ERL Annual Founders Meeting 05/23/2019

Earth Resources Laboratory

Finite Element Modeling of Planar Fault Slip: Fracturing vs. Friction

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Motivation

Joint theory of friction and fracturing:

Friction along the fault + crack tip processes.

Induced earthquakes:

Mineralization of parts of the fault, slip propagation includes breaking of locked sections - fracturing.

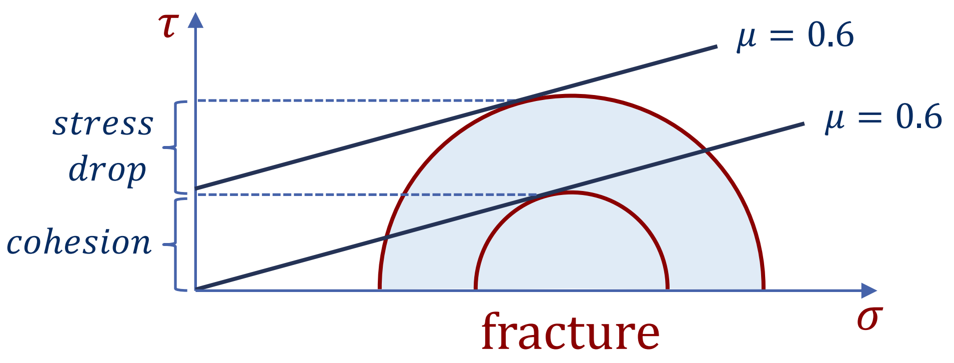
Rupture velocity for different mechanisms:

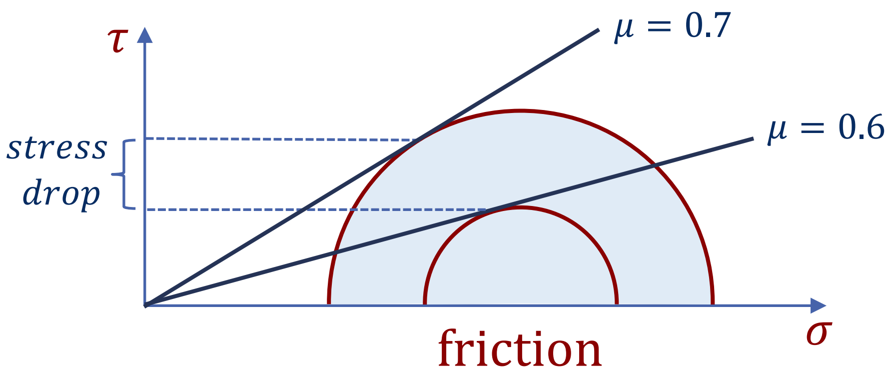
Slow slip events vs. seismic events.

Problem and solution

Problem: stress drop for fracturing vs. friction

Absence of joint theory of fracturing and friction that would be able to describe both brittle cracking and frictional sliding along the fault.





Solution:

- Finite element numerical simulations
- Observing slip-weakening instability propagation
- Observing rupture propagation, described by fracture energy criterion
- Observing similarities and differences in stress, slip, friction coefficient, slip rate etc., trying to link fracture and friction theories

Fracturing modes

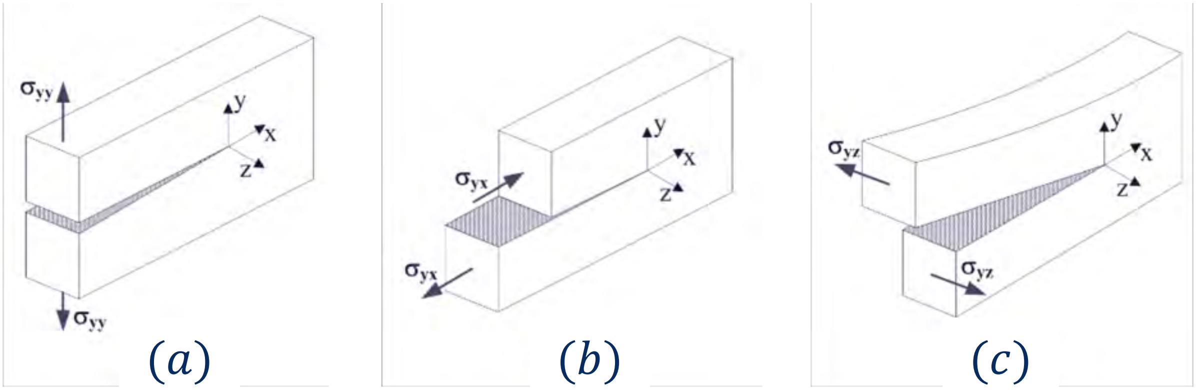


Figure 1. Crack modes: (a) – mode I; (b) – mode II; (c) – mode III

$$\left.\begin{aligned} K_I &= \lim_{r\to 0}\sqrt{2\pi r}\,\sigma_{yy}(r,0) \\ K_{II} &= \lim_{r\to 0}\sqrt{2\pi r}\,\sigma_{yx}(r,0) \end{aligned}\right\}$$ – stress intensity factors

Mode II is preferred if (Melin 1985):

$$\kappa = \frac{K_{IImax}}{K_{Imax}} > \frac{K_{IIc}}{K_{Ic}} = \kappa_c$$

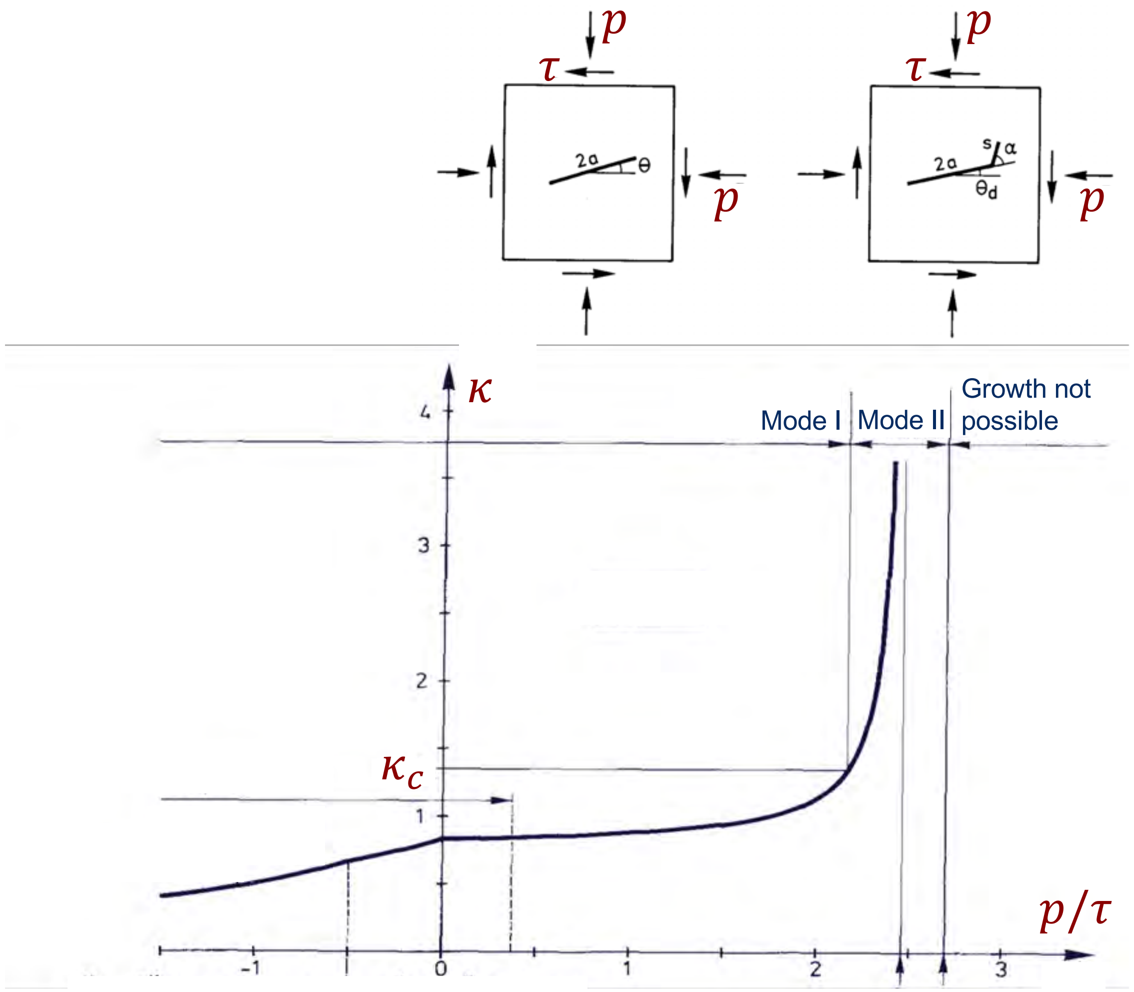


Figure 2. Preferred crack modes (Melin 1985)

Mode I vs. Mode II fracture

- Preexisting horizontal flaw (no cohesion, static friction)
- Abaqus XFEM (eXtented Finite Elements Method)
- Maximum principal stress propagation criterion vs. material weaker in shear

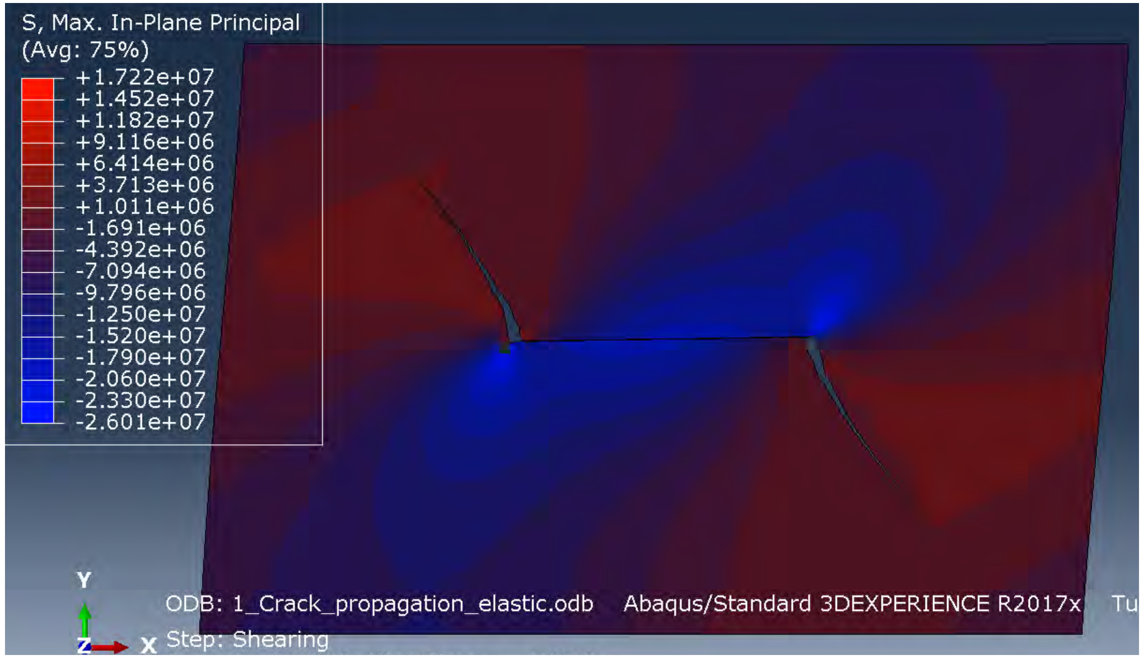


Figure 3. Mode I crack

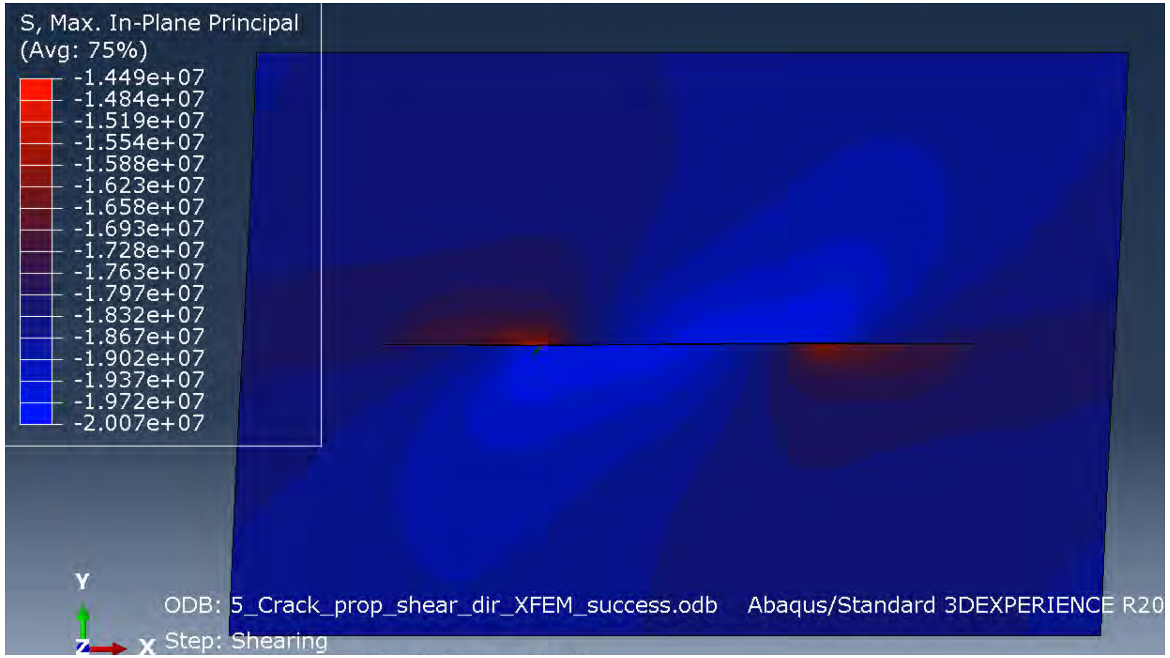


Figure 4. Mode II crack

Slip-weakening friction

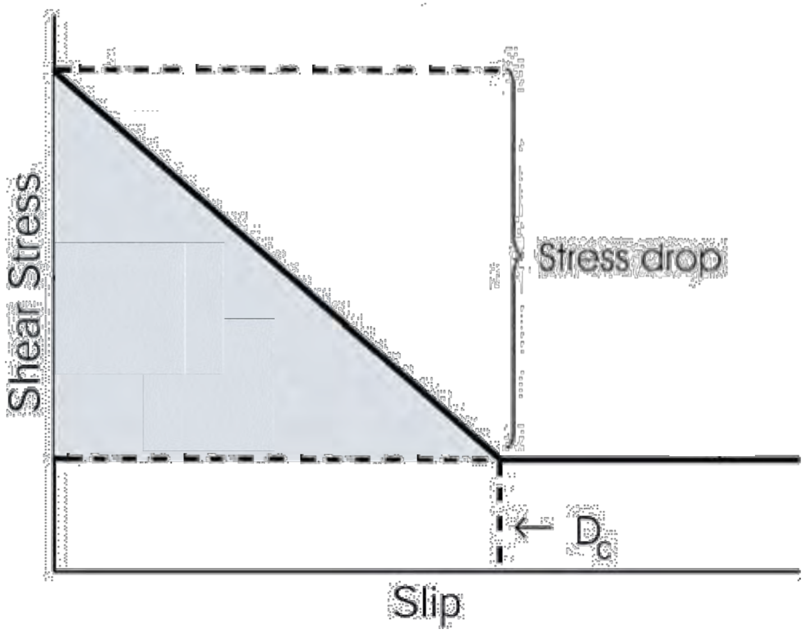


Figure 5. Shear stress for slip weakening friction

The slip-weakening friction law was first proposed by Ida (1972) and Andrews (1976):

$$\tau = \begin{cases} \tau_c - \left(\mu_s - (\mu_s - \mu_d)\dfrac{D}{D_c}\right)\sigma & D \leq D_c \\ \tau_c - \mu_d \sigma & D > D_c \end{cases}$$

where μ_s and μ_d – static and dynamic friction coefficients, D – slip magnitude, σ – normal stress, D_c – critical slip distance, and τ_c – cohesive stress.

In our simulations fault healing is enforced between cycles.

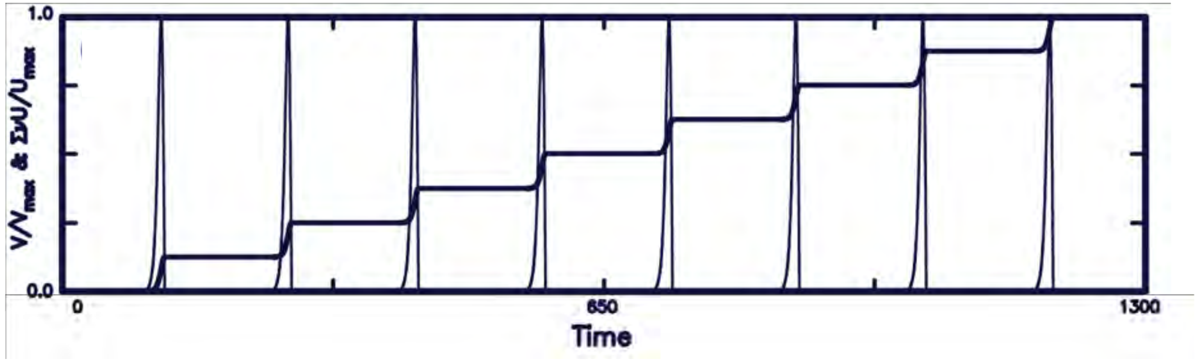
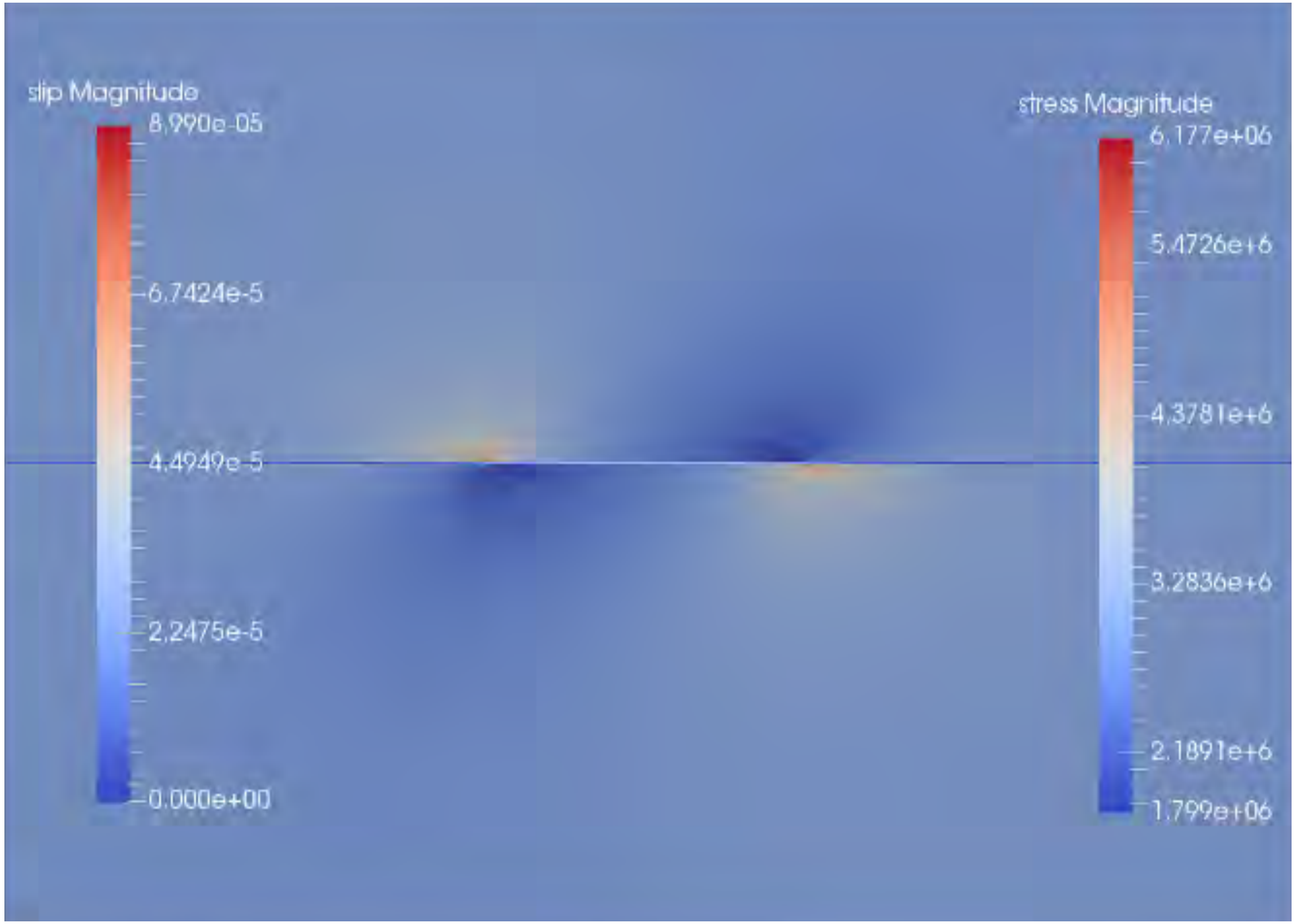


Figure 6. Slip and slip rate as a function of time for slip weakening friction

Earthquake cycle model



- 2D, plane strain
- Linear elastic material
- Boundary conditions: lithostatic compression and shear
- 3 fault sections: middle section – static friction $\mu = 0.6$; sides – slip-weakening $\mu_d = 0.6$, $\mu_s = 0.7$
- Time scale: years for quasi-static part, seconds for dynamic part

Figure 7. Model geometry

Quasi-static cycle - friction

- 3 sections: middle – creeping $\mu = 0.6$, sides – slip-weakening $\mu_d = 0.6$, $\mu_s = 0.7$

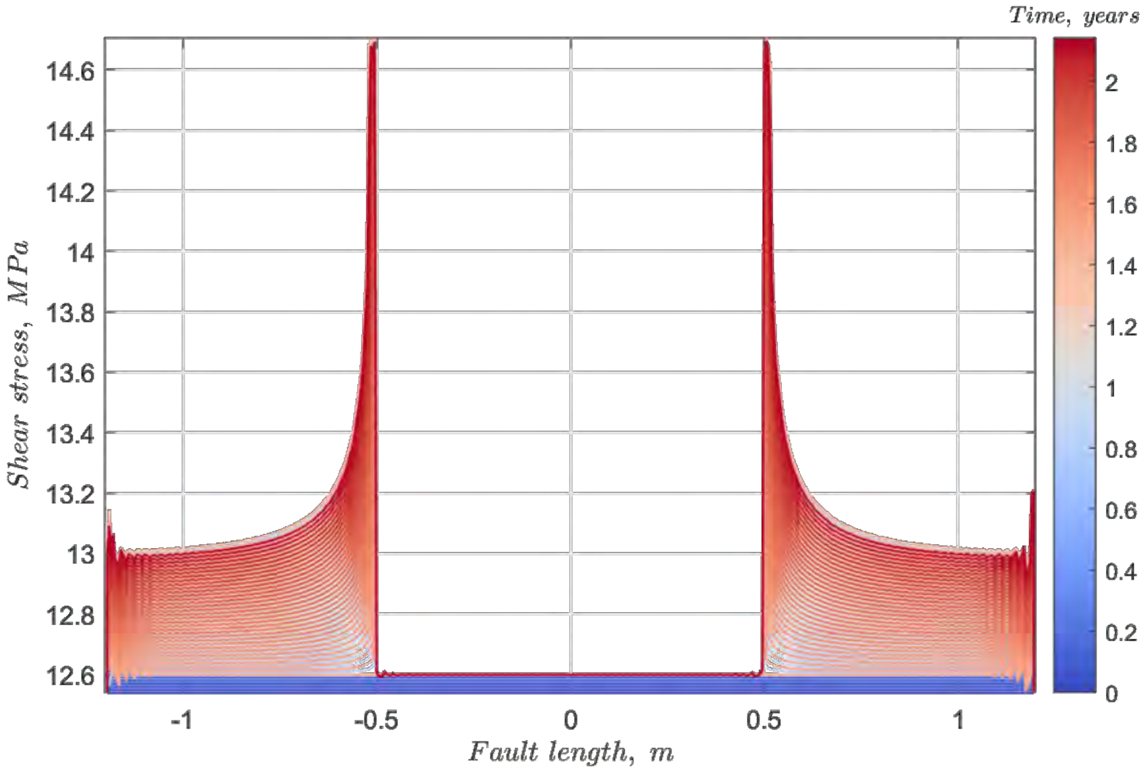


Figure 8. Shear stress on the fault

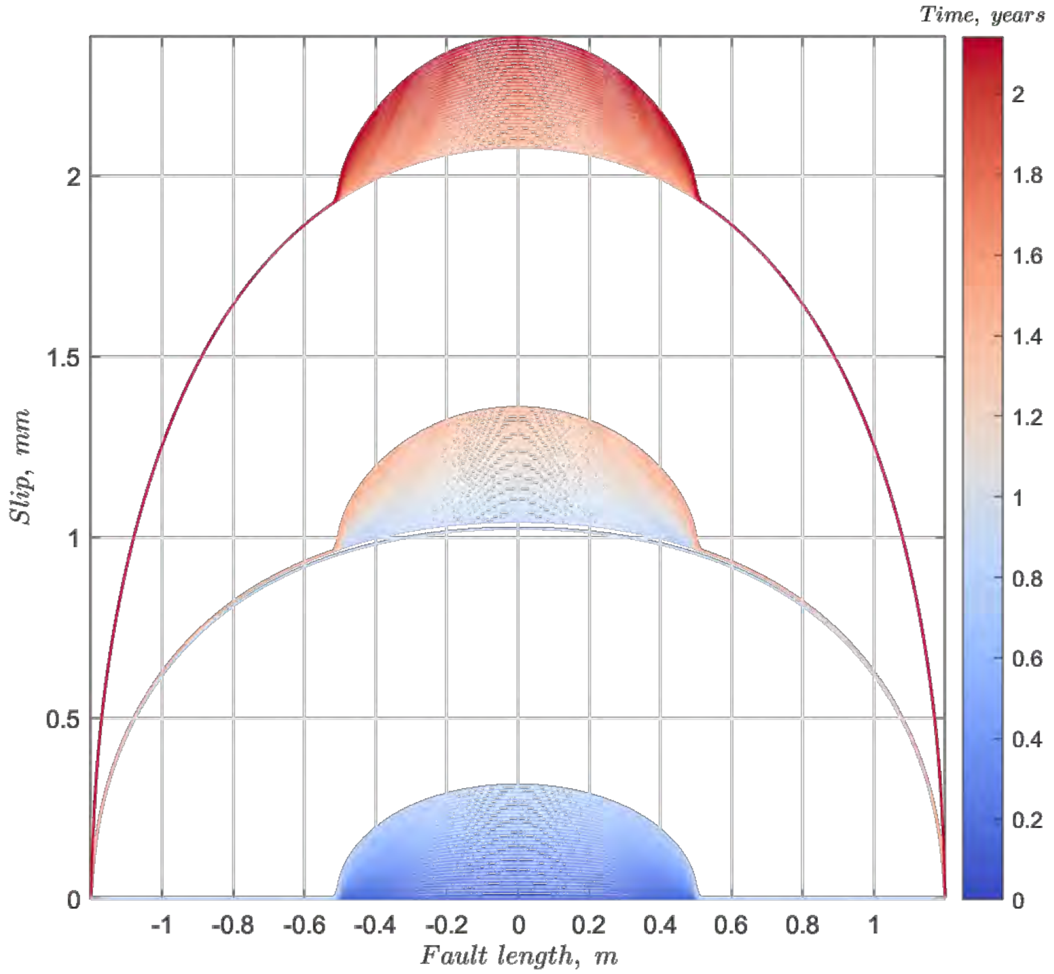


Figure 9. Slip on the fault

Dynamic cycle - friction

- Initial conditions – output of the last step of quasi-static simulation

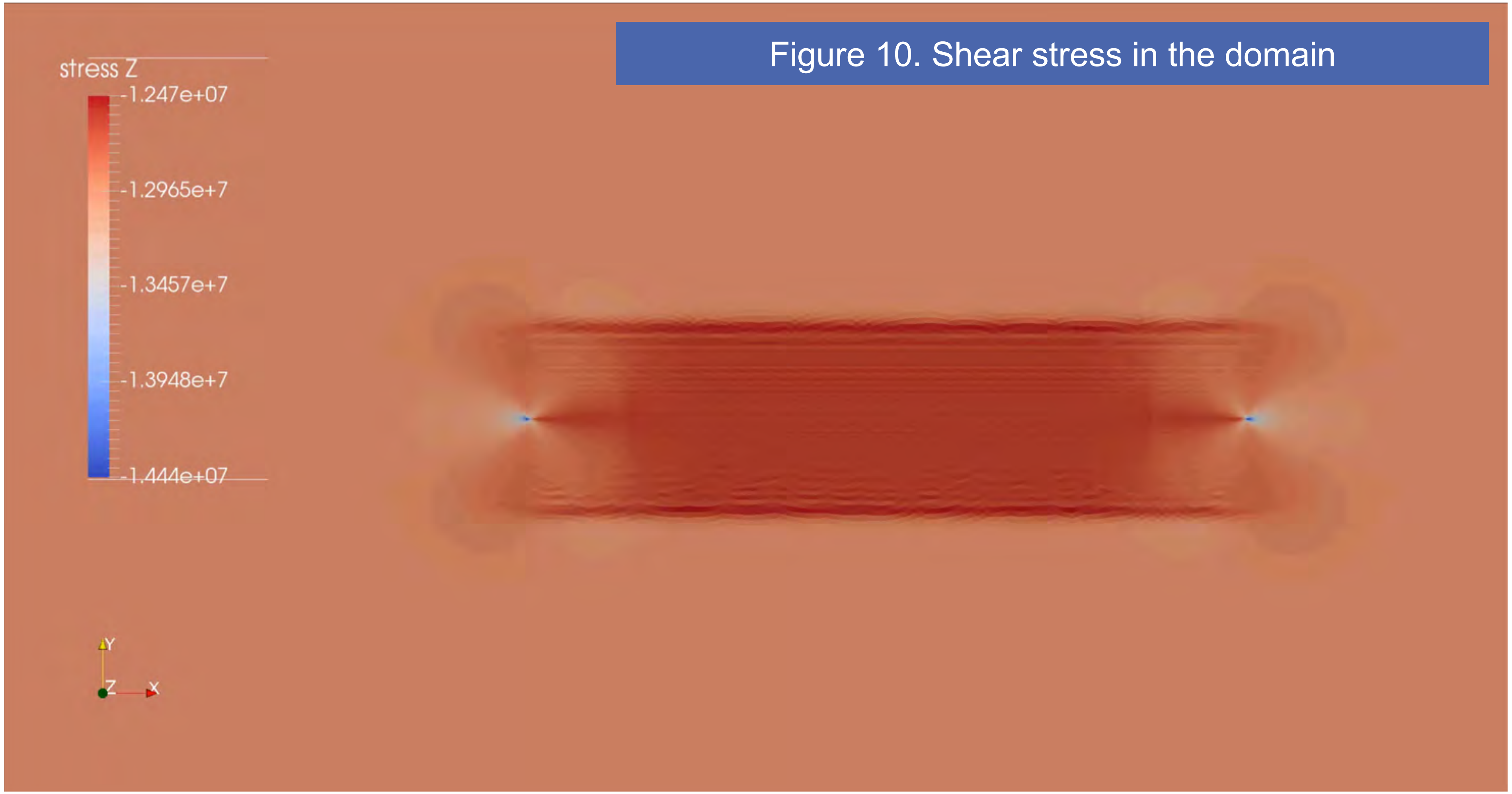


Figure 10. Shear stress in the domain

Dynamic cycle - fracture

Brittle crack propagation occurs along predefined surfaces. Power law model (Wu 1965):

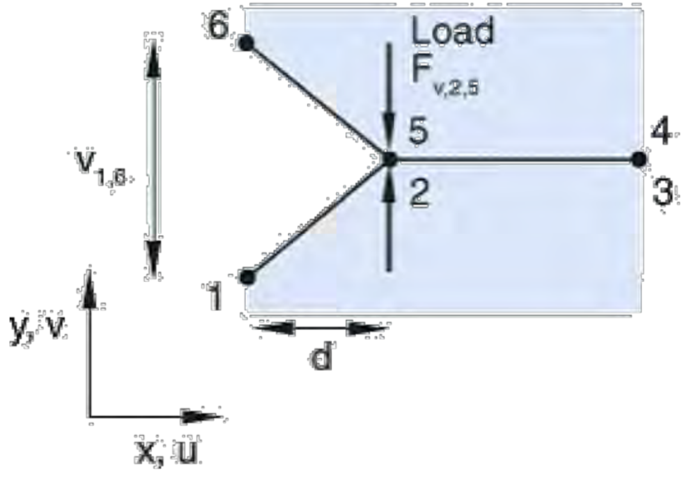
$$\left(\frac{G_I}{G_{IC}}\right)^{a_I} + \left(\frac{G_{II}}{G_{IIC}}\right)^{a_{II}} + \left(\frac{G_{III}}{G_{IIIC}}\right)^{a_{III}} \geq 1$$

Where G_I, G_{II}, and G_{III} strain energy release rates for different fracturing modes and G_{IC}, G_{IIC}, and G_{IIIC} are critical energy release rates necessary for the fracture to propagate.

For plane strain:

$$G_{II} = \frac{(1-\nu^2){K_{II}}^2}{E}$$

Similarly for G_I, and G_{III}.



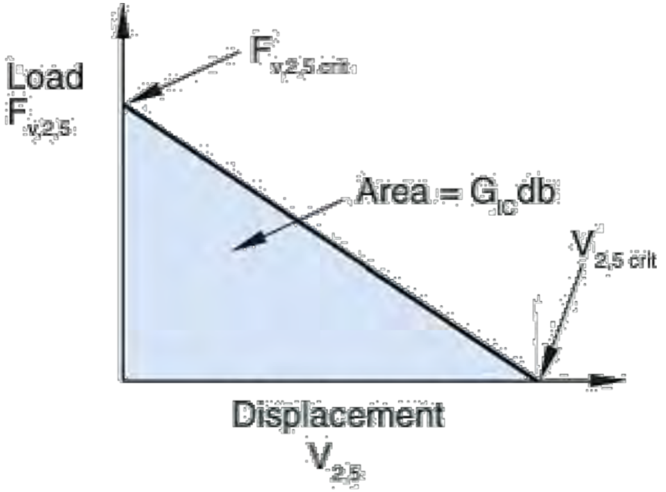


Figure 11. Strain energy release rate fracture propagation criterion

Dynamic cycle - fracture

- Initial conditions – output of the last step of quasi-static simulation

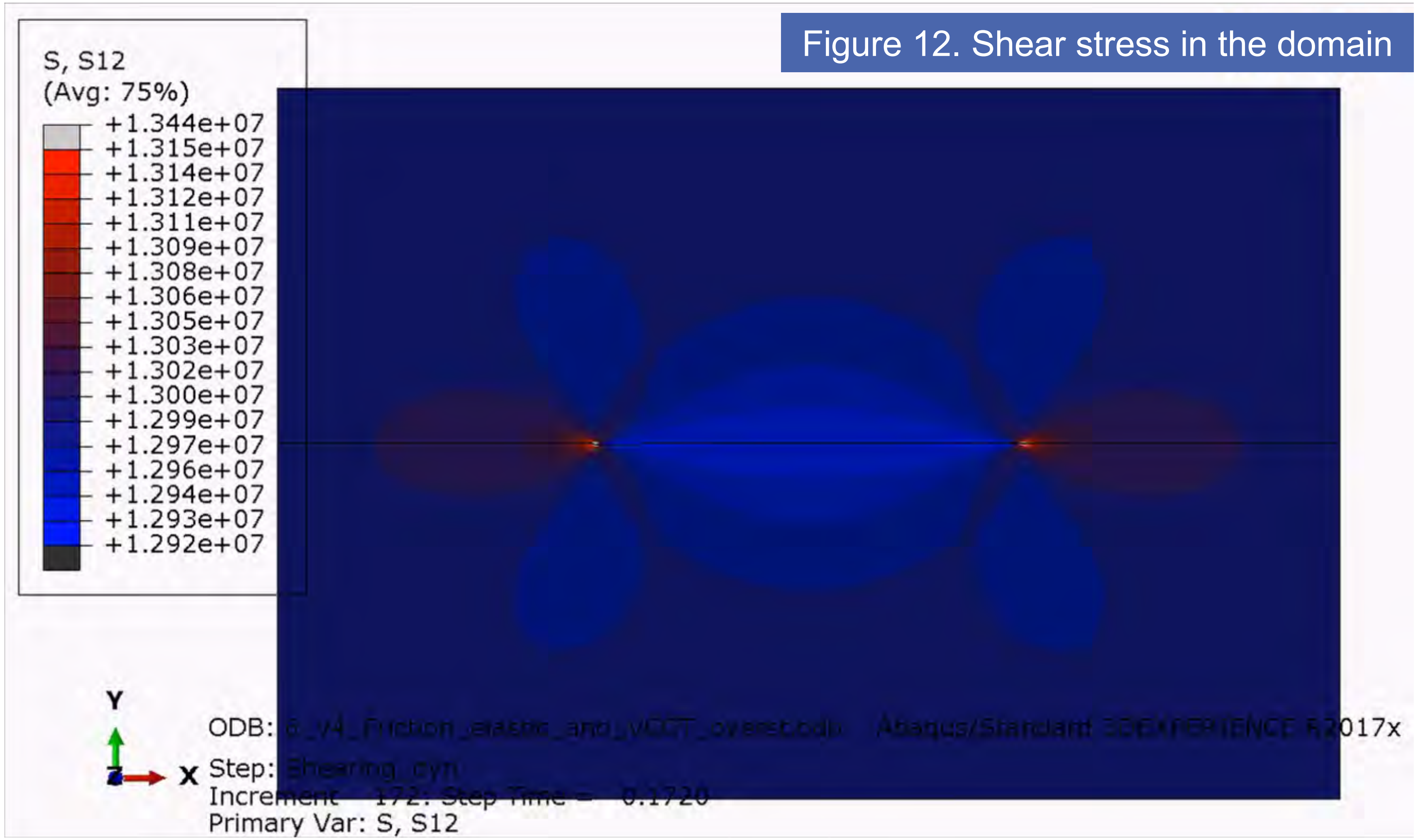


Figure 12. Shear stress in the domain

Friction vs. fracture

- Slip distribution for dynamic rupture propagation

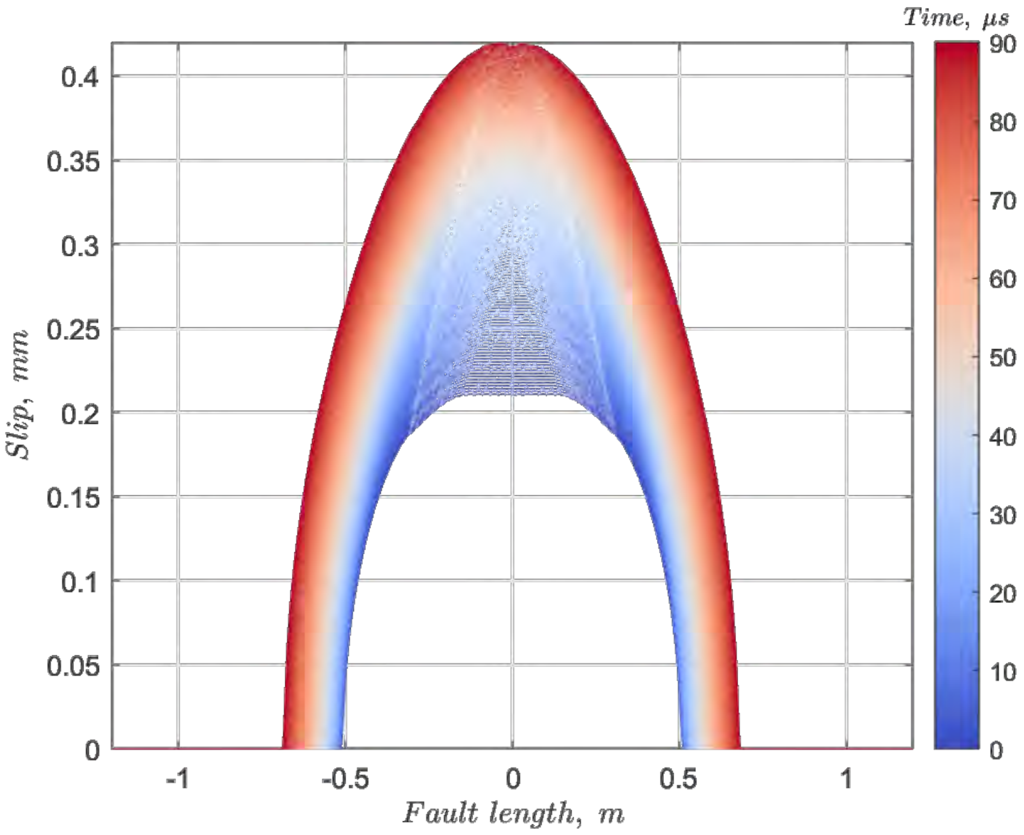


Figure 13. Slip on the fault - Slip-weakening friction
$\mu_d = 0.6, \mu_s = 0.7$

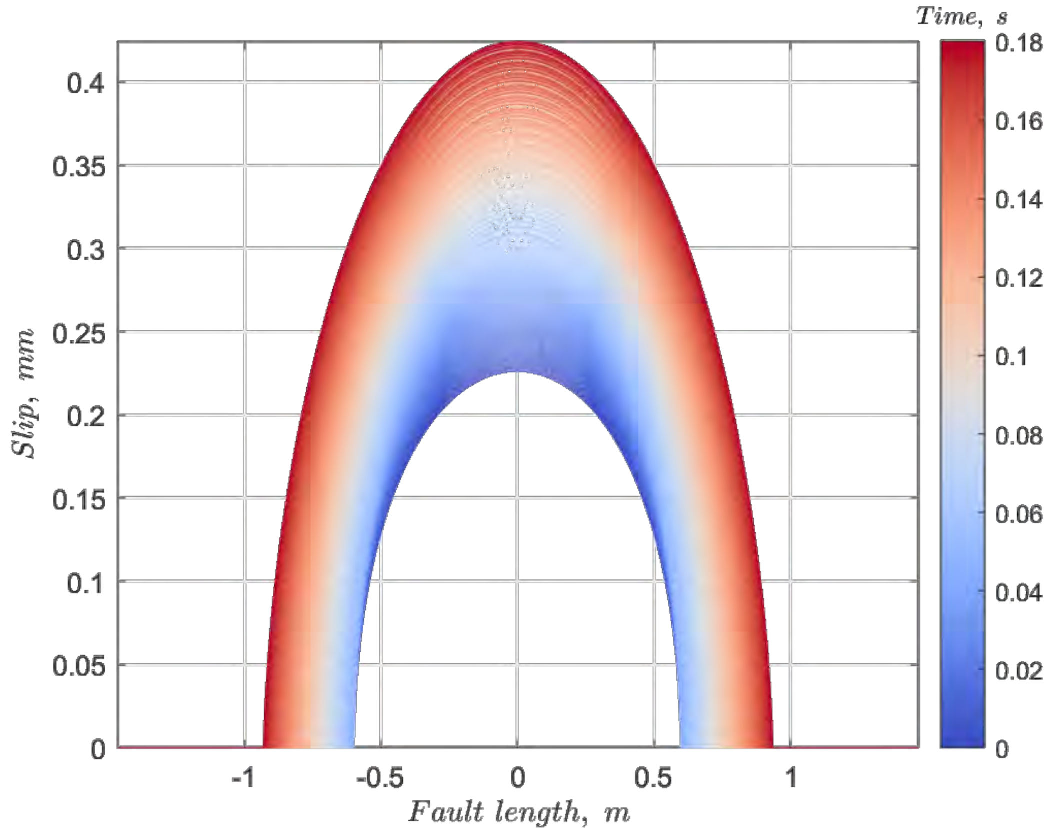


Figure 14. Slip on the fault – strain energy release fracture criterion

Friction vs. fracture

- Shear stress distribution for dynamic rupture propagation

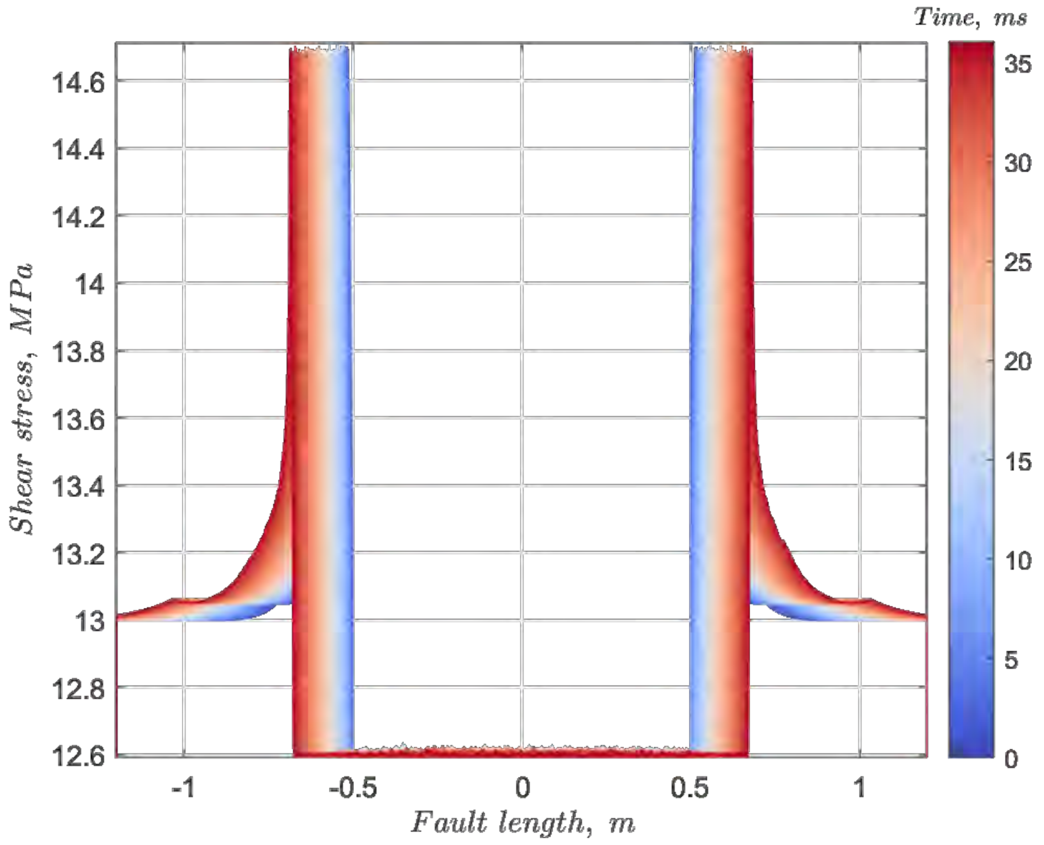


Figure 15. Traction on the fault - Slip-weakening friction $\mu_d = 0.6, \mu_s = 0.7$

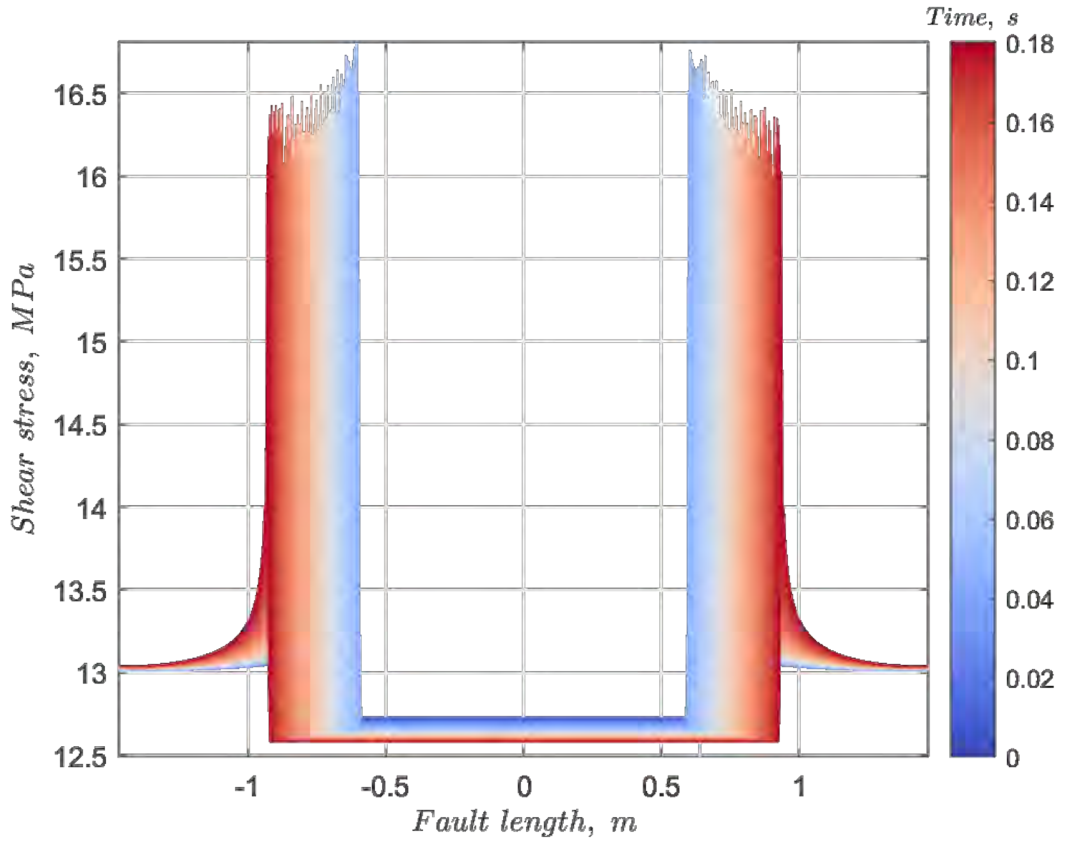


Figure 16. Traction on the fault – strain energy release fracture criterion

Friction vs. fracture

- Slip rate on the fault for dynamic rupture propagation

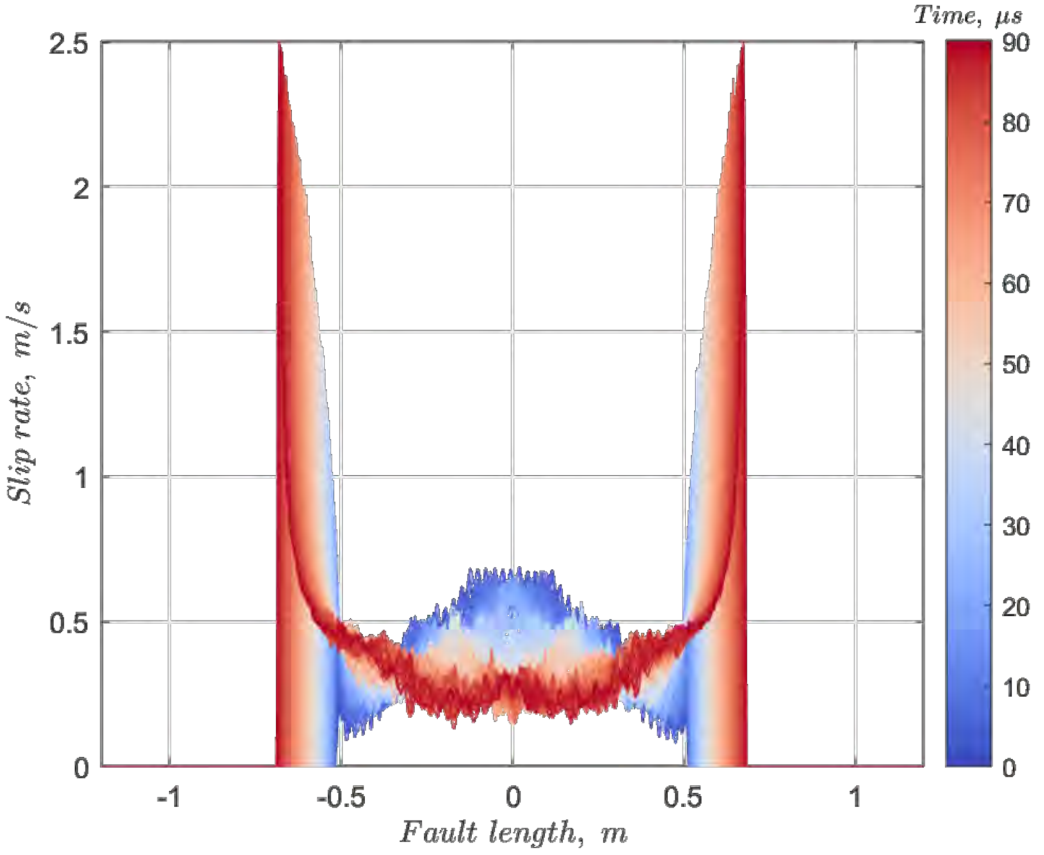


Figure 17. Slip rate on the fault - Slip-weakening friction $\mu_d = 0.6$, $\mu_s = 0.7$

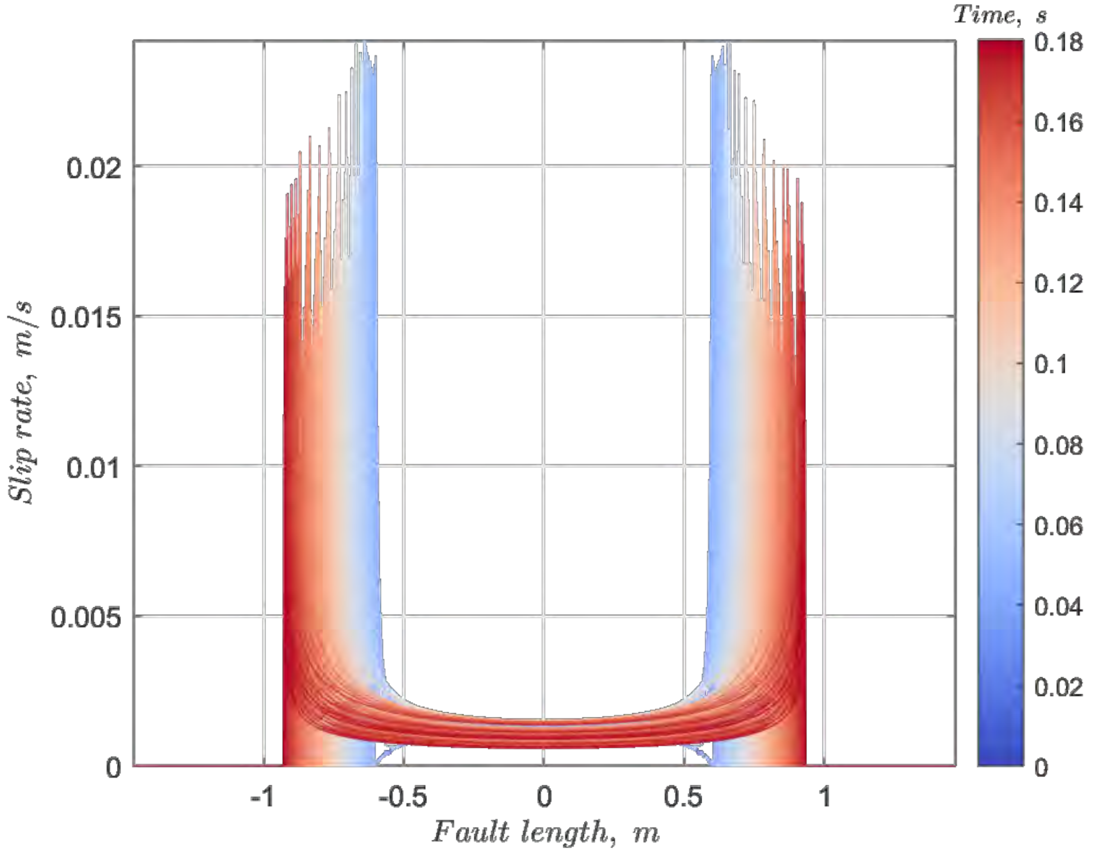


Figure 18. Slip rate on the fault – strain energy release fracture criterion

Conclusions

Current state:

- Successful modeling of out-of-plane mode I and mode II shear cracks with Abaqus XFEM
- Quasi-static models of earthquake cycle (slip-weakening instability) with fault healing between the cycles
- Dynamic part of the cycle modeled with Abaqus fracture propagation capability and Pylith slip-weakening subroutine
- Qualitative comparison of fault parameters for the two approaches: similar stress and slip distribution, different time scale and slip rate (potentially seismic vs. aseismic slip)

Work in progress:

- Analytical expressions to link fracturing and frictional parameters
- Implement cohesive strength with slip-weakening friction in Pylith
- Rate-and-state friction in a similar context

Questions?

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Back-up slides

References

- Dieterich, J.H. 1979. "Modeling of rock friction 1. Experimental results and constitutive equations." Journal of Geophysical Research 84 2161-2168.
- Ruina, A. 1983. "Slip instability and state variable friction laws." Journal of Geophysical Research 88 10,359-10,370.
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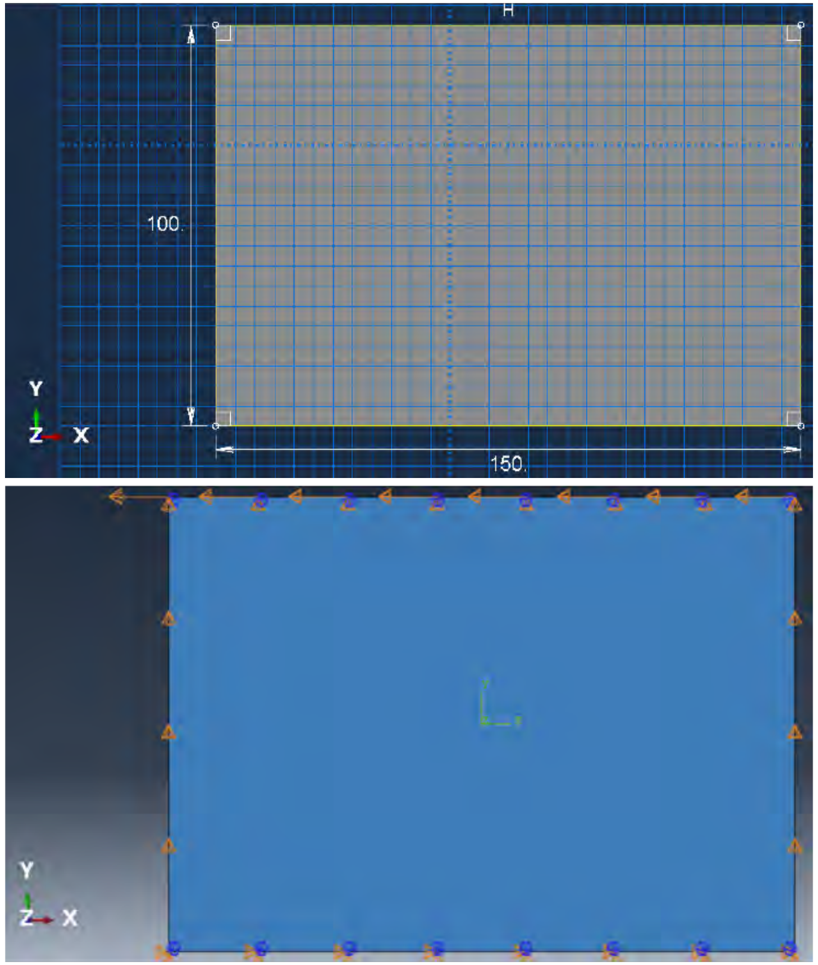
Model description



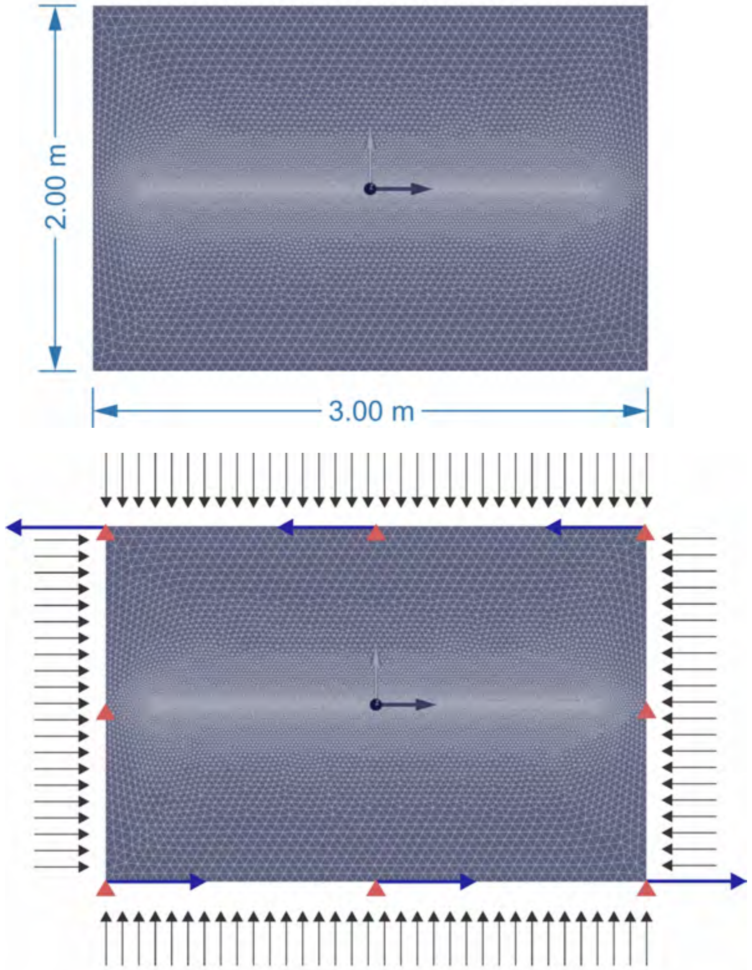


Figure 4. Abaqus model and BC

Figure 5. PyLith model and BC

- Method: FEM (Abaqus and PyLith)
- 2D, plane strain
- Size: meters
- Boundary conditions: lithostatic compression + shear
- Material: linear elastic
- Fault rheology: static friction with cohesion, slip-weakening friction

Plastic simulations

- Orientation and size of plastic zones depend on material dilation angle

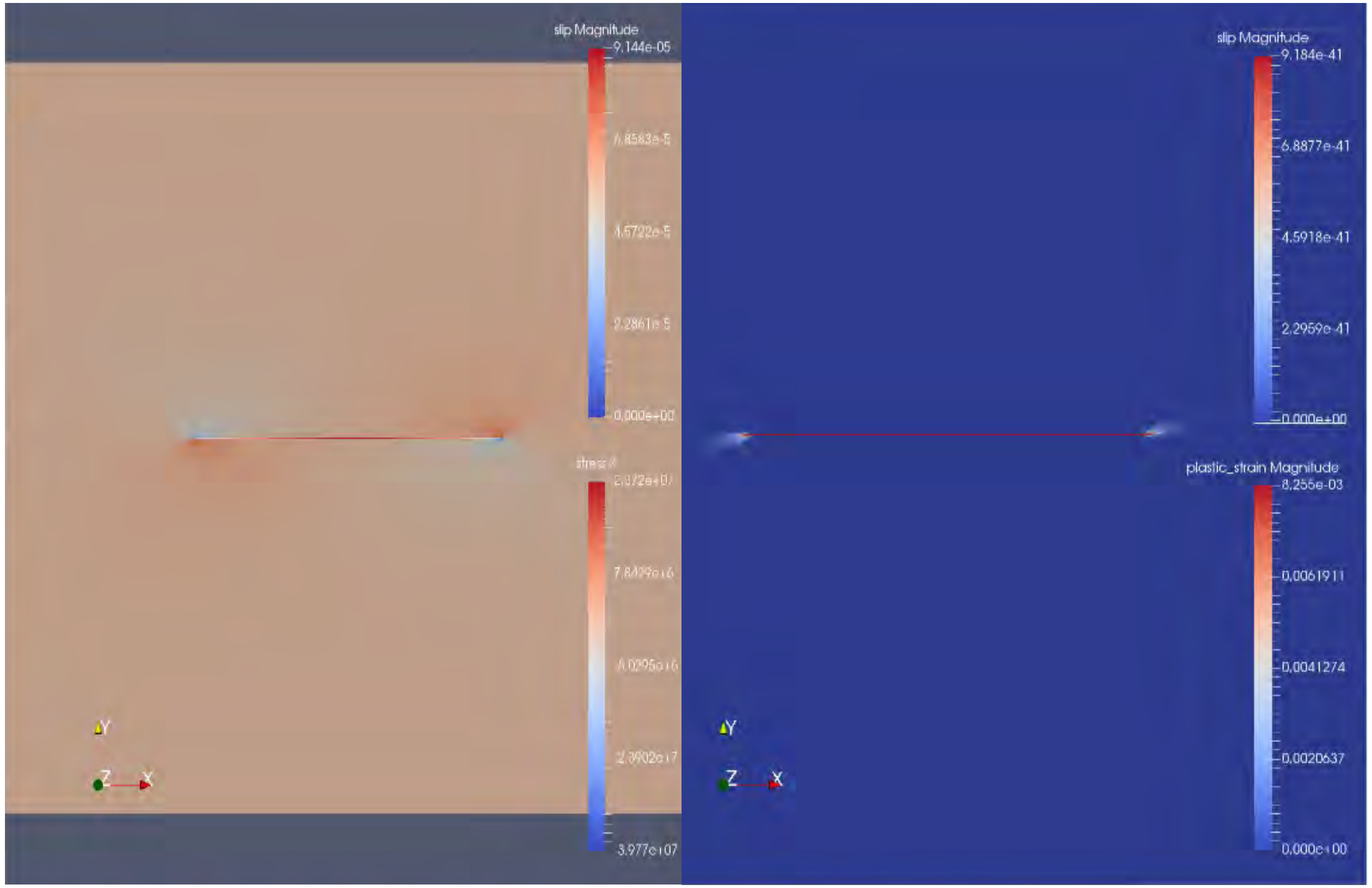


Figure 6. PyLith stress and plastic strain

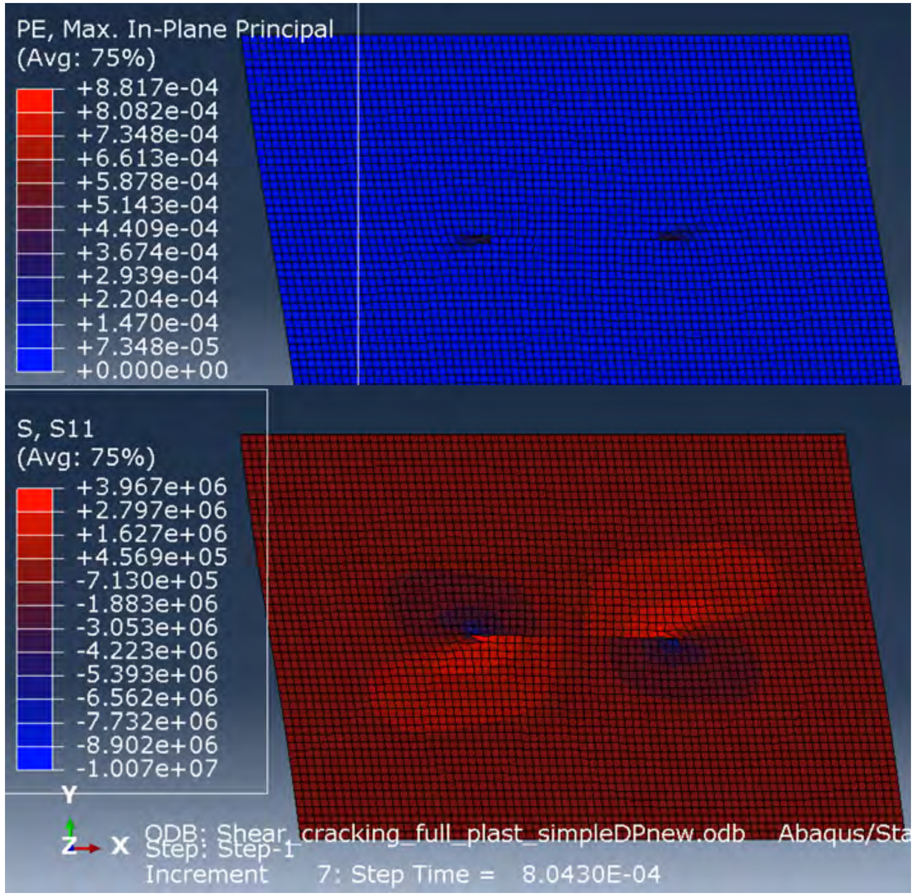


Figure 7. Abaqus stress and plastic strain

Rate-and-state friction

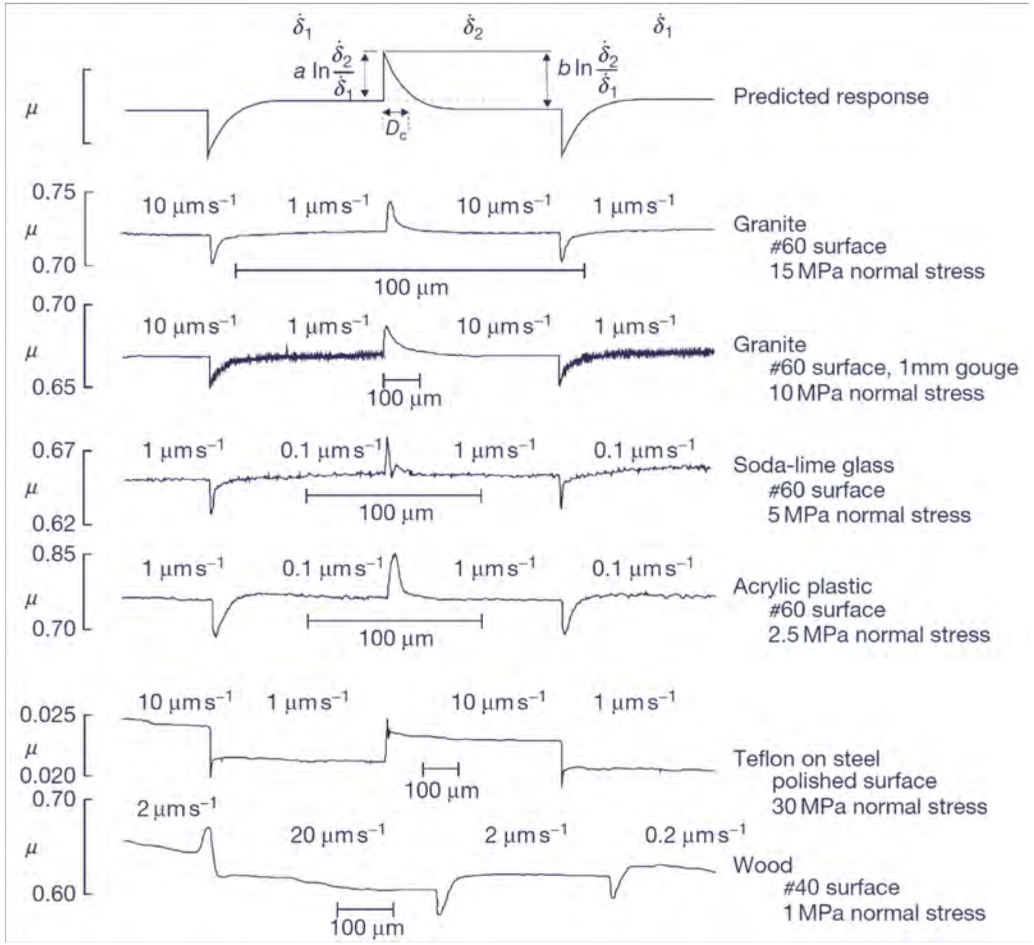


Figure 3. Rate-and-state model and experiments

Empirical frictional constitutive law (Dieterich 1979, Ruina 1983):

$$\tau = \tau_0 + A ln\left(\frac{V}{V_0}\right) + B ln\left(\frac{V_0\theta}{D_c}\right),$$

where $B = b\sigma$ and $A = a\sigma$ rate-and-state parameters, θ – state variable, D_c is the critical slip distance, and τ_0 and V_0 are reference stress and slip rate values.

Dieterich ageing (or slowness) law:

$$\dot{\theta} = 1 - \frac{V\theta}{D_c},$$

State variable evolves even when slip rate is 0.

Stability modelling

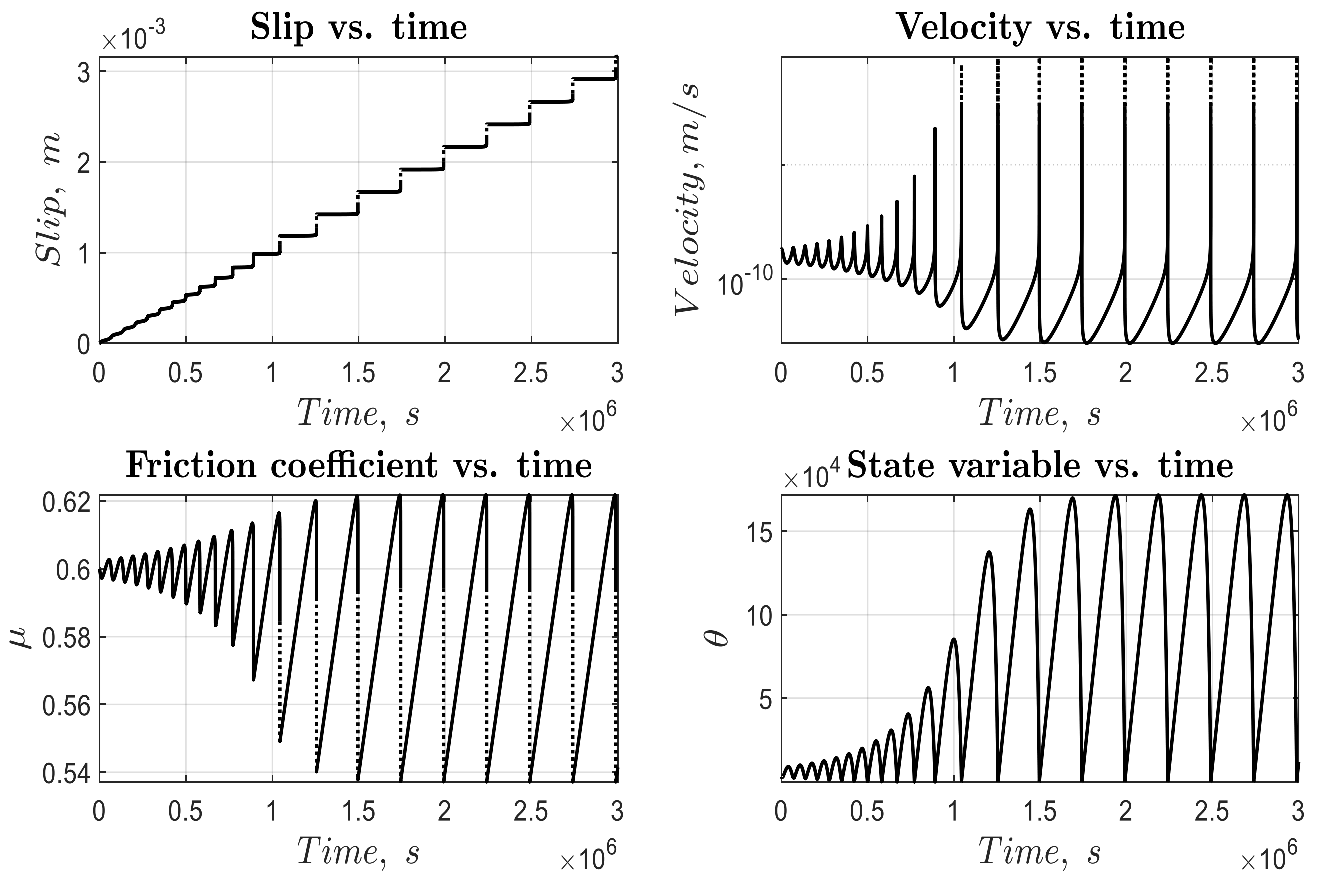
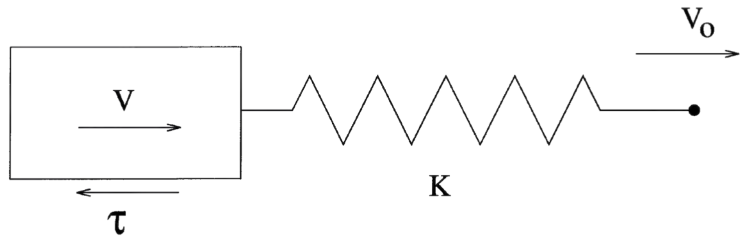


Figure 12. Periodic cycles



Introducing mass into the system (Erickson, Birnir and Lavalee 2008):

$$\dot{\theta} = -\left(\frac{V}{D_c}\right)\left(\theta + Bln\left(\frac{V}{V_0}\right)\right)$$

$$\dot{u} = V - V_0$$

$$\dot{v} = -\frac{1}{M}\left(Ku + \theta - Aln\left(\frac{V}{V_0}\right)\right)$$

Critical nucleation length (Dieterich 1992):

$$L_c = \frac{G\eta D_c}{\xi\sigma}$$

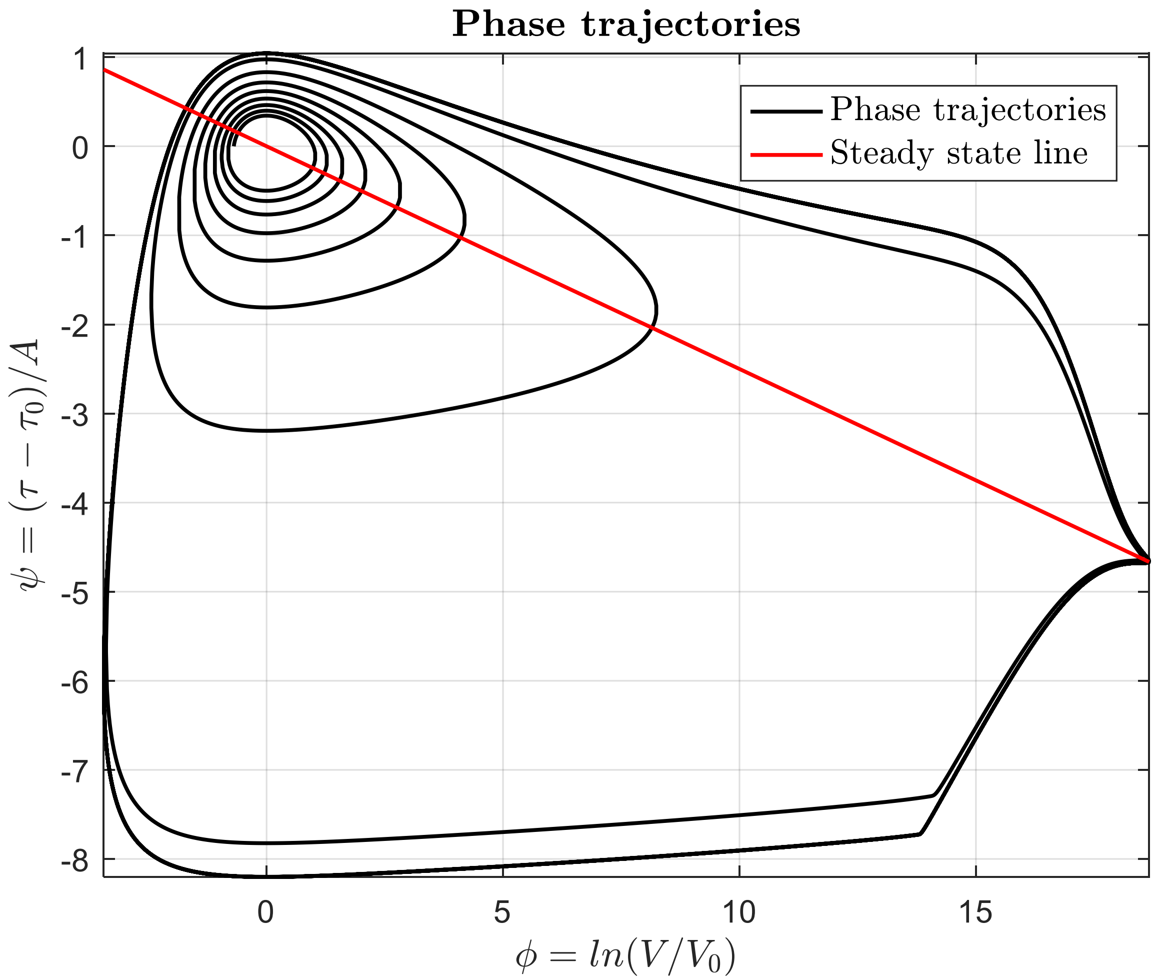
Steady state and stiffness

Steady state line with:

$V = V_0$ and $\tau = \tau_0 - (B - A)ln\left(\frac{V}{V_0}\right)$.

$$K_{crit} = \frac{\xi \sigma_n}{D_c} = \frac{(B - A)\sigma_n}{D_c}$$

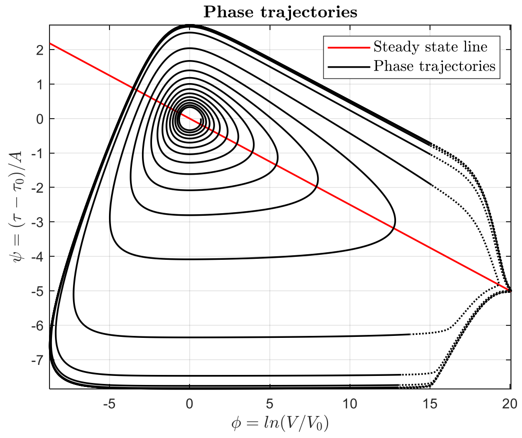
$$K = \frac{G\,\eta}{l}$$



Comparison of the two evolution laws

Dieterich ageing law:

$$\dot{\theta} = 1 - \frac{V\theta}{D_c}$$



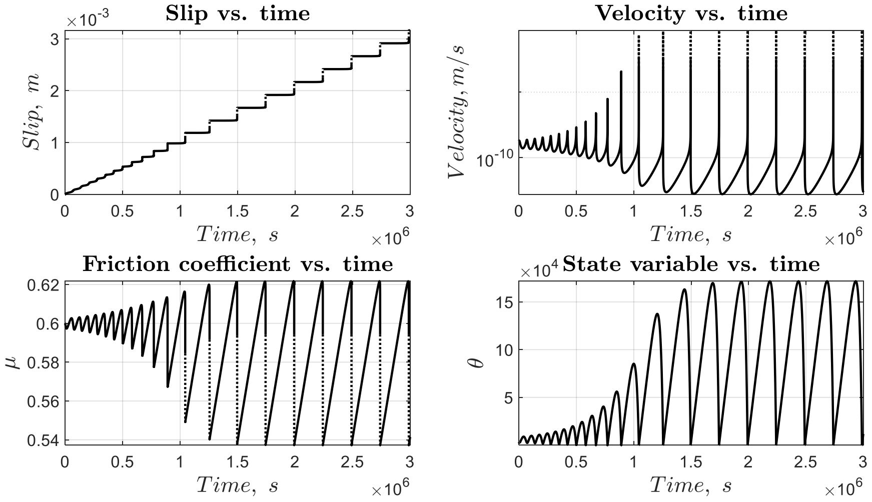
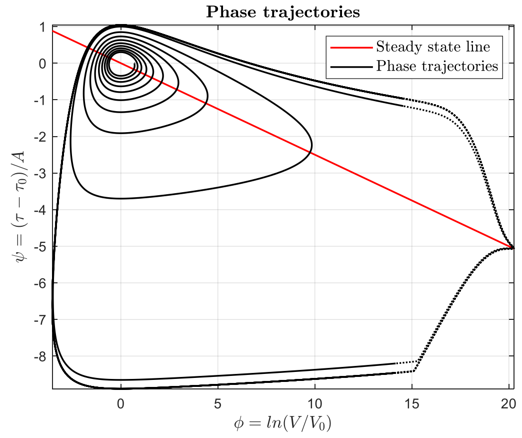


Figure 13. Phase diagram (top) and parameters vs. time for ageing law (bottom)

Ruina slip law:

$$\dot{\theta} = -\frac{V\theta}{D_c} ln\left(\frac{V\theta}{D_c}\right)$$



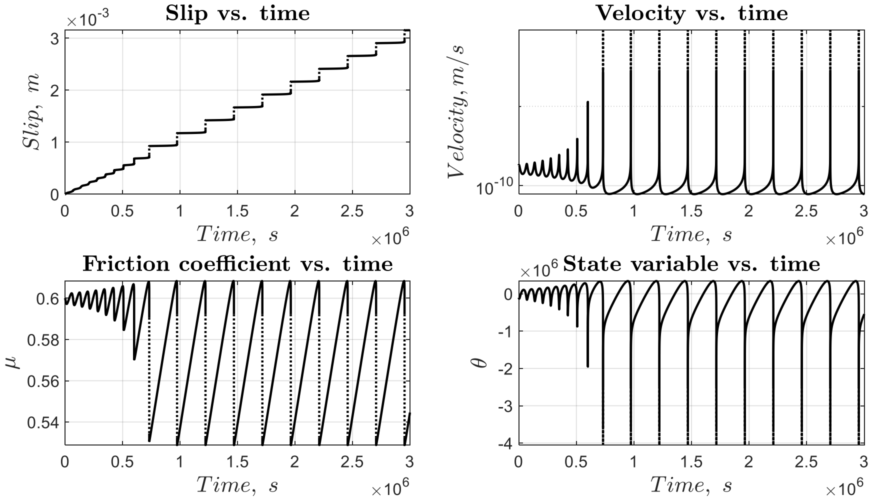


Figure 14. Phase diagram (top) and parameters vs. time for slip law (bottom)

θ mapping for Ruina

$$\dot{\theta} = -\frac{v}{d_c}\left(\theta + B \ln \frac{v}{v_*}\right)$$

$$\dot{\hat{\theta}} = -\frac{\hat{\theta} v}{d_c} \ln \frac{\hat{\theta} v}{d_c}$$

$$\theta = B \ln \frac{\hat{\theta}}{\hat{\theta}_*}, \quad \text{where} \quad \hat{\theta}_* = \frac{d_c}{v_*},$$

PyLith rate-and-state implementation

$$T_f = \begin{cases} T_c - \mu_f T_n & T_n \leq 0 \\ 0 & T_n > 0 \end{cases}$$

$$\mu_f = \begin{cases} \mu_0 + a \ln\left(\frac{V}{V_0}\right) + b \ln\left(\frac{V_0 \theta}{L}\right) & V \geq V_{linear} \\ \mu_0 + a \ln\left(\frac{V_{linear}}{V_0}\right) + b \ln\left(\frac{V_0 \theta}{L}\right) - a\left(1 - \frac{V}{V_{linear}}\right) & V < V_{linear} \end{cases}$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L}$$

To avoid significant variations in the coefficient of friction for slip rates on the same order as the residual tolerance we regularize the rate-and-state friction model by imposing a linearization of the variation of the coefficient of friction with slip rate when the slip rate drops below a cutoff slip rate:

$$\theta(t+\Delta t) = \theta(t) \exp\left(\frac{-V(t)\Delta t}{L}\right) + \frac{L}{V(t)}\left(1 - \exp\left(-\frac{V(t)\Delta t}{L}\right)\right)$$

First two terms of Taylor series:

$$\theta(t+\Delta t) = \begin{cases} \theta(t) \exp\left(-\frac{V(t)\Delta t}{L}\right) + \Delta t - \frac{1}{2}\frac{V(t)\Delta t^2}{L} & \frac{V(t)\Delta t}{L} < 0.00001 \\ \theta(t) \exp\left(-\frac{V(t)\Delta t}{L}\right) + \frac{L}{V(t)}\left(1 - \exp\left(-\frac{V(t)\Delta t}{L}\right)\right) & \frac{V(t)\Delta t}{L} \geq 0.00001 \end{cases}$$

PyLith rate-and-state

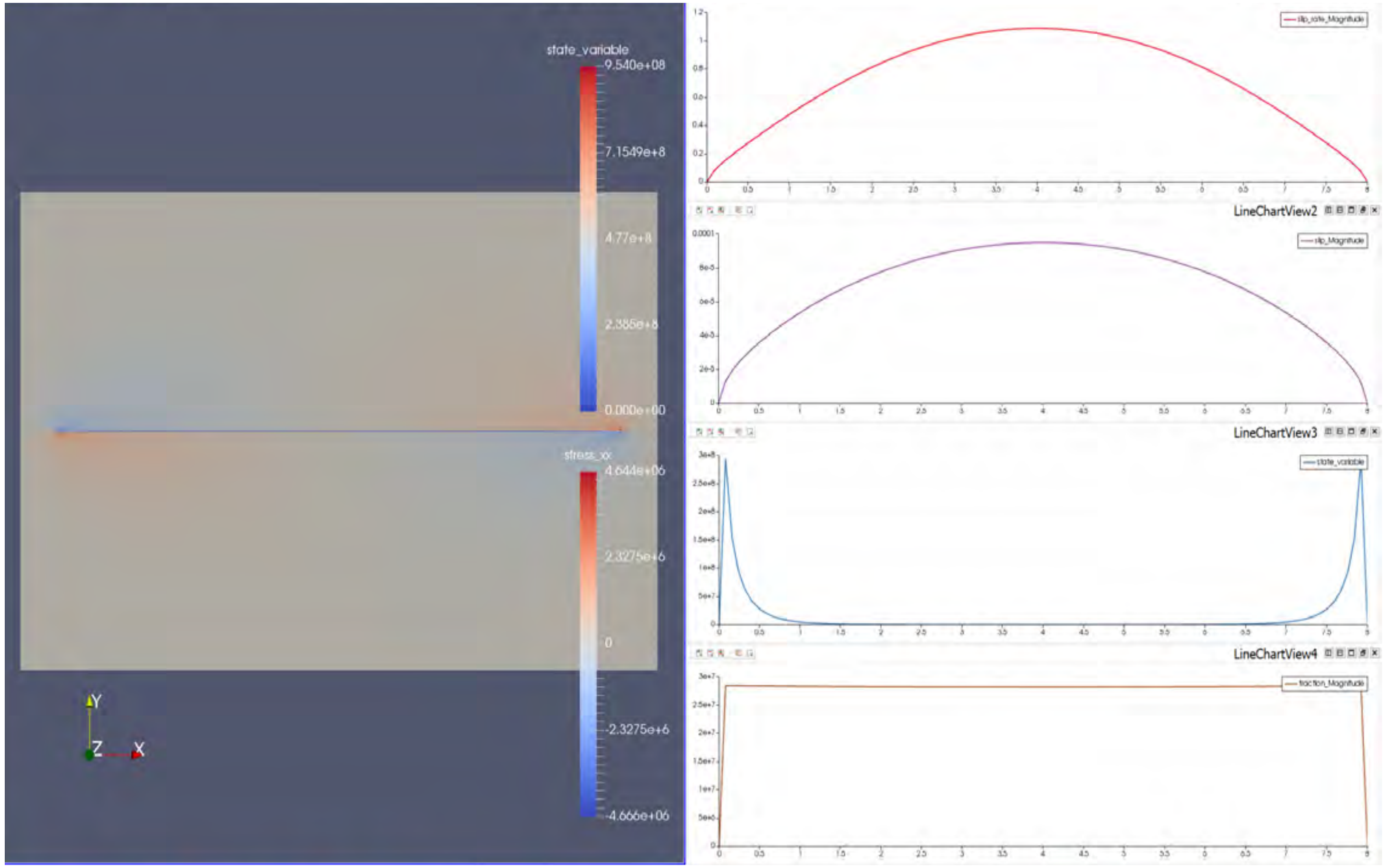


Figure 15. Slip rate, slip, state variable, traction along the fault

Parameters:

$D_c = 10^{-5} m$, $a = 0.008$, $b = 0.010$, $\sigma \sim 27\ MPa$, $\tau \sim 18\ MPa$ and reference friction coefficient $\mu_0 = 0.6$

Observations:

Slip rate up to 100 m/s, time to instability ~ 0.5 year

Issues:

Time-resolution

Comparison with Dieterich 1992

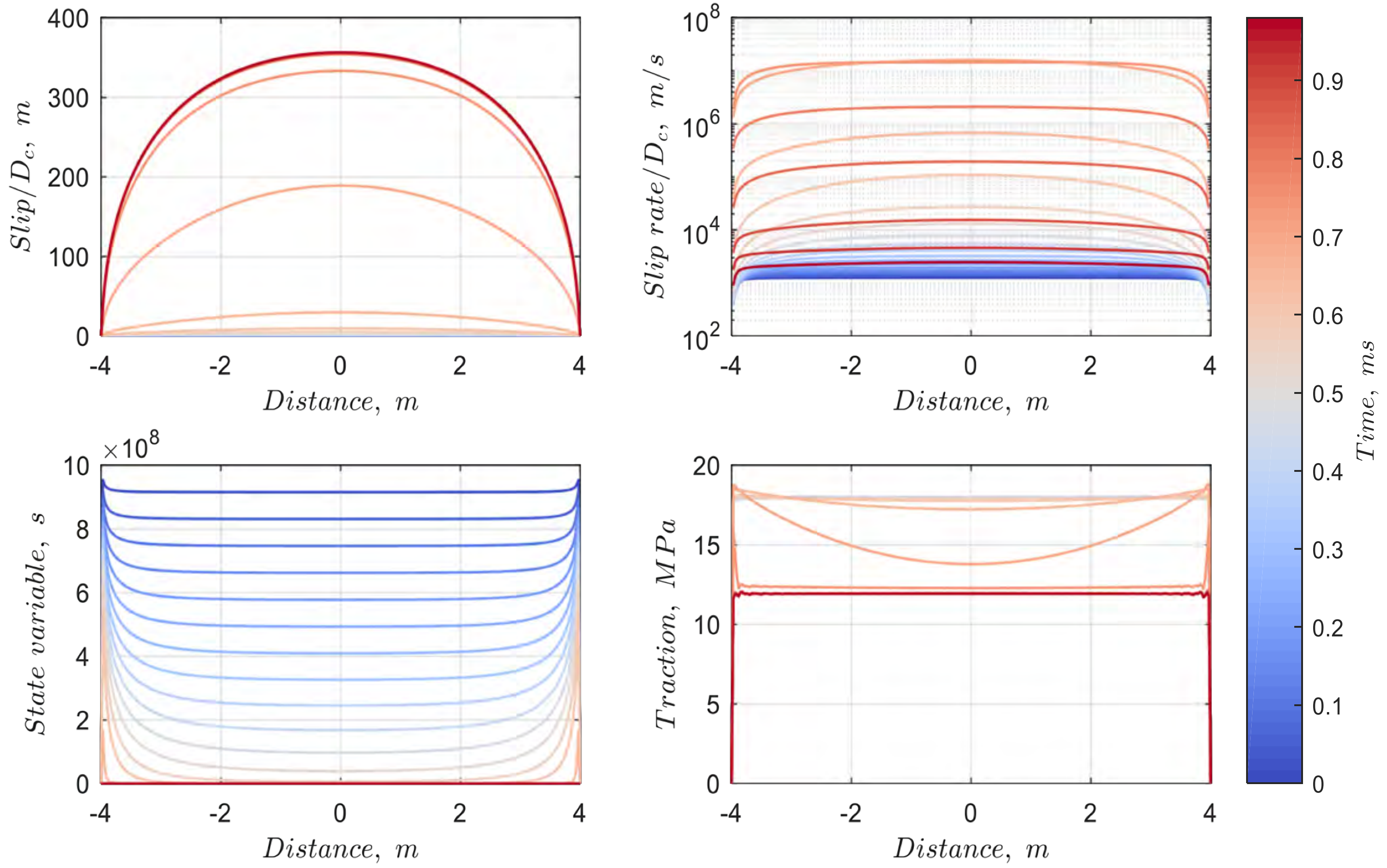


Figure 16. Time dependence for instability

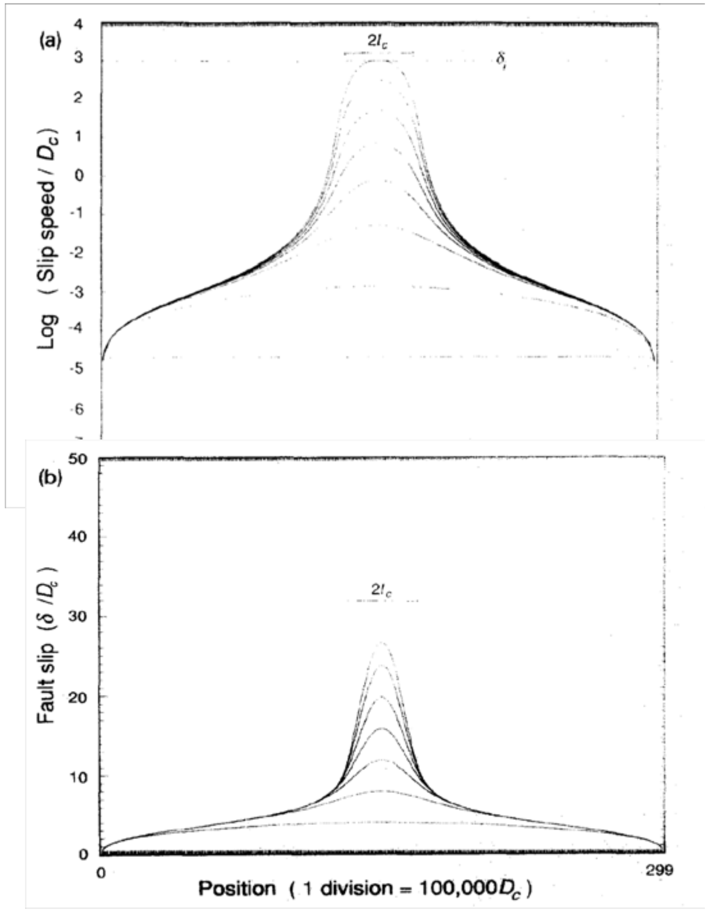
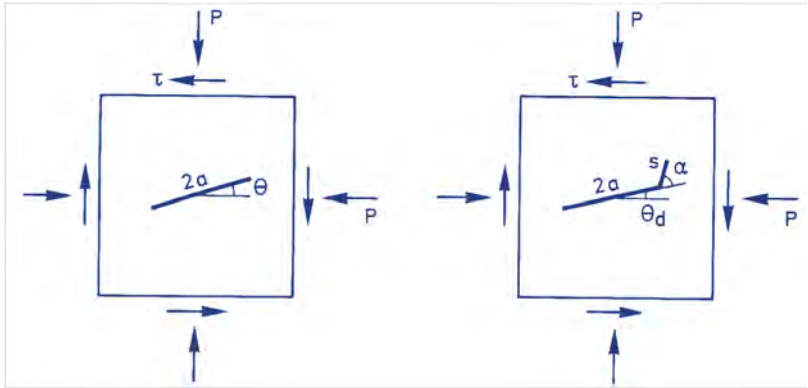


Figure 17. Slip and slip rate (Dieterich 1992)

Mode II stress intensity



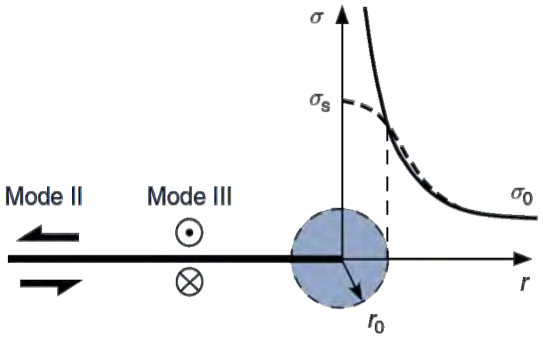
The maximum value of K_{II} appears for

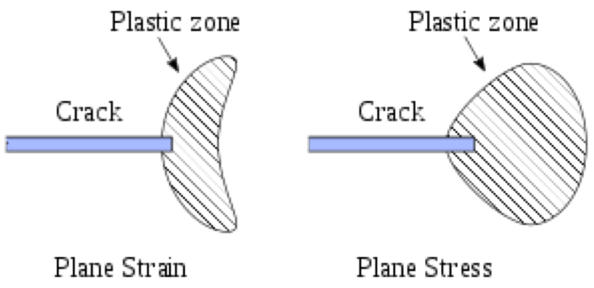
$$\theta = \theta_d = \begin{cases} 0 & \text{if } p/\tau \leqslant 0 \\ \frac{1}{2}\sin^{-1}(p/\tau) & \text{if } 0 < p/\tau \leqslant \mu/(1+\mu^2)^{1/2} \\ \frac{1}{2}\tan^{-1}\mu & \text{if } \mu/(1+\mu^2)^{1/2} < p/\tau < (1+\mu^2)^{1/2}/\mu . \end{cases} \quad (5)$$

A crack originally situated in this direction will grow in mode II without change of direction with the stress intensity factor [1]

$$K_{\text{IImax}}/(\pi a)^{1/2} = \begin{cases} \tau & \text{if } p/\tau \leqslant 0 \\ (\tau^2 - p^2)^{1/2} & \text{if } 0 < p/\tau \leqslant \mu/(1+\mu^2)^{1/2} \\ -\mu p + \tau(1+\mu^2)^{1/2} & \text{if } \mu/(1+\mu^2)^{1/2} < p/\tau < (1+\mu^2)^{1/2}/\mu \end{cases} \quad (6)$$

Crack tip plasticity





Plastic zone length (Irwin 1957):

Plastic zone length (plane stress):

$$2r_1 = \frac{1}{\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

In plane strain, increasing of σ_Y : Irwin suggested $\sqrt{3}\,\sigma_Y$ in place of σ_Y

$$2r_1 = \frac{1}{3\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

Stress Intensity Factor corresponding to the effective crack of length $a_{eff} = a + r_1$

$$K_I\left(\sigma_\infty, a + r_1\right) \equiv K_{eff} = \sigma_\infty\sqrt{\pi\left(a + r_1\right)} \quad \text{(effective SIF)}$$