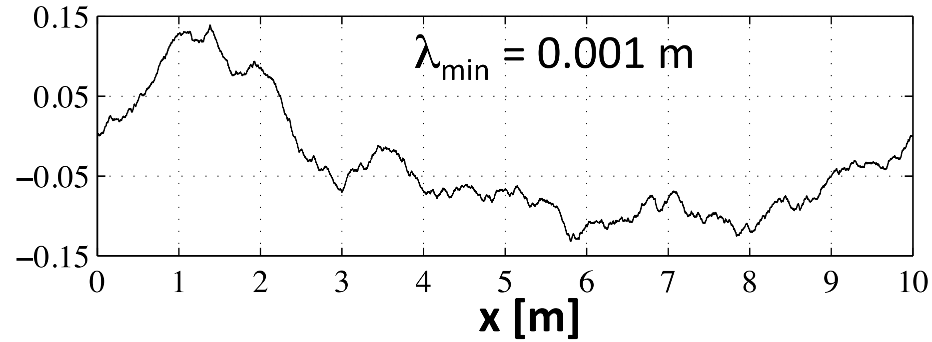
Conceptual approach

μ (u, v, θ, σ)



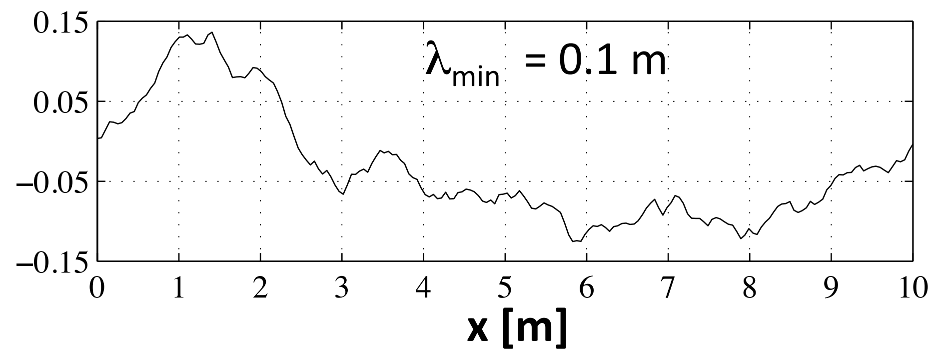
L ~ 10 cm

L ~ 10 m – 100 m



Experimental data

Gouge thickness

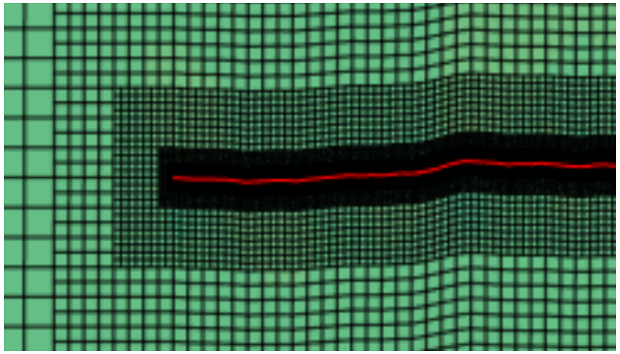


Numerical challenges and approach

Larger fault with fixed minimum wavelength => larger range in wavelengths

1. Computationally expensive

   => Hanging nodes



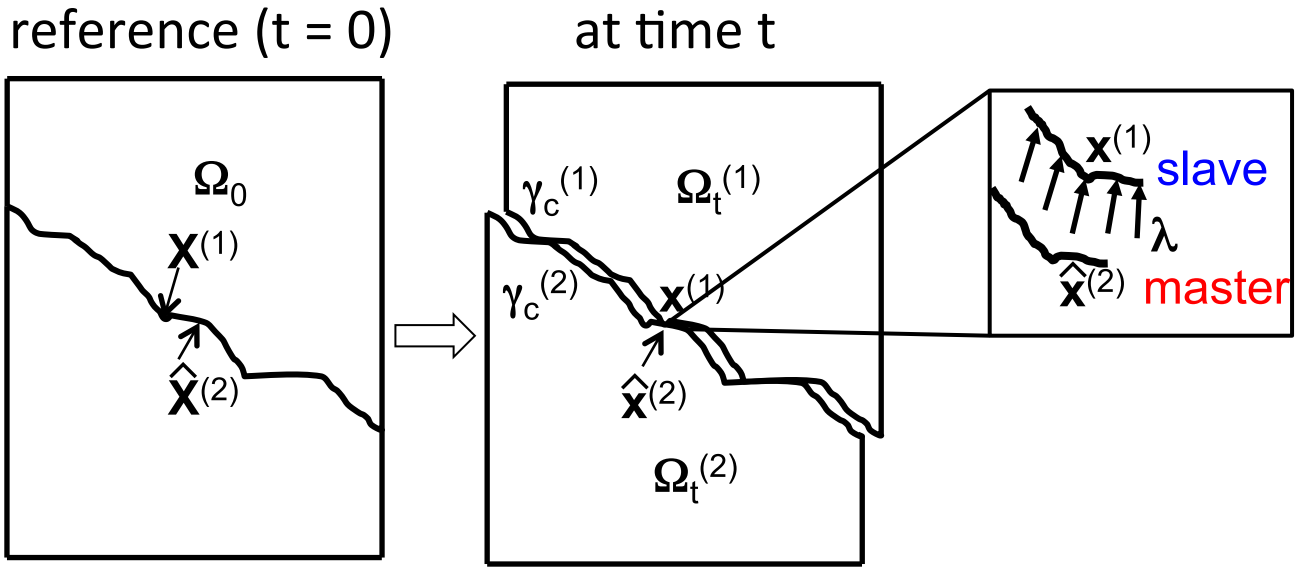
2. The assumption of small slip relative to the size of the elements may not be valid

   => **Mortar Finite Element Method**

   Continuity in an integral sense

Implementing slip weakening and rate and state friction laws

Mortar Finite Element



1. Finite element discretization

Virtual work: $\delta\Pi(\mathbf{u},\delta\mathbf{u}) = \delta\Pi_{\text{int,ext}}(\mathbf{u},\delta\mathbf{u}) + \delta\Pi_{\text{c}}(\mathbf{u},\delta\mathbf{u})$

Total Lagrangian formulation (Bathe, 1996)

Lagrange multipliers (λ)

$$\delta\Pi_{\text{c}} = \int_{\gamma_c^{(1)}} \boldsymbol{\lambda}\cdot\left(\delta\mathbf{u}^{(1)} - \delta\mathbf{u}^{(2)}\right)d\gamma$$

Mortar Finite Element

2. Contacts are updated each time step

3. Contact constraints

Normal direction: non-penetration condition

$$\int_{\gamma_c^{(1)}} \delta\lambda_n g_n d\gamma \geq 0, \qquad \lambda_n \geq 0, \qquad \lambda_n g_n = 0$$

Tangential direction: Coulomb's law

$$\int_{\gamma_c^{(1)}} \delta\lambda_t \big(v_{\mathrm{t,rel}} - \beta\lambda_t\big) d\gamma = 0,$$

$$\psi \coloneqq |\lambda_t| - \mu|\lambda_n| \leq 0, \qquad \beta \geq 0, \qquad \psi\beta = 0$$

g_n- gap function, $v_{t,rel}$ - relative velocity,

μ – friction coefficient

Mortar Finite Element

4. Variable time step
 - Quasi-static stages: backward Euler
 - Dynamic stages: implicit Newmark

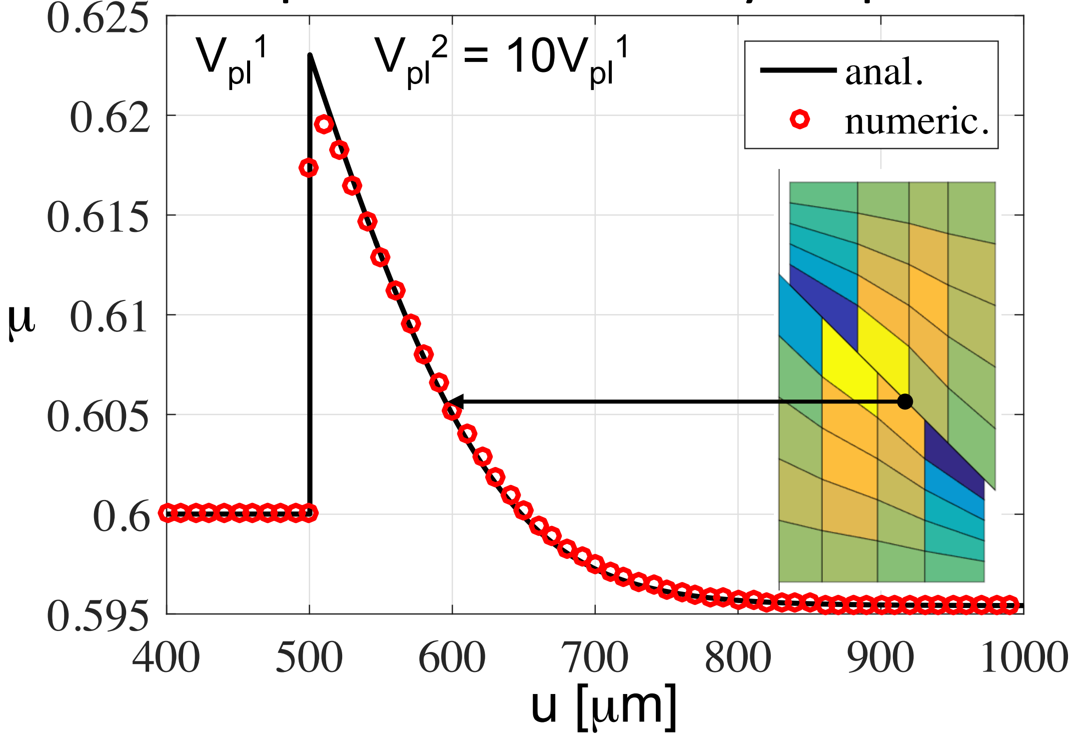
5. A primal-dual active set strategy
 - Dividing slave nodes into non-active, stick, and slip node sets
 - Replacing the inequality contact constraints by complementarity functions (Hueber and Wohlmuth, 2005)
 - Consistent linearization to give iterative semi-smooth Newton scheme
 - Static condensation of the Lagrange multipliers (Wohlmuth, 2000)

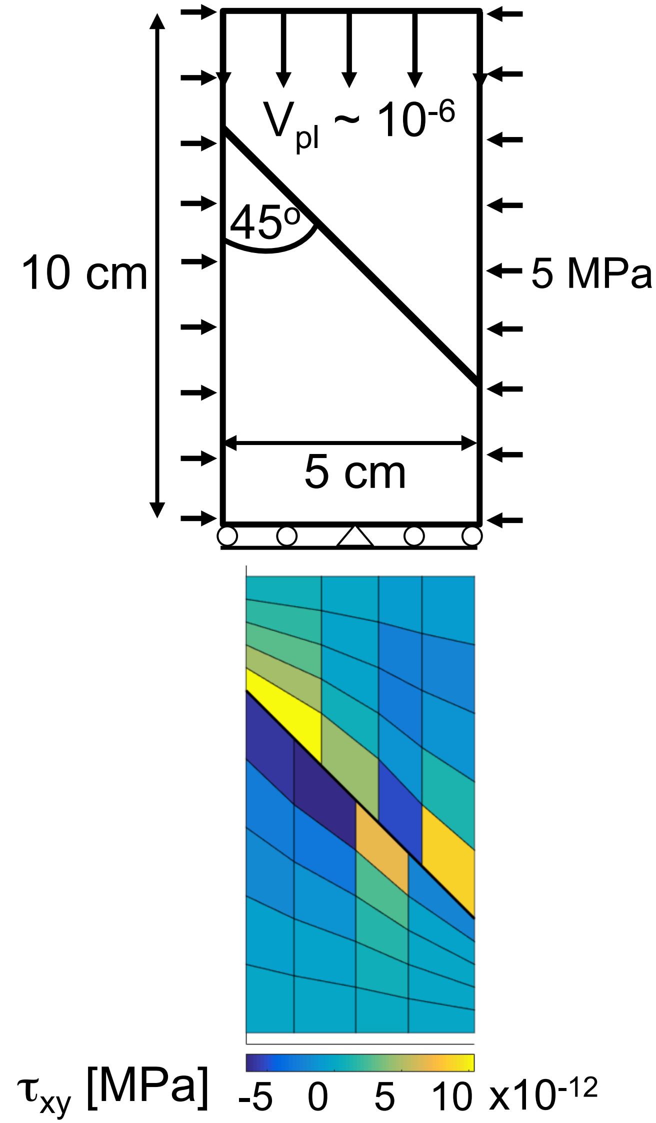
Quasi-static benchmark for rate and state

$$\mu = \mu^* + a\ln\left(\frac{v_{t,\mathrm{rel}} + v_{th}}{v^*}\right) + b\ln\left(\frac{\theta}{\theta^*}\right)$$

$$\dot{\theta} = 1 - \frac{\theta\left(v_{t,\mathrm{rel}} + v_{th}\right)}{d_c}$$

Response to a velocity step

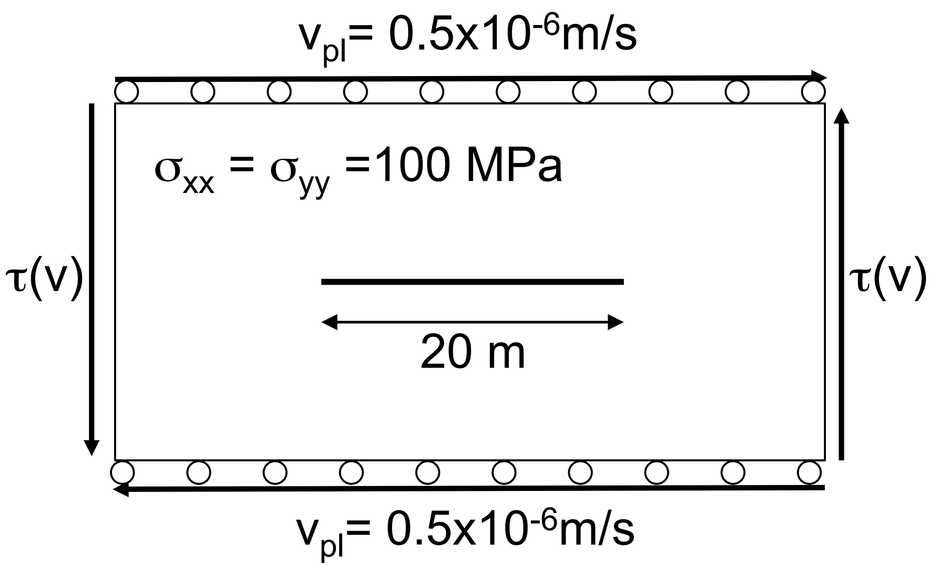


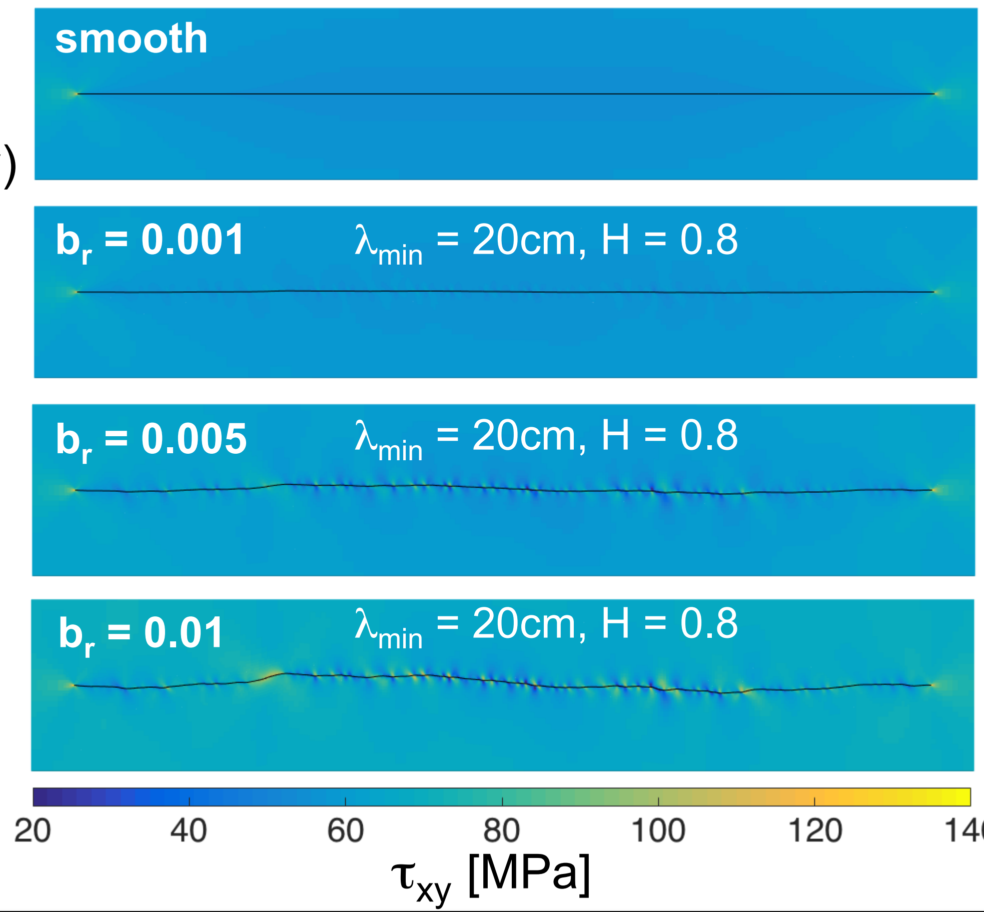


Example: Finite fault in a continuous domain

$$P(k) = Ck^{-(2H+1)} \Longrightarrow h(L) = b_r L^H$$

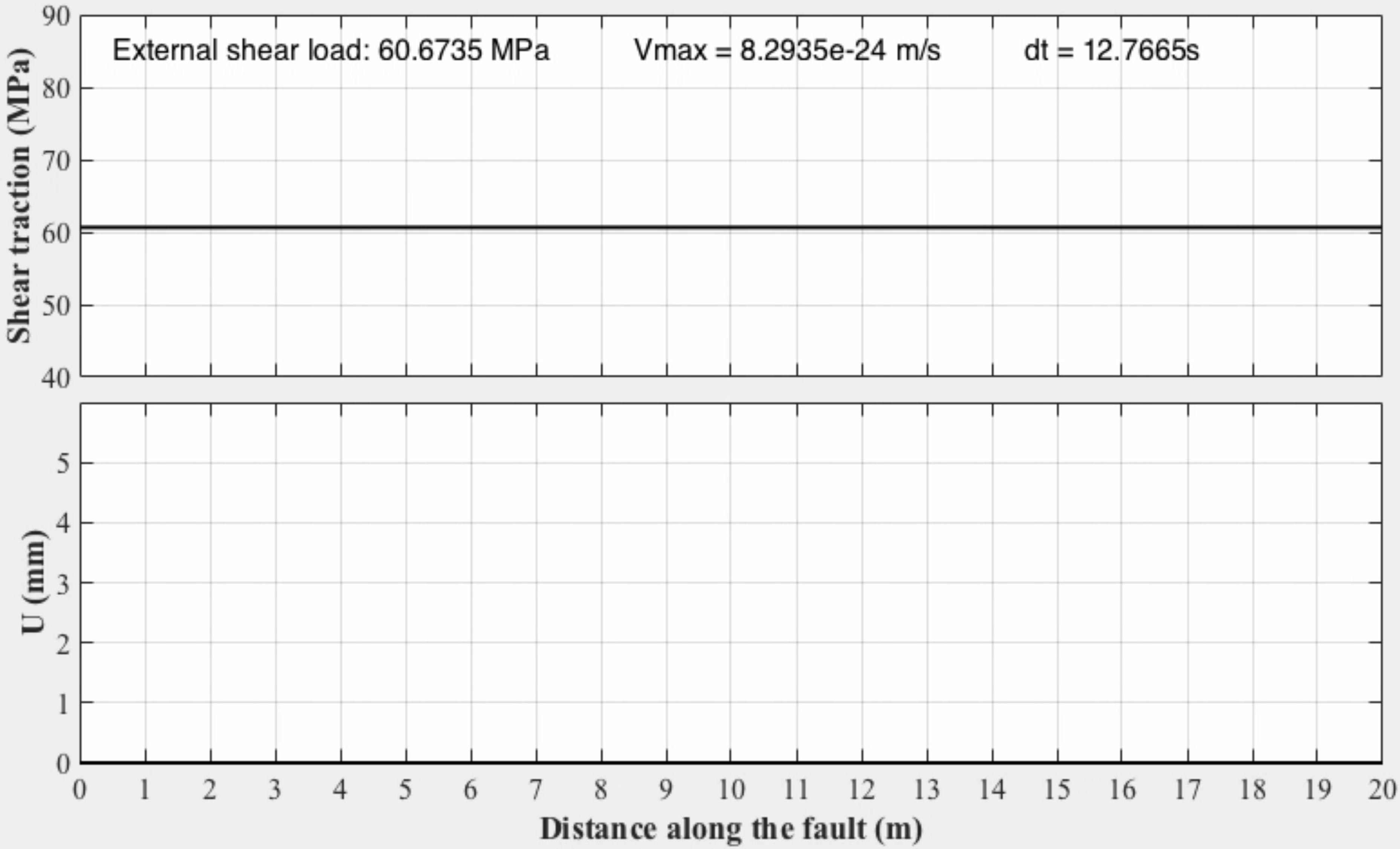
h - RMS height
L - Profile length
b_r - pre-factor
H - Hurst exponent



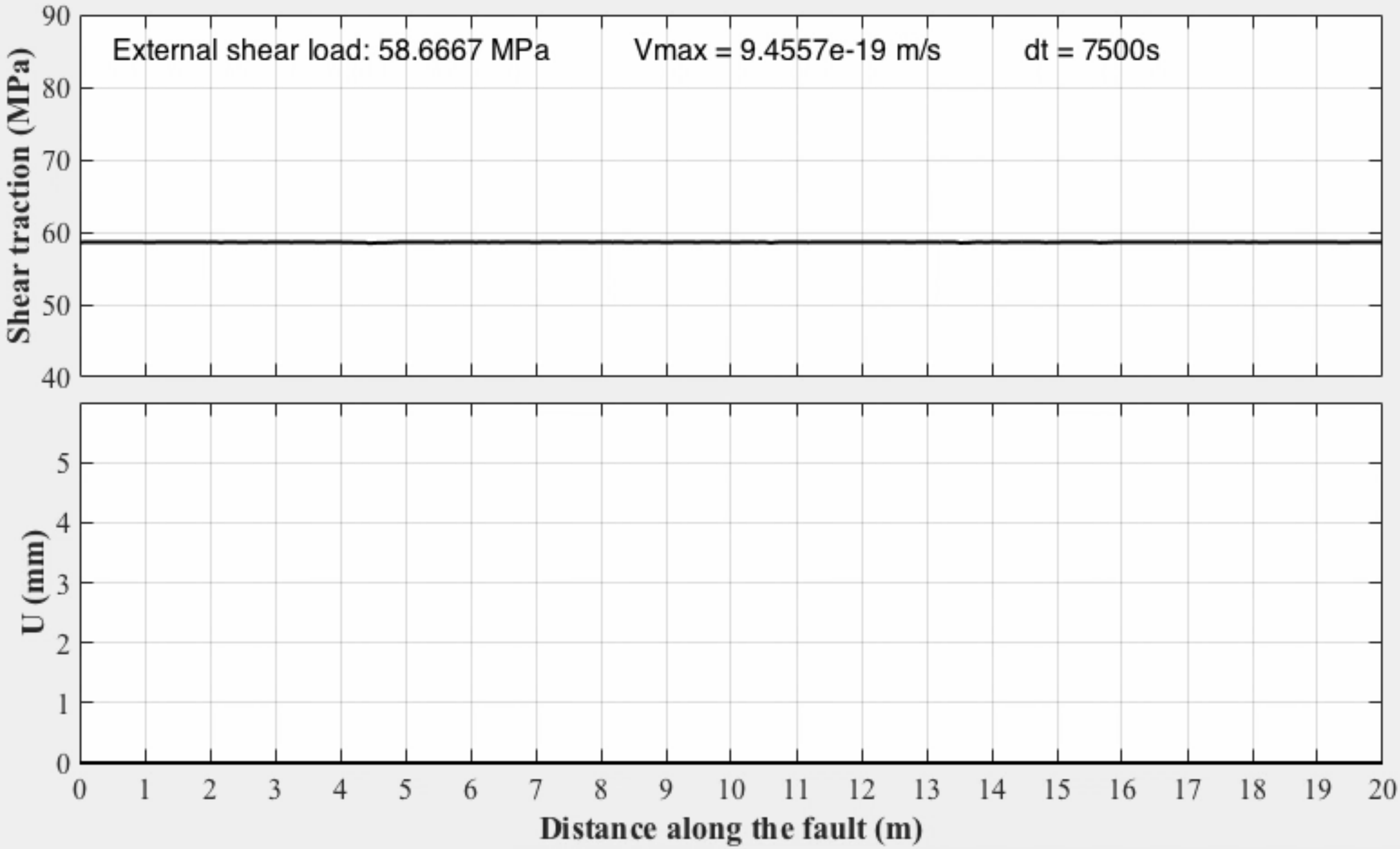


Rate-state parameters:
μ^* = 0.6
a = 0.01
b = 0.012
d_c = 20 μm
V^* = 1x10^{-6} m/s

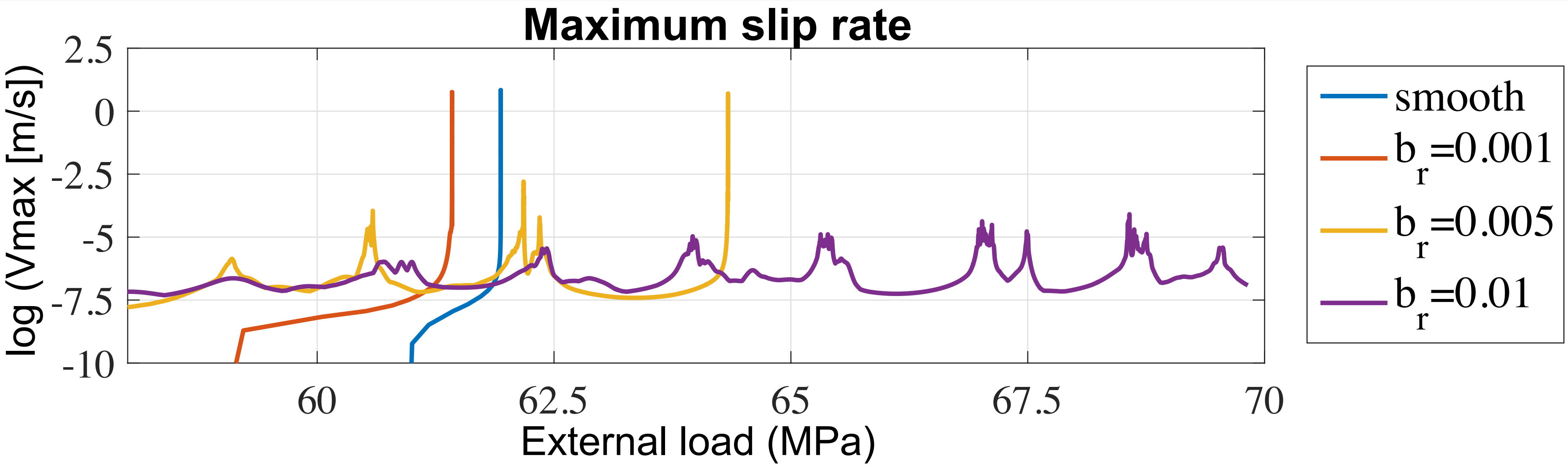
Smooth fault

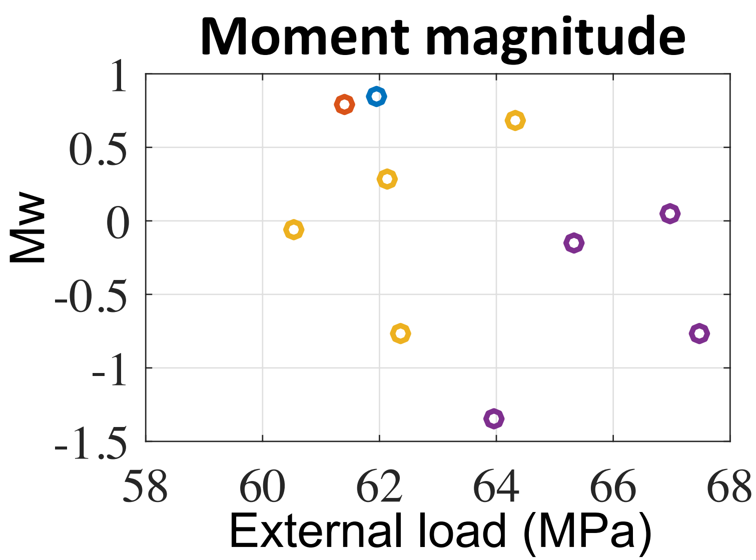


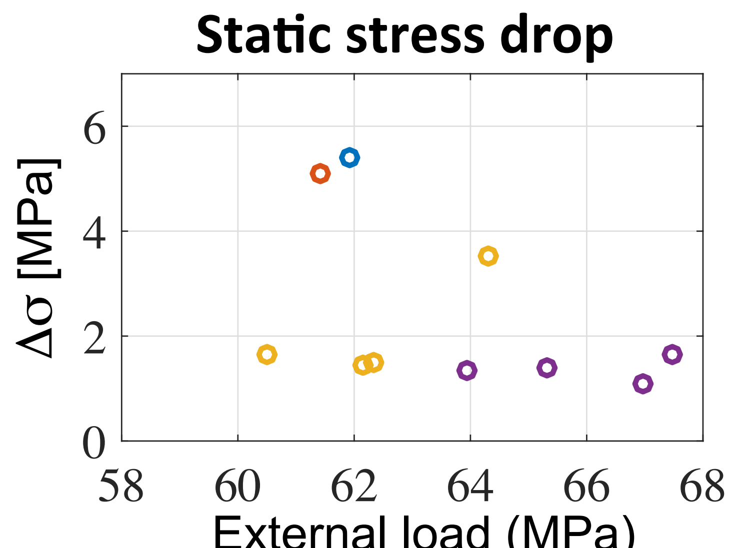
Rough fault: b_r = 0.001



The effect of roughness amplitude







Conclusions

- By implementing friction laws into the Mortar Finite Element method, we model the nucleation and propagation of shear rupture along rough faults with a large range in wavelengths.

- We numerically observe and quantify the significant effect of roughness on the following quantities: (1) Seismic moment; (2) Stress drop; (3) Slip rate; and (4) Nucleation and propagation properties such as nucleation length, rupture velocity, and breakdown zone.

Thank You!