Study of Induced Seismicity for Reservoir Characterization

by

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Abstract

The main goal of the thesis is to characterize the attributes of conventional and unconventional reservoirs through passive seismicity. The dissertation is comprised of the development and applications of three new methods, each of which focuses on a different aspect of fractures/faults and the resulting seismicity. In general, the thesis work discusses reservoir characterization from two aspects: 1) understanding fractures and faults in reservoirs as seismic sources with induced seismicity, and then inferring other properties of the reservoirs, such as stress regime and velocity structure (Chapters 2, 3, 4); 2) understanding the fractures in reservoirs as seismic scatterers (Chapter 5).

First, I introduce a new method to determine the source mechanisms of the induced earthquakes by incorporating high frequency waveform matching, first P-arrival polarities and average S/P amplitude ratios. The method is applied to 40 induced earthquakes from an oil/gas field in Oman monitored by a sparse near-surface seismic network and a deep borehole seismic network. The majority of the events have a strike direction parallel with the major NE-SW faults in the region, and some events trend parallel with the NW-SE conjugate faults. The results are consistent with the in-situ well breakout measurements and the current knowledge of the stress direction of this region. The source mechanisms of the studied events together with the hypocenter distribution indicate that the microearthquakes are caused by the reactivation of preexisting faults.

Then I introduce a new method to locate microseismic events induced by hydraulic fracturing with simultaneous anisotropic velocity inversion using differential arrival times and differential back azimuths. We derive analytical sensitivities for the elastic moduli ($C_{ij}$) and layer thickness $L$ for the anisotropic velocity inversion. The method is then applied to a microseismic dataset monitoring a Middle Bakken completion in the Beaver Lodge area of North Dakota. Our results show: 1) moderate-to-strong anisotropy exists in all studied sedimentary layers, especially in both the Upper Bakken and Lower Bakken shale formations, where the Thomsen parameters ($\epsilon$ and $\gamma$) can be over 40%; 2) all events selected for high signal-to-noise ratio and used for the joint velocity inversion are located in the Bakken and overlying Lodgepole formations, i.e., no strong events are located in the Three Forks formation below the Bakken; 3) more than half of the strong events are in two clusters at about 100 and 150 meters above the Middle Bakken. Reoccurrence of strong, closely clustered events suggests activation of natural fractures or faults in the Lodgepole formation.

Finally, I introduce a new hybrid method to model the shear (SH) wave scattering from arbitrarily shaped fractures embedded in a heterogeneous medium by coupling the boundary element method (BEM) and the finite difference method (FDM) in the frequency domain. The hybrid method can calculate scattering from arbitrarily shaped fractures very rapidly, thus Monte Carlo simulations for characterizing the statistics of fracture attributes can be performed efficiently. The advantages of the hybrid method are demonstrated by modeling waves scattered from tilted fractures embedded in complex media. Interesting behaviors of the scattered waves, such as frequency shift with the scattering order and coherent pattern of scattered waves through strong heterogeneities, are observed. This method can be used to
analyze and interpret the scattered coda waves in the microseismic observations, e.g., the reverberating multiples in the Bakken microseismic data which cannot be explained by the determined layered anisotropic velocity model alone.

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Chapter 1  Introduction

1.1  Motivation

Induced seismicity is a common phenomenon associated with human activities that change the crustal stress status. In conventional and unconventional oil/gas production, enhanced geothermal systems, mining, carbon sequestration, waste water disposal, impoundments of water reservoirs, etc, induced seismicity is often observed. With the advent of a large number of unconventional oil/gas operations, which utilize hydraulic fracturing to increase the fluid/gas flows in impermeable shale, induced seismicity has attracted increased attention and concerns both from the technologists and from the public (Hitzman et al., 2012).

In this thesis, I analyze the attributes of induced seismicity and use the information to characterize conventional and unconventional oil/gas reservoirs. First, I present a method to determine reliably the source mechanisms of induced earthquakes by using more information from full waveform seismograms. Then I present a method to locate accurately the induced microseismic events by improving relative locations among events and by simultaneously inverting for the velocity structure, in isotropic and anisotropic media. Finally, I present a hybrid method to model scattering of seismic energy from fractures and fractured zones. I characterize the fractures/faults from different aspects, as seismic sources and as seismic scatterers, and then infer important reservoir properties, such as the distribution of fractures and reactivation of faults, as well as the stress regime and velocity structure of reservoirs.

Most of the induced seismicity is of very small (moment magnitude $M_w$ ranging from -4 to -2) or small ($M_w$ ranging from -2 to 1) magnitude and is not felt by human beings.
Occasionally, critically stressed pre-existing faults with extensive area are reactivated and induced seismicity of moderate magnitude can occur ($M_w$ ranging from 1 to 4). Damage to production equipment or to local surface buildings may occur, and concerns from the public are drawn (Suckale, 2010; Warpinski et al., 2012).

On the other hand, seismic signals from induced seismicity also carry very valuable information of the crustal structure. By analyzing recorded seismic signals in detail, we can infer the trending and distribution of fractures and faults, as well as how formation discontinuities dislocate under the local stress regime due to stress perturbations in reservoirs (Rutledge et al., 1998; Eisner et al., 2010; Li et al., 2011). The relation between oil/gas production and potential seismic hazards can consequently be better understood. Also, seismic records from passive seismicity can help determine the properties of the reservoirs where active seismic surveys lack of penetration (e.g., Zhang et al., 2009). In small unconventional oil/gas operations, induced seismicity can also provide important structural information, including anisotropy, when active seismic surveys and well loggings are not conducted or can only provide incomplete information of velocities (Grechka et al., 2011; Li et al., 2012b). Moreover, mapping of the fractures through induced seismicity is a crucial constraint for designing fracturing jobs, providing parameters such as injection rate, fluid volume, and well spacing. (Maxwell et al., 2010).

For oil/gas production, especially in reservoirs with low or extremely low permeability, such as tight sandstones or shales, good understanding of the fracture network attributes is crucial since it relates closely to eventual production (Mayerhofer et al., 2008). Fractures act as
seismic sources when activated due to stress perturbation; in the meantime, they are also wave scatterers for the incident seismic waves. The incident waves on fractures not only originate from active sources such as dynamite or vibroseismic sources (Fang, 2013), but also can originate from other neighboring induced seismic events (Miksat & Müller, 2009; Warpinski, per. comm., 2012). Seismic waves propagating through fractured rock formations are affected by the mechanical properties of the fractures, thus the scattered waves from the fractures can be used to determine the fracture attributes (e.g., Willis et al., 2006), such as fracture orientation, compliance, spacing and dimension. Thus, fractures and even faults (Zhang et al., 2009) can also be characterized through scattered waves from active sources or induced seismicity. To understand the fractures as seismic scatterers, accurate theoretical and numerical descriptions of the scattered seismic waves are critical.

1.2 Occurrence of induced seismicity: fractures and faults as sources

1.2.1 Induced Seismicity in Conventional Oil/Gas Fields

The occurrence of induced seismicity in the conventional oil and gas fields has been noticed and studied for many decades. Induced seismicity is a common phenomenon in oil/gas reservoirs accompanying changes in internal stress due to water injection or water/oil/gas extraction. (Rutledge & Phillips, 2003; Suckale, 2010; Maxwell et al., 2010). For example, the gas/oil extraction can cause reservoir compaction and reactivate preexisting faults and induce microearthquakes (e.g., Chan & Zoback, 2007; Miyazawa et al., 2008; Sarkar et al., 2008), or injection of water causes the decrease of effective stress and slippage along preexisting faults.
(Grasso, 1992). Either in the conventional oil and gas fields or in other cases discussed in the following sections, induced seismicity caused by injection of fluid that increases the pore pressure or by extraction of fluid that decreases the pore pressure produces a combination of the following conditions (Hitzman et al., 2012):

1. significant change in net pore pressure in a reservoir;
2. activation of faults that are critically stressed and have favorable orientations;
3. create faults or fractures by brittle failure.

The reactivation of preexisting faults is very likely responsible for the sheared casings of production wells in some fields (Maury et al., 1992) or is a serious source of wellbore instability during drillings (Willson et al., 1998; Zoback & Zinke, 2002). In recent years, using induced seismicity to monitor reservoir dynamics has attracted increasing interest (Suckale, 2010). For instance, induced seismicity can help to characterize the reservoir compaction, which leads to well casing deformation (Kristiansen et al., 2000; Sarkar, 2008), and consequently provides valuable information for well constructions. Also, induced seismicity can help to map the drained fractures that are resituated with brine (Rutledge et al., 1998). In general, felt seismicity is uncommon compared to the large number of operating oil and gas fields worldwide, and the majority of the events are less than magnitude 4.0 (Hitzman et al., 2012).

1.2.2 Induced Seismicity in Unconventional Oil/Gas Fields

Unlike in the conventional reservoirs, induced seismicity in the unconventional reservoirs is usually considered beneficial for gas and oil production due to low or extremely
low matrix permeability. Hydraulic fracturing activities lead to fracture openings and closures (Baig & Urbancic, 2010), and also increase pore pressures, inducing slippage along preexisting fractures. New fractures can also be initiated by highly pressurized water (Warpinski et al., 2009). With the recent flourishing of shale gas/oil production in the U.S., the induced seismicity associated with hydraulic fracturing has drawn increasing public awareness and concerns and the associated safety issues have been carefully evaluated (Warpinski et al., 2012; Hitzman et al., 2012). Given their extremely low matrix permeability, tight sandstones and oil/gas shales require successfully engineered fracture networks that generate flow paths for economical oil and gas production. Usually, the microseismic events associated with the hydraulic fracturing are of very small or small magnitude (ranging from -4 to 1), and can only be observed by dedicated seismic networks onsite, preferably networks deployed at depth in boreholes near the injection wells for improved signal-to-noise ratio. Depending on the geology, the local stress regime, petrology, heterogeneities, and pumping rates, the hydraulically fractured networks can vary significantly from one place to another (e.g., Cipolla et al., 2010). Therefore, mapping of the fractures is a crucial constraint for designing fracturing jobs, controlling injection rate and fluid volume, directing horizontal well direction and spacing between wells, especially in areas where few jobs have been performed before (e.g., Rutledge & Phillips, 2003; Maxwell et al., 2010). Especially, induced seismicity mapping can help to detect loss and diversion of fluid by preexisting tectonic faults. Faults near the injection well or even at a distance are often reactivated due to increased pore pressures and generate seismic signals with different b-values compared to events related to fracture activation (Wessels et al., 2011).
Although induced seismicity accompanies most oil/gas production, the study of it, however, faces many difficulties. First and foremost, monitoring instruments are only deployed in a small fraction of all exploration and production operations due to deployment costs and to interference with production. Even in operations with deployment, the monitoring networks are usually very sparse for comprehensive monitoring of the seismicity; second, the signals of induced seismicity usually are weak and signal-to-noise ratio is not favorable for seismic analysis; third, even when there are adequate monitoring stations and events with satisfactory signal-to-noise ratio, oftentimes the lack of local structure and velocity information hinders the study of induced seismicity.

1.2.3 Induced Seismicity Related to Other Human-Activities

In Enhanced Geothermal Systems (EGS), pressurized fluid is used to hydraulically fracture hot dry regions at depth and then circulates back to the surface to deliver the captured heat for power generation and other uses, while sustaining the open fractures (Hitzman et al., 2012). The injection of pressurized fluid results in induced seismicity, which is sometimes deemed as hazardous by the communities near the EGS sites. Most induced seismicity in the EGS is of small magnitude. However, a few events larger than magnitude 4 have been recorded at The Geysers field in Northern California. To understand the induced seismicity at EGS, many studies have been conducted at various EGS sites, including The Geysers in the U.S., Cooper Basin in Australia, Berlín in El Salvador, Soultz-Sous-Forêts in France and Basel in Switzerland (Majer et al., 2007). Due to public concerns from the local communities on the induced
seismicity, supplemental injection and large-scale hydraulic stimulations have been banned at The Geysers and at Soultz-Sous-Forêts. The project in Basel was first suspended and was eventually canceled, immediately after two earthquakes larger than magnitude 3.0 occurred.

Mining induced seismicity was first reported in 1738 at South Stanford coalmine in England (Li et al., 2007). Excavation of large volumes of rock results in stress redistribution, leading to reactivation of preexisting fractures or faults due to increases in the shear stress or decreases in normal stress acting on the fault planes, or both. During the past decade, Australia, South Africa, China and Russia among some other countries have started monitoring induced seismicity related to mining for safety reasons. In South Africa, where the monitoring of mining-induced seismicity has been carried out for several decades, 2.5 million microseismic events are recorded per year by 1500 channels from 30 monitoring systems (Gibowicz, 2009). Mining in the 50 underground coalmines in Upper Silesian Basin in Poland, where the tectonic stress concentration is high, generated more than 56000 seismic events with local magnitude larger than 1.5 during the period of 1974 and 2005. In general, mining-induced seismicity has a multimodal distribution, i.e., there are two Gutenberg-Richter relations, one associated with geological features (moderate magnitudes) and the other associated with fracturing under very high concentrated stress ahead of the stope faces (small magnitudes).

Induced seismicity also occurs due to other activities, including waste water disposal by injection, impoundments of water reservoirs and carbon capture and sequestration (CCS). The waste water disposal usually involves injection at relatively low pressures into large areas of aquifers with high porosity (Hitzman et al., 2012). Most injections are unlikely to pose a hazard
by inducing seismicity, except for a very limited number of wells among the tens of thousands of disposal wells in the U.S. However, unlike in conventional or unconventional oil/gas exploration, EGS or mining, waste water wells are often drilled without a comprehensive geological survey of the surrounding formation structure. Thus, the vicinity of unidentified faults may pose unexpected risks.

Reservoir-induced seismicity has been observed at over seventy different reservoirs worldwide since the causal relation between the impoundment of Lake Mead and seismicity was established in the early 1940s. Most reservoir-induced seismicity follows the impoundment, large lake-level changes, or water filling above the historic lake-level (Talwani, 1997; Gupta, 2002). The induced seismic events are often observed both beneath the deepest part of the reservoirs and in the surrounding areas, and are considered to be highly related to preexisting faults, activated by the water loading and pore pressure diffusion. Usually, the delay between the beginning of filling and the larger events varies from months to years, depending on the reservoir and local geology.

The CCS projects, which usually involve injection of a large volume of fluid of a long period, inevitably deform the injected reservoir, and may cause significant induced microseismicity. However, few CCS projects with limited injection volume have been conducted, thus the potential for induced seismic hazard is still to be monitored (Hitzman et al., 2012). It should be noted that enhanced oil recovery with CO\textsubscript{2} injection is also considered as one approach of CCS. As of 2007, over 600 million tons of CO\textsubscript{2} have been injected in about 13000 wells in the U.S.
1.3 Characterization of Fractures as Scatterers

Fractures and faults can be considered as seismic sources, of which the locations and rupturing process are analyzed. In comparison, in the study of fractures with scattered waves, the dynamic responses of the fractures to external excitations are considered (e.g., Grandi Karam, 2008). The characterization of fractures though active seismic surveys is not only applicable to oil/gas reservoirs, but also applicable to geothermal reservoirs, where the fracture networks are crucial for the efficiency of heat extraction, and applicable to CO₂ sequestration, waste water disposal, etc., where fracture networks may hinder the containment of the injected fluid (Hitzman et al., 2012).

In the past, the characterization of subsurface fracture networks was difficult due to the lack of direct observation. Oftentimes, a reservoir was first developed before the role of the fracture network was realized by observing the discrepancies between the observed and expected productions (Bansal, 2007). Fractures can be characterized by core samples, but the integrity of the samples can often be compromised. Logging tools can also be used to characterize fractures, but only in the vicinity of the boreholes. In addition to these two approaches, fractures can be characterized by active seismic surveys without the compromises and limitations of core sampling and well logging (e.g., Shen et al., 2002; Hall and Kendall, 2003). Since the seismic waves propagating through fractured rock formations are affected by the mechanical properties of the fractures, the scattered waves from the fractures can be used to determine the fracture properties, e.g., the fracture orientation, compliance, spacing and dimension, etc. If the fracture dimension and spacing are small relative to the seismic wavelength, the scattered waves from the top and bottom of the fracture zone display
amplitude variations with offset and azimuth (AVOA). In contrast, if the fracture dimension and spacing are close to the seismic wavelength, the scattered waves exhibit a complex, reverberating pattern with multiple scattering in the seismic codas (Willis et al., 2006). Nevertheless, important information of the fractures is carried by the scattered waves in both cases.

Fractures, as scatterers of seismic energy from nearby induced events, can also be characterized through the coda waves in the induced seismicity records. For instance, Miksat & Müller (2009) found that the seismogram envelopes of induced seismicity can be related to the fractal properties of the fracture network; Warpinski (per. comm., 2012) found the seismogram amplitudes of induced seismicity vary considerably with direction, i.e., the amplitudes can be much smaller if waves propagate through the hydraulically fractured zones than if they do not.

To understand and interpret the observed coda waves from fractures, accurate numerical modeling of the scattering phenomenon is crucial. The embedded fractures are usually modeled as equivalent anisotropic media (e.g., Coates & Schoenberg, 1995; Willis et al., 2006), and the finite-difference method (FDM) is often used to compute the scattered waves. If the fractures are planar and aligned with the discretization grids, the equivalent anisotropy can be handled conveniently by standard staggered grid FDM (Lavender, 1988). However, if the fractures are non-planar or not aligned with the discretization grids, the equivalent anisotropy becomes monoclinic or beyond and cannot be handled by staggered grid FDM without compromise in accuracy and dramatic increase in computational cost. Rotated staggered grid FDM (Saenger et al., 2000) can handle complex equivalent anisotropy, but this method comes
with numerical noises that can overshadow the weak scattering from fractures (Fang, 2013). Therefore, to find a method that can accurately model the scattering from fractures without these limitations and compromises is important to further our understanding of the fractured reservoirs.

1.4 Importance of the Studies

My thesis focuses on the characterization of reservoirs by understanding fractures and faults from two aspects: 1) understanding the fractures and faults in reservoirs as seismic sources with induced seismicity, and then inferring other properties of the reservoirs (Chapters 2, 3, 4); 2) evaluating the fractures in reservoirs as seismic scatterers (Chapter 5).

In the first area, my research focuses on the determination of the source locations and mechanisms of the induced seismicity, as well as on the determination of the anisotropic velocity model of the reservoir with the induced events. In conventional and unconventional reservoirs, fault and fracture systems can be identified via the induced seismicity. The response of the reservoir to fluid/gas extraction or water injection under the local in-situ stress regime can be determined from the mapping of the induced events and the analysis of their source mechanisms (Rutledge et al, 1998; Maxwell et al., 2010; Li et al., 2011; Song & Toksöz, 2011). In unconventional reservoirs, induced seismicity can characterize the fracture growth and determine the Stimulated Reservoir Volume (SRV), which is the decisive factor for the production in unconventional reservoirs and a critical criterion for evaluating hydraulic fracturing jobs (Mayerhofer et al., 2008). In hydraulic fracturing operations, induced seismicity
mapping can provide dynamic and real-time feedbacks for the optimization of the hydraulic fracturing job and is invaluable for the eventual oil/gas productions (Liu et al., 2009). For public safety, induced seismicity can be used to check whether fracture networks grow into aquifers and contaminate near surface water sources (Hitzman et al., 2012; Warpinski et al., 2012).

It should be emphasized that successful event location and attribute analysis usually need an accurate velocity model. In shale gas/oil operations, the velocity models can be calibrated by perforation shots, string shots, etc. (Maxwell et al., 2010), but these calibrations are only valid when the subsequent hydraulic-fracturing events occur in the vicinity. This, however, is often not the case since the hydraulic pressure can propagate, e.g., through rock deformation and water conduits, and then initiate events at a distance (e.g., Warpinski et al., 2008; O’Brien et al., 2011). Also, additional complexity caused by strong rock anisotropy render the calibrated velocity models even less accurate for the induced events (Warpinski et al., 2009). Without an accurate velocity model, the induced seismicity cannot be correctly mapped and the SRV cannot be properly determined (Li et al., 2012a). Additionally, an accurate velocity model also helps to better understand the reservoir and plan for future drilling. Therefore, using the passive induced seismicity to build a velocity model that accurately reflects the structures between the events and the receivers is of great importance.

In the second area, my research focuses on the accurate modeling of scattered waves from fractures with arbitrary shapes in heterogeneous reservoirs by a new hybrid modeling method. Accurate modeling of the scattered waves can provide a solid base for interpreting the observed data, and is essential for the understanding of reservoir fracture networks from
scattered waves. This is because in conventional reservoirs most fractures contributing to the reservoir permeability are not (re)activated by production to generate seismic signals. This is also the case in unconventional reservoirs when they are hydraulically fractured. Moreover, aseismic creep accounts for a significant percentage of all hydraulically activated fractures in unconventional reservoirs, especially in ductile shales with more clay contents (Zoback, 2012). To characterize fractures in these cases, active seismic waves can be sent to the fracture zones and the resulting scattered waves are recorded for fracture attribute analysis (e.g., Fang, 2013). Also, seismic waves from an induced event are also scattered by neighboring fractures, thus interpretations of the fracture networks can benefit from understanding the scattered coda signals. Fractures in heterogeneous reservoirs, however, are often irregularly shaped and have complicated boundary conditions (Chen et al., 2011), thus the scattered waves are difficult to model with any numerical scheme alone. Our hybrid method takes advantages of both the finite-difference method and the boundary-element method and avoids the shortcomings of both methods, providing a new tool for understanding the scattered waves from complex fractures in heterogeneous reservoirs.

1.5 Thesis Outline

In chapter 2, I introduce a new method to determine the source mechanisms of the induced earthquakes by incorporating high frequency waveform matching, first P-arrival polarities and average S/P amplitude ratios. An objective function is constructed to include all criteria. The method is applied to induced earthquakes from an oil/gas field in Oman monitored
by a sparse near-surface shallow seismic network and a deep borehole seismic network. Source mechanisms of 40 events monitored by both seismic networks are determined. The majority of the events have a strike direction parallel with the major NE-SW faults in the region, and some events trend parallel with the NW-SE conjugate faults. The results are consistent with the in-situ well breakout measurements and the current knowledge of the stress direction of this region. The source mechanisms of the studied events together with the hypocenter distribution indicate that the microearthquakes are caused by the reactivation of preexisting faults. Chapter 2 has been published in *Geophysical Journal International* and in *Geophysics*;

In chapter 3, I present a new method to locate microseismic events induced by hydraulic fracturing with simultaneous anisotropic velocity inversion by using differential arrival times and differential back azimuths. The velocity inversion is constrained to a 1-D layered VTI structure to improve inversion stability given the limited passive seismic data. We derive analytical sensitivities for the elastic moduli \(C_{ij}\) and layer thickness \(L\) for the anisotropic velocity inversion. In the two tests with synthetic data, our method provides more accurate relative locations than the traditional methods, which only use absolute information. With fast speed and high accuracy, our inversion scheme is suitable for real-time microseismic monitoring of hydraulic fracturing. This chapter lays the theoretical bases for the application to the field microseismic data in chapter 4. It has been submitted to *Geophysical Journal International* for publication.

In chapter 4, following the theoretical derivations in the previous chapter, the method is applied to a microseismic dataset monitoring a Middle Bakken completion in the Beaver Lodge
area of North Dakota. Our results show: 1) moderate-to-strong anisotropy exists in all studied sedimentary layers, especially in both the Upper Bakken and Lower Bakken shale formations, where the Thomsen parameters ($\epsilon$ and $\gamma$) can be over 40%; 2) all events selected for high signal-to-noise ratio and used for the joint velocity inversion are located in the Bakken and overlying Lodgepole formations, i.e., no strong events are located in the Three Forks formation below the Bakken; 3) more than half of the strong events are in two clusters at about 100 and 150 meters above the Middle Bakken. Reoccurrence of strong, closely clustered events suggests activation of natural fractures or faults in the Lodgepole formation. Using an accurate anisotropic velocity model is important to correctly assess height growth of the hydraulically induced fractures in the Middle Bakken. This chapter is to be submitted to *Geophysics* for publication soon.

In chapter 5, I introduce a hybrid method to model the shear (SH) wave scattering from arbitrarily shaped fractures embedded in a heterogeneous medium by coupling the boundary element method (BEM) and the finite difference method (FDM) in the frequency domain. FDM is used to propagate SH waves from the source through heterogeneities to fractures embedded in small local homogeneous domains surrounded by artificial boundaries. According to Huygens’ Principle, the points at these artificial boundaries can be regarded as ‘secondary’ sources and their amplitudes are calculated by FDM. A numerical iterative scheme is also developed to account for the multiple scattering between different sets of fractures. The hybrid method can calculate scattering from different fractures very fast, thus Monte Carlo simulations for characterizing the statistics of fracture attributes can be performed efficiently. The advantages of the hybrid method are demonstrated by modeling waves scattered from tilted fractures embedded in complex media. Interesting behaviors of the scattered waves such as frequency
shift with the scattering order and coherent patterns through strong heterogeneities are observed.

Chapter 6 gives the conclusion of my research.
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Chapter 2  Focal Mechanism Determination Using High Frequency Waveform Matching and Its Application to Small Magnitude Induced Earthquakes in a conventional reservoir

Summary

A new, relatively high frequency, full waveform matching method is used to study the focal mechanisms of small, local earthquakes induced in an oil field, which are monitored by a sparse near-surface network and a deep borehole network. The determined source properties are helpful for understanding the local stress regime in this field. During the waveform inversion, we maximize both the phase and amplitude matching between the observed and modeled waveforms. We also use the polarities of the first P-wave arrivals and the average S/P amplitude ratios to better constrain the matching. An objective function is constructed to include all four criteria. An optimized grid search method is used to search over all possible ranges of source parameters (strike, dip and rake). To speed up the algorithm, a library of Green’s functions is pre-calculated for each of the moment tensor components and possible earthquake locations. Optimizations in filtering and cross-correlation are performed to further speed the grid search algorithm. For different hypocenters and source types, comprehensive synthetic tests show that our method is robust to determine the focal mechanisms under the current array geometries, even when there is considerable velocity inaccuracy. The application to several tens of induced microseismic events showed satisfactory waveform matching between modeled and observed seismograms. The majority of the events have a strike direction parallel with the major NE-SW faults in the region, and some events trend parallel
with the NW-SE conjugate faults. The results are consistent with the *in-situ* well breakout measurements and the current knowledge on the stress direction of this region. The source mechanisms of the studied events together with the hypocenter distribution indicate that the microearthquakes are caused by the reactivation of preexisting faults. We observed that the faulting mechanism varies with depth, from strike-slip dominance at shallower depth to normal faulting dominance at greater depth.

### 2.1 Introduction

Induced seismicity is a common phenomenon in oil/gas reservoirs accompanying changes in internal stress due to water injection or water/oil/gas extraction, etc. (e.g., Suckale, 2010; Maxwell et al., 2010). For example, the gas/oil extraction can cause reservoir compaction and reactivate preexisting faults and induce microearthquakes (e.g., Chan & Zoback, 2007; Miyazawa et al., 2008; Sarkar et al., 2008), or injection of water can cause the decrease of effective stress and slippage along preexisting faults (Grasso, 1992). The reactivation of preexisting faults is very likely responsible for the sheared casings of production wells in some fields (Maury et al., 1992) or is a serious source of wellbore instability during drillings (Willson et al., 1998; Zoback & Zinke, 2002). Also, the hydraulic fracturing activities in an enhanced geothermal system or in shale gas extraction can result in crack openings and closures and induce microseismicity (Baig & Urbancic, 2010). Through the studying of locations and source characteristics (e.g., focal mechanism) of the induced seismicity over an extended time period, temporal and spatial changes of the stress in the fields may be reconstructed; this can help to understand the intrinsic response of geological formations to the stress disturbance.
Microearthquakes usually have small magnitudes and are generally recorded at sparse local stations. As a result, it is difficult to obtain enough seismic waveforms with high signal to noise ratio for picking the polarity information of first P-wave arrivals. Therefore, it is challenging to use only the P-wave polarity information (even when adding S/P amplitude ratios) as used in conventional methods to constrain the focal mechanisms of the induced earthquakes (e.g., Hardebeck & Shearer, 2002, 2003), especially when there are only a limited number of stations. Waveform matching has been used to determine earthquake focal mechanisms on a regional and global scale using low frequency waveform information (e.g., šílený et al., 1992; Zhao & Helmberger, 1994; Tan & Helmberger, 2007). šílený et al. used waveform matching to determine the best-fit focal mechanism, source time function and source depth. Zhao & Helmberger (1994) allowed time-shift in the synthetic seismograms to account for the imperfect Green’s functions when matching the synthetic with observed seismograms. Tan & Helmberger (2007) matched the direct P-arrival phases (the first one cycle after initial P-arrival) between synthetic and observed seismograms to determine the focal mechanisms. However, in the case of induced seismicity, waveforms usually have higher frequencies. There have been many studies on determining the focal mechanism of the induced seismicity in the cases of enhanced geothermal system development, mining and hydraulic fracturing. Godano et al. (2011) used the direct amplitudes of P, SV and SH to study the focal mechanisms of induced microearthquakes in a geothermal site using full-space homogeneous velocity models. Nolen-Hoeksema et al. (2001) used the first half cycle after the first arrivals from the observed seismograms and synthetics from full-space Green’s functions to determine the focal mechanisms of several hydraulic fracture events. Julian et al. (2007) used first arrival polarities
and amplitude ratios from 16 three-component borehole stations and 14 three-component surface stations to determine the full moment tensors of the induced events and studied the volume change accompanying the geothermal process. High frequency waveform matching, in addition to polarity information, has been used to determine the focal mechanisms of induced earthquakes in a mine with a dense network of 20 stations (Julià et al., 2009). Julià et al. used a full-space homogeneous model to calculate the Green’s functions, and they performed the focal mechanism inversion in the frequency domain without phase information in a least square sense between the synthetic and filtered observed data generally below 10 Hz. The simplification to full-space homogeneous model is valid when the receivers are deployed deep in the subsurface and close to the induced events, such as deploying borehole monitoring sensors in the vicinity of the hydraulic well, or when complexities in rock structure are not large compared to the frequencies recorded.

To retrieve reliable solutions, we developed a method to use high frequency, full waveform information (both P and S) to determine the focal mechanisms of small earthquakes (Li et al., 2011). Using the known velocity model (one-dimensional layered model in this study), we calculate the Green’s functions for all moment tensor components of the source at each location (hypocenter) and then the synthetic seismograms by convolving them with the source time function. To find the best match between the observed and synthetic seismograms, we formulate an objective function that incorporates information from different attributes in the waveforms: the cross correlation values between the modeled waveforms and the data, the $L_2$ norms of the waveform differences, the polarities of the first P arrivals and the S/P average amplitude ratios. Compared to previous studies, our method uses more attributes of
seismograms to better determine the focal mechanisms of induced seismicity. The “high frequency” referred to in our study (several Hertz for the shallow network and tens of Hertz for the deep network) is a relative term: it is much higher than the frequency band (0.05-0.5 Hz) often used in the study of large earthquakes (e.g., Tan & Helmberger, 2007), but it is lower than the frequency band often used for exploration seismic imaging (e.g., Etgen et al., 2009). Essentially, the frequency bands used in our study include a considerable portion of the energy radiated from the source, thus the waveforms have good signal-to-noise ratio (SNR) and can reflect the characterizations of the source rupture.

Compared with full waveform tomography or migration techniques, which focus on improving the knowledge of the subsurface structures illuminated by simple active sources with known signatures (e.g., explosion or vibration source with known location and origin time; similar frequency, amplitude, radiation pattern etc. are expected for all shots), the source mechanism determination method assumes the velocity model input, and focuses on determining the complicated source signature associated with the events. For induced seismicity in oil and gas fields, the velocity model is generally known from seismics and well logs. Also, comprehensive synthetic tests with random velocity perturbations are performed to examine the robustness of our algorithm in the presence of the velocity uncertainties.

Previously, we tested our newly developed focal mechanism determination method on induced microearthquakes monitored by a five-station surface network at an oil field in Oman (Li et al., 2011). The field, operated by Petroleum Development Oman (PDO), was discovered in 1962 and put into production in 1969. An official program to monitor induced seismicity using a
surface station network in the field commenced in 1999, and a borehole network was installed in February of 2002. The primary objective of this passive seismicity monitoring program was to locate the events and to correlate them with production and injection activities in order to understand and monitor the cause of induced seismicity in the field. In this paper, we applied the newly developed focal mechanism determination method to data from the borehole network. The source mechanisms determined using the borehole network are compared to those determined using the surface network. The robustness of the method is tested extensively on synthetic datasets generated for both the surface and borehole networks using a randomly perturbed velocity model.

2.2 Induced Microearthquake Dataset

The petroleum field discussed in this paper is a large anticline created by deep-seated salt movement (Sarkar, 2008). The dome is about $15 \times 20$ km in size with a northeast-southwest axial elongation that is probably a result of regional deformation. The structure is dominated by a major central graben and two systems of faulting with two preferred directions (southeast-northwest and northeast-southwest) that affect the trapping mechanism in the oil reservoir. The northeast-southwest major network of faults and fractures partially connects all parts of the fields together (Figure 2-1 and Figure 2-2). The main oil production is from the Lower Cretaceous Shuaiba chalk overlain unconformably by Nahr Umr shale, while gas is produced from the shallower Natih Formation overlain by the Fiqa shale Formation (Sarkar, 2008; Zhang et al., 2009).
Since 1996, increasing seismic activity has been reported by the staff working in the field. Significant surface subsidence in the center of the field has also been observed by InSAR, GPS and leveling surveys, and has been attributed to compaction of the Natih formation (Bourne et al., 2006). To monitor the induced seismicity in the field, PDO first deployed a surface array of monitoring stations in 1999 (Figure 2-1). The stations are instrumented with SM-6B geophones with a natural frequency \( f_n \) of 4.5 Hz. In 2002, another network, independent of the shallow network, was installed in the field as part of a Shell/PDO collaborative study (Figure 2-2). Unlike the surface array/shallow network, this network had borehole installations of seismic sensors (SM-7m, \( f_n = 30 \) Hz) at multiple levels, roughly ranging from depths 750 m – 1250 m. The instrumentation for this network was much deeper than that of the surface network and, therefore, this monitoring network is referred to as the “borehole network.” A schematic diagram of the wells and sensor positions is shown in Figure 2-2. The borehole network consisted of 5 closely spaced monitoring wells in the most seismically active part of the reservoir and covered a much smaller area than the surface network. Due to sensor positions at depths, the ability to acquire data at much higher frequencies and the proximity to the two producing units (Natih gas and Shuaiba oil), the deep network recorded much smaller magnitude events than the shallow network, resulting in a greatly increased detectability of induced seismicity (roughly about 25 times more induced events per day) compared to the shallow network. The borehole network was operational for about 18 months starting in February 2002; however, only microseismic data from the last 11 months (October 2002 – August 2003) were available for this study. During that 11 month monitoring period, about 15,800 events were identified with an average rate of \(~ 47/\text{day}\), out of which we analyzed and
located about 5,400 events (Sarkar, 2008). Attempts were made to select common events detected during this period by both (deep and shallow) networks for a joint location analysis, however, due to clock synchronization problems and difference in sensor frequency bands between the two networks, the common events could not be identified, and hence the task could not be accomplished. Some research indicated that by carefully identifying the largest events in different networks, synchronization between networks sometimes can be achieved by shifting the origin times in one network with a constant time (Eisner et al., 2010). A similar strategy will be adopted in the future.

During the period of 1999 to 2007, over 1500 induced earthquakes were recorded by the surface network, and their occurrence frequency was found to be correlated with the amount of gas production (Sarkar, 2008). The distribution of induced events in the field recorded by the surface network is shown in Figure 2-1 (Sarkar, 2008; Sarkar et al., 2008; Zhang et al., 2009). All the events have a residual travel time of less than 30 ms, indicating they are well located. Figure 2-2 shows the microearthquake locations determined using the deep borehole network and the double-difference tomography method (Zhang et al., 2009). The root-mean-square travel time residual is around 10 ms (Zhang et al., 2009). In the map view, the earthquakes can be found mainly distributed along the mapped two NE-SW fault systems. This earthquake distribution suggests that most of the earthquakes are induced by the reactivation of the existing faults in the field. Figure 2-3 and Figure 2-4 show typical events and their spectrograms recorded by the surface network and borehole network, respectively. Because of the proximity of the earthquake source to the deep borehole network, the frequency content of the recorded waveform by the borehole network is much higher than by
the surface network. For the waveforms recorded by the surface network, there is a considerable amount of energy in the frequency range of 3 to 9 Hz (Figure 2-3). For the deep borehole network, the recorded waveforms contain significant energy between 15 to 35 Hz (Figure 2-4).

2.3  Focal Mechanism Determination Method

The focal mechanism can be represented by a 3 by 3 second order moment tensor with six independent components (Aki & Richards, 2003). Here we assume the focal mechanism of the small induced events can be represented by pure double couples (Rutledge & Phillips, 2002), though it is possible that a volume change or Compensated Linear Vector Dipoles (CLVD) part may also exist, especially in hydraulic fracturing cases, and the non-double-couple components are informative for understanding the rock failure under high-pressure fluid (Ross & Foulger, 1996; Jechumtálová & Eisner, 2008; Šílený et al., 2009; Song & Toksoz, 2010). The constraining of focal mechanism as double couple (DC) can eliminate the spurious non-DC components in the inversion raised by modeling the wave propagation in anisotropic medium with isotropic Green’s functions or inaccuracy of the velocity model (Šílený & Vavryčuk, 2002; Godano et al., 2011). However, if strong non-DC components actually exist in the source rupture process, the determined fault plane may be biased (e.g., Jechumtálová & Šílený, 2001; Jechumtálová & Šílený, 2005). In our analysis, we describe the DC focal mechanism of seismic source in terms of its strike (Φ), dip (δ) and rake (λ), and determine double couple components from these three parameters. The simplification of the source is supported by the observation that almost all the detected microearthquakes occurred along preexisting faults, i.e., reactivated faults slipping
along preexisting weak zones would not cause significant volumetric or CLVD components (Julian et al., 1998). For each component of a moment tensor, we use the Discrete Wavenumber Method (DWN) (Bouchon, 1981, 2003) to calculate its Green’s functions $G^n_{ij,k}(t)$ for the horizontally layered medium. The appendix gives the modified reflectivity matrix for computing the seismograms when the receiver is deeper than the source, such as in the borehole monitoring case. It should be noted that if the full moment tensor needs to be determined, e.g., in the hydraulic fracturing cases, the seismic source should be described with six independent tensor components, which will increase the cost in searching for the best solution. The structure between the earthquake and the station is represented as a 1-D horizontally layered medium, which can be built from 1) averaging borehole sonic logs across this region, or 2) extracting the velocity structure between the source and the receiver from the 3-D velocity model from double-difference seismic tomography for passive seismic events (Zhang et al., 2009).

The modeled waveform from a certain combination of strike, dip, and rake is expressed as a linear combination of weighted Green’s functions:

$$V^n_i = \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk} G^n_{ij,k}(t) * s(t)$$  \hspace{1cm} (2-1)

where $V^n_i$ is the modeled $i^{th}$ (north, east or vertical) component at station $n$; $m_{jk}$ is the moment tensor component and is determined by the data from all stations; $G^n_{ij,k}(t)$ is the $i^{th}$ component of the Green’s functions for the $(j, k)$ entry at station $n$, and $s(t)$ is the source time function. In this study, a smooth ramp is used for $s(t)$, the duration of which can be estimated
from the spectra of the recorded seismograms (Bouchon, 1981). The source time functions are found to be insensitive to the waveform fitting, as both the synthetic and observed seismograms are low-pass filtered before comparisons (Zhao et al., 2006). Using reciprocity by strain Green’s tensors can improve the efficiency of calculating the Green’s functions, especially when the sources greatly outnumber the stations (Eisner & Clayton, 2001; Zhao et al., 2006). For instance, only one numerical simulation with reciprocity (e.g., finite difference method), by setting a source at a station, is needed to calculate the Green’s functions for all six components of the moment tensor between anywhere in the field and one component at the station in a 3-D heterogeneous medium.

Earthquake locations are usually provided by the travel time location method. However, due to uncertainties in velocity model and arrival times, the seismic event locations may have errors, especially in focal depth determined from the surface network. While matching the modeled and observed waveforms, we also search for an improved location \((x, y, z)\) around the catalog location.

Before the grid search is performed, we build a Green’s functions library. We pre-calculate all \(G_{y,k}^n(t)\)’s for all possible event locations at all stations and store them on the disk. When we perform the grid search, we simply need to do a linear combination of \(G_{y,k}^n(t) * s(t)\), each of which is weighted by \(m_{jk}\).

To determine the best solution, we construct an objective function that characterizes the similarity between the modeled and observed waveforms. We use the following objective function, which evaluates four different aspects of the waveform information:
maximize \( J(x, y, z, \Phi, \delta, \lambda, t_s) = \)
\[
\sum_{n=1}^{N} \sum_{j=1}^{3} \left[ \alpha_1 \max (\tilde{d}^n_j \otimes \tilde{v}^n_j) - \alpha_2 \left\| \tilde{d}^n_j - \tilde{v}^n_j \right\| \right]
\]
\[
+ \alpha_3 f(\text{pol}(\tilde{d}^n_j), \text{pol}(\tilde{v}^n_j)) + \alpha_4 h(\text{rat}(\frac{S(d^n_j)}{P(d^n_j)}), \text{rat}(\frac{S(v^n_j)}{P(v^n_j)})) \}
\]

Here \( \tilde{d}^n_j \) is the normalized data and \( \tilde{v}^n_j \) is the normalized modeled waveform; \( x, y, \) and \( z \)
are the event hypocenter that will be re-determined by waveform matching; \( t_s \) is the time shift
which gives the largest cross correlation value between the observed and synthetic
seismograms (1st term). Since it is difficult to obtain accurate absolute amplitudes due to site
effects in many situations, we normalize the filtered observed and modeled waveforms before
comparison. The normalization used here is the energy normalization, such that the energy of
the normalized wave train within a time window adds to unity. Compared to peak amplitude
normalization, energy normalization is less affected by site effects, which may cause
abnormally large peaks due to focusing and other factors. In a concise form, this normalization
can be written as:

\[
\tilde{d}^n_j = \frac{d^n_j}{\sqrt{\int_{t_1}^{t_2} (d^n_j)^2 dt}}
\]

where \( t_1 \) and \( t_2 \) are the boundaries of the time window.

The objective function \( J \) in Equation 2-2 consists of 4 terms. \( \alpha_1 \) through \( \alpha_4 \) are the
weights for each term. Each weight is a positive scalar number and is optimally chosen in a way
such that no single term will over-dominate the objective function. The weights \( \alpha_1 \) through \( \alpha_4 \) in
the objective function (Equation 2-2) were tried with different values, and we selected ones that balance different terms. We used $\alpha_1=3$, $\alpha_2=3$, $\alpha_3=1$ and $\alpha_4=0.5$ for the synthetic tests and real events. We also found that the final solutions are not very sensitive to small changes in the weights. The first term in Equation 2-2 evaluates the maximum cross correlation between the normalized data ($\tilde{d}_j^n$) and the normalized modeled waveforms ($\tilde{v}_j^n$). From the cross correlation, we find the time-shift ($t_s$) to align the modeled waveform with the observed waveform. In high frequency waveform comparisons, cycle-skip is a special issue requiring extra attention: over-shifting the waveform makes the wiggles in the data misalign with wiggles of the next cycle in the modeled waveforms. Therefore, allowed maximum time-shift should be predetermined by the central frequency of the waveforms. The second term evaluates the $L_2$ norm of the direct differences between the aligned modeled and observed waveforms (note the minus sign of the 2$^{nd}$ term in order to minimize the amplitude differences, which measures both phase and amplitude differences). The reason for maximizing the cross correlation value and minimizing the direct difference between the observed and modeled waveforms is to match both the similarity of the waveforms and the actual amplitudes. The first two terms are not independent of each other, however, they have different sensitivities at different frequency bands and by combining them together the waveform similarity can be better characterized. The third term evaluates whether the polarities of the first P-wave arrivals as observed in the data are consistent with those in the modeled waveforms. $pol$ is a weighted sign function which can be $\{\beta, -\beta, 0\}$, where $\beta$ is a weight reflecting our confidence in picking the polarities of the first P-wave arrivals in the observed data. Zero (0) means undetermined polarity. $f$ is a function that penalizes the polarity sign inconsistency in such a way that the polarity consistency gives a
positive value while polarity inconsistency gives a negative value. The matching of the first P-wave polarities between modeled and observed waveforms is an important condition for determining the focal mechanism, when the polarities can be clearly identified. Polarity consistency at some stations can be violated if the polarity is not confidently identified (small $\beta$) and the other three terms favor a certain focal mechanism. Therefore, the polarity information is integrated into our objective function in a flexible way. By summing over the waveforms in a narrow window around the arrival time and checking the sign of the summation, we determine the polarities robustly for the modeled data. For the observed data, we determine the P-wave polarities manually.

The S/P amplitude ratio is also very important in determining the focal mechanism. The fourth term in the objective function is to evaluate the consistency of the average S/P amplitude ratios in the observed and modeled waveforms (Hardebeck & Shearer, 2003). The “$\text{rat}$” is the ratio evaluation function and it can be written as:

$$\text{rat} = \frac{\int^{T_2}_{T_1} |r_j^d(t)| dt}{\int^{T_2}_{T_1} |r_j^v(t)| dt} \quad (2-4)$$

where $[T_1 \ T_2]$ and $[T_2 \ T_3]$ define the time window of P- and S-waves, respectively, and $r_j^n$ denotes either $d_j^n$ or $v_j^n$. The term $h$ is a function which penalizes the ratio differences so that the better matching gives a higher value. Note that here we use the un-normalized waveforms $d_j^n$ and $v_j^n$. 

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In general, the amplitudes of P-waves are much smaller than those of S-waves. To balance the contribution between P- and S-waves, we need to fit P- and S-waves separately using the first two terms in Equation 2-2. Also, by separating S- from P-waves and allowing an independent time-shift in comparing observed data with modeled waveforms, it is helpful to deal with incorrect phase arrival time due to incorrect Vp/Vs ratios (Zhu & Helmberger, 1996). Here we allow independent shifts for different stations as well as for P- and S-waves. We calculate both the first P- and S-arrival times by the finite difference Eikonal solver (Podvin & Lecomte, 1991). The wave train is then separated into two parts at the beginning of the S wave. To reduce the effect of uncertainty in the origin time, we first align the modeled and observed data using first arrivals and then define the alignment by cross-correlation. The window for the P-wave comparison is from the first arrival to the beginning of the S-wave, and the window for the S-wave comparison is proportional to the epicenter distance. It should be noted that the full wave train is not included as later arrivals, usually due to scattering from heterogeneous media, cause larger inaccuracies in waveform modeling.

The processing steps can be summarized as follows:

1. Use the known velocity structure to generate a Green’s function library;

2. Calculate the first P- and S-wave arrival times using the finite-difference travel time solver (Podvin & Lecomte, 1991), and separate the S-wave segment from the P-wave segment according to the travel time information;

3. For separated P- and S-wave segments
   a. Determine the time-shift by cross-correlation between modeled and observed data.
b. Evaluate the maximum cross correlation value and $L_2$ norm between the aligned modeled and observed data

c. Identify the first arrival polarities

d. Calculate the average amplitudes of the P-wave and S-wave segments;

4. Determine the best fit mechanism by maximizing the objective function.

To find the similarity between modeled and observed waveforms, we do two kinds of basic computations: filtering and cross-correlation. These two computations are very time-consuming when millions of modeled traces are processed. To expedite the computation we use the following manipulations:

\[
v_i^n = F \ast V_i^n = F \ast \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk} G_{ij,k}^n(t) \ast s(t) \\
= \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk}^n [F \ast (G_{ij,k}^n(t) \ast s(t))] \\
\]

(2-5)

\[
d_i \otimes v_i = d_i \otimes \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk} F \ast G_{ij,k} \ast s(t) \\
= \sum_{j=1}^{3} \sum_{k=1}^{3} m_{jk} [d_i \otimes (F \ast G_{ij,k} \ast s(t))] \\
\]

(2-6)

where $F$ denotes the impulse response of a filter; "$\ast$" denotes time domain convolution; $d_i^n$ and $V_i^n$ denote the $i^{th}$ component of the filtered observed and modeled data at station $n$, respectively; "$\otimes$" denotes the cross-correlation. These two equations indicate that we can apply the filtering and cross correlation into the summation to avoid filtering and cross
correlation repetitively during the search over all strikes, dips and rakes. A large amount of time is thereby saved, and the searching speed is boosted by an order of magnitude.

By pre-calculating the library of Green’s functions and manipulating the filtering and cross correlation, we greatly speed up the grid search process. Searching through all possible X, Y, and Z for location and strikes, dips and rakes for focal mechanisms often results in over 10 million different waveforms to be compared with the data. Since the grid search can be easily parallelized, it can be done on a multicore desktop machine within 10 minutes. In some cases, when we have more confidence in some stations, e.g., stations with short epicenter distance, or stations deployed on known simpler velocity structure, we can give more weight to those stations by multiplying $\alpha_1 - \alpha_4$ with an additional station weight factor.

The comparison algorithm (Equation 2-2) is optimized such that it can be performed on a multicore desktop machine usually within 30 minutes, even when tens of millions of synthetic traces are compared with the data. The computation of the Green’s function library using DWN takes more time, but it only needs to be computed once.

The passive seismic tomography only provides a detailed 3-D velocity model close to the central area of the field due to the earthquake-station geometry (Zhang et al., 2009). Therefore, for the focal mechanism determination through the surface network, of which most stations are not placed within the central area (Figure 2-1), we use the 1-D layered velocity model from the averaged sonic logs (Sarkar, 2008; Zhang et al., 2009). Considering that we use a frequency band of 3-9 Hz (Figure 2-3) in our waveform matching for this surface network, corresponding to a dominant P-wave wavelength of 800 m and S-wave wavelength of 400 m, the velocity
model should satisfy our modeling requirement. The deep network consists of five boreholes with eight-levels of receivers at different depths in each borehole (Figure 2-2). Due to the proximity of borehole receivers to the seismicity, we were able to record the seismograms of very small induced seismicity. Waveforms between 15-35 Hz are used to determine the focal mechanisms (Figure 2-4). To better model the waveforms, we replaced part of the 1-D average layered velocity model with the extracted P- and S-wave velocities from the 3-D tomographic model between 0.7 km and 1.2 km in depth, where it has the highest resolution and reliability. Note that the updated 1-D velocity model between the earthquake and each station becomes different for the deep borehole network.

2.4 Synthetic Tests for the Surface and Deep Borehole Networks

We first test the robustness of the method by adding noise to the synthetic data. We add white spectrum Gaussian noise to each trace, with zero mean and a standard deviation of 5% of the maximum absolute amplitude of that trace. This level corresponds to the typical noise level we encounter for real data. Figure 2-5 shows the focal mechanism determined using waveform information and only three first P arrival polarities (we assume two polarities out of five are not identifiable due to noise contamination). The best solution here (#1) matches the correct solution. Figure 2-6 shows the comparison between the modeled and synthesized waveforms with noise contamination. The “shift” in the title of each subplot indicates the time shifted in the data to align with the synthetic waveforms. The reasons for having some time shift are as follows: 1) we introduced some artificial error in arrival time by manually picking the first P arrival in the synthetic data; 2) scattering noise can change the maximum cross correlation
position (Nolet et. al., 2005). In the left column, the “+” or “-” signs indicate the first arrival polarities of P-waves in the data and those in the synthetics; the upper ones are signs for the synthetic data while the lower ones are signs for the modeled data. The modeled traces all have the identical polarities as their counterparts in the synthetic data. Note that for the evaluation of the polarities, we use the unfiltered waveforms, as filtering usually blurs or distorts the polarities. In the right column, the number to the left of the slash denotes the S/P ratio for the data, and the number to the right of the slash denotes the ratio for the modeled waveform. They are quite close in most cases.

We further tested the robustness of the method in the presence of velocity uncertainty. To account for the uncertainty of the 1-D velocity model, an 8% uncertainty in the velocity model is applied. We test different cases for different focal mechanisms and event locations. We first use the station configuration of the surface network in our test, as it provides a considerable challenge due to the large epicenter distance and the relative inaccuracy in the computation of Green’s functions by using the 1-D averaged velocity model from several sonic logs. We choose three different epicenters (E1, E2 and E3), and for each epicenter we choose three different depths (D1=1000 m, D2=1200 m and D3=1700 m), corresponding to shallow, medium and deep events in this field, respectively. At each depth, we test three different focal mechanisms, which yield 27 different synthetic tests in total. The different focal mechanisms and widely distributed hypocenters in the synthetic test give a comprehensive robustness test for the focal mechanism determination in this region. The station configuration and the hypocenter distribution are shown in Figure 2-1. At each hypocenter, three distinct mechanisms are tested, namely M1: $\Phi=210^\circ$, $\delta=50^\circ$, $\lambda=-40^\circ$; M2: $\Phi=50^\circ$, $\delta=60^\circ$, $\lambda=-70^\circ$; and M3: $\Phi=130^\circ$,
$\delta=80^\circ, \lambda=80^\circ$ (Table 2-1). Three or four first P-arrival polarities are used in each synthetic test, resembling the measurements we have for real data for this surface network. Because the auxiliary plane solution and the fault plane solution give the identical waveform, this means that half of the model space $[0^\circ \leq \text{strike} \leq 360^\circ; 0^\circ \leq \text{dip} \leq 90^\circ; -180^\circ \leq \text{rake} \leq 180^\circ]$ is redundant. Therefore, by constraining the model space in $[0^\circ \leq \text{strike} \leq 360^\circ; 0^\circ \leq \text{dip} \leq 90^\circ; -90^\circ \leq \text{rake} \leq 90^\circ]$ (Zhao & Helmberger, 1994), we can eliminate the redundancy and further shorten the search time by half. In real cases, as inevitable differences exist between the derived velocity model and the true velocity model, we need to examine the robustness of our method under such circumstances. We add up to 8% of the layer’s velocity as the random velocity perturbation to the reference velocity model in each layer (Figure 2-7) and use the perturbed velocity models to generate synthetic data. The perturbation is independent for five stations, i.e., the velocity model is path-dependent and varies among different event-station pairs to reflect the 3-D velocity heterogeneities in the field. Also, the perturbation is independent for the P-wave and S-wave velocities in a specific velocity model for an event-station pair. The Green’s functions (modeled data) are generated with the reference velocity model. Figure 2-8 shows the modeled seismograms with offset using the reference velocity model. The predicted travel times by the eikonal equation and the first arrivals in the waveforms are matched well. It should also be noted that the P-wave and S-wave velocity perturbation from one station to another can reach up to 800 m/s in some layers. Considering that this reservoir consists mainly of sedimentary rocks, the magnitude of the random lateral velocity perturbation should reflect the upper bounds of the local lateral velocity inhomogeneity. The density is not perturbed in this test, as the velocity perturbation is dominant in determining the characteristics of the waveforms. The
test results are summarized in Table 2-1. Although the perturbation can change the waveform characteristics to a very large extent, the synthetic test shows that our method can still find a solution very close to the correct one by including information from different aspects of the waveforms, even when only records from five vertical components are used. Figure 2-9 shows a waveform match between the synthetic data and the modeled data. The best solution found is (230°, 60°, -40°), close to the correct solution (210°, 50°, -40°) in comparison. The synthetic event is at 1220 m in depth.

In general, the focal mechanisms are reliably recovered (Table 2-1). To quantify the recoverability, we define the mean recovery error for the focal parameters:

$$\Delta \varphi_{m,e} = \frac{1}{3} \sum_{i=1}^{3} |\varphi'_{m,d,e} - \varphi_{m}^{e}|$$

(2-7)

where $\varphi'_{m,d,e}$ is the recovered strike, dip or rake for epicenter $e$, with mechanism $m$ at depth $d$, where $e, m, d \in \{1,2,3\}$, and $\varphi_{m}$ is the reference (true) focal parameter for mechanism $m$. It is found that $\Delta \varphi$ is only a weak function of epicenter, with marginally smaller value for E1 than for E2 or E3 in general. Also, we found that for each individual depth $\Delta \varphi$ ($d=1$, 2 or 3) is marginally smaller for shallower earthquakes (D1 and D2) than for deeper earthquakes (D3) (results not tabulated). Due to our use of only vertical components, we found that the uncertainty in strike is slightly larger than that in dip or rake. In general, no distinct variation of $\Delta \varphi$ is found against the hypocenter or faulting type. Therefore, we conclude that our method is not very sensitive to the faulting type, to the azimuthal coverage of the stations, or to the hypocenter position within a reasonable range for the array geometries studied.
For the borehole network, we perform a similar synthetic test to check the reliability of our method for the deep network configuration. As we have shown that the reliability of our method is not very sensitive to the azimuthal coverage of the stations or to the depth of the event in a reasonable range, we only perform synthetic experiments at two hypocenters with three different mechanisms, respectively, for the deep borehole network (Table 2-2). Nine to eleven receivers are used for each case. The frequency band is the same as we use for the real data set (15 – 35 Hz). A typical waveform comparison for the synthetic test is shown in Figure 2-10. It is also found that the method is robust with the borehole receiver configuration using higher frequency seismograms.

2.5 Application to Field Data

We applied this method to study 40 microearthquakes using surface and deep borehole networks. The instrumental responses have been removed before processing. An attenuation model with Q value increasing with depth (Table 2-3) was used for the waveform modeling. In general, we consider the attenuation larger (smaller Q) close to the surface due to weathering, and the attenuation for S-waves larger than for P-waves at the same depth. The attenuation model is built from empirical knowledge of the local geology, and we also tested that reasonable deviation from our Q model (50%) causes only small changes in our synthetic waveforms. Figure 2-11 shows the beachballs of the nine best solutions out of millions of trials for a typical event recorded by the surface network. Our best solution (the one at the bottom right, reverse strike-slip) has a strike of 325°, which is quite close to the best known orientation 320° of the NW-SE conjugate fault (Figure 2-1). Figure 2-12 shows the comparison between the
modeled and the observed data for this event. The waveform similarity between the modeled and observed data is good. Typically, the cross correlation coefficient is greater than 0.7. Additionally, the S/P waveform amplitude ratios in the modeled and observed data are quite close, and the first P arrival polarities are identical in the modeled and observed data for each station. In this example, all four criteria in Equation 2-2 are evaluated, and they are consistent between the modeled and observed data.

For the deep borehole network, we use the frequency band 15~35 Hz, which includes enough energy in the spectra to provide good SNR, for determining the focal mechanisms of these small magnitude earthquakes from the borehole network data (Figure 2-4). The lower frequency here is limited by the bandwidth of the borehole instrumentation (f_c=20 Hz), and the frequency contents below the corner frequency f_c may suffer from an increased noise level. As there is also uncertainty in the orientations of the horizontal components, we use only the vertical components of the 4-C sensors configured in a proprietary tetrahedral shape for each level (Jones, et al., 2004). Although there are in total 40 vertical receivers, we often only use about 10 seismograms in determining each event due to the following reasons:

1) Some receivers are only separated by ~30 m vertically and therefore do not provide much additional information for determining the source mechanism;

2) Some traces show peculiar, unexplainable characteristics in seismograms and are, therefore, discarded. Also, the SNR for some traces is very poor.
In our selection of seismograms, we try to include data from different wells to provide a better azimuthal coverage, as well as from different depths spanning a large vertical range, providing waveform samplings at various radiation directions of the source.

Figure 2-13 shows the comparison between the observed and modeled seismograms for a typical event recorded by the deep borehole network. Eleven receivers from four boreholes are used in this determination. Among the eleven seismograms, five first P-wave arrival polarities are identified and then used in this determination. The waveform similarities, average S/P amplitude ratio and consistency in the P-wave arrival polarities are satisfactory. Comparing Figure 2-13 with Figure 2-12, we found the fewer matched cycles in the deep borehole case. Similar comparison can also be found between the shallow and deep borehole synthetic tests (Figure 2-9 and Figure 2-10), where focal mechanisms close to the correct solutions were still found in both synthetic cases.

Using this method, we have studied 40 earthquakes distributed across this oil field from both the surface network and the borehole network. Among these studied events, 22 events are recorded by the surface network, 18 events are from the borehole network. Figure 2-14 shows that the majority of the events primarily have the normal faulting mechanism, while some have the strike-slip mechanism, and some have a reverse faulting mechanism. The strike directions of most events are found to be approximately parallel with the NE trending fault, suggesting the correlation of these events with the NE trending fault. However, some events also have their strikes in the direction of the conjugate NW trending fault, suggesting that the reactivation also occurred on the conjugate faults. Although the number of studied events is
small compared to the total recorded events, their mechanisms still provide us with some insights on the fault reactivation in this field: 1) The hypocenter distribution and the determined source mechanisms (e.g., strikes) indicate that the reactivation of preexisting faults is the main cause of the induced microearthquakes in this field, and both the NE trending fault and its conjugate fault trending in the NW direction are still active. Interestingly, we note that the strike directions of the normal faulting events (red) are slightly rotated counterclockwise with respect to the mapped fault traces from the 3-D active seismic data and are consistent with the trend of the located earthquake locations (Figure 2-1 and Figure 2-2). The counterclockwise rotation may be due to the non-planar geometry of the fault, i.e., the strike of the shallow part of the fault as delineated by the surface seismic survey does not need to be the same as the deeper part of the fault, where most induced seismicity is located. 2) Most strike-slip events (Cyan) are shallow, suggesting that the maximum horizontal stress ($S_{H\text{max}}$) is still larger than the vertical stress ($S_v$) at this depth range. However, deeper events (e.g., red, blue) mainly have a normal faulting mechanism, suggesting $S_v$ exceeds $S_{H\text{max}}$ when depth increases beyond ~1km in this region. The dominance of normal faulting is consistent with the study by Zoback et al. (2002) on the Valhall and Ekofisk oil fields, where reservoir depletion induced normal faulting in and above the productive horizon. In this oil field, most induced earthquakes occurred above the oil layer, which is located around 1.5 km below the surface. 3) Assuming $S_{H\text{max}}$ is parallel with the strike of normal faulting events, perpendicular to the strike of reverse events, and bisects the two fault planes of the strike-slip events (Zoback, 2007), the majority of the determined events then suggest a $S_{H\text{max}}$ trending NE or NNE, which is consistent with the well breakout measurement and local tectonic stress analysis in the region (Al-Anboori,
The observations indicate that the regional preexisting horizontal stress and the vertical stress played an important role in the reactivation of these preexisting faults.

### 2.6 Discussion

Although we only applied our method to a particular oil/gas field, the method is applicable to any microseismic monitoring case, especially to cases when the monitoring stations are sparse. We only used the vertical components in our study, but the waveform comparison can be easily expanded to include three components. Considering each component at a station contains different information in the radiation pattern (Aki & Richards, 2003), the incorporation of multi-component observations should further reduce the solution uncertainty.

The attenuation needs to be taken into account in the synthetic waveform modeling. Not only is the amplitude changed, but frequency-dependent phase-shift also occurs as the phase velocity becomes dependent on frequency due to the attenuation effect (Aki & Richards, 2003). It should be noted that the attenuation-induced phase-shift is in addition to any phase-shift related to the wave propagation, e.g., guided wave effect. In our waveform modeling, compared to the pure elastic case we have observed notable waveform change in the frequency band of observation when moderate attenuation is included. Attenuation tomography (e.g., Quan & Harris, 1997) should be considered to construct an attenuation model if receivers are not located in the vicinity of the microseismic events.

Our synthetic test indicates that when there are errors in the velocity model, the inverted mechanisms are affected and can deviate from the true ones. Therefore, it is difficult
to tell whether the oscillation in the inverted strike, dip and rake is true or if it is caused by our limited observations and errors in the velocity model.

In general, we find the inversion results are not sensitive to the weighting parameters that are within reasonable ranges. The rule-of-thumb is to choose a parameter set that balances the contribution from each term in the objective function. The weighting parameters used in our study may not be optimal in other fields and need to be determined for individual data sets.

Although tens of millions of synthetic seismograms are usually compared with observed seismograms in the global grid search, some manipulation in the cross correlation and filtering (Li et al., 2011) can be used to greatly reduce the time consumption. Additionally, our inversion algorithm can be easily parallelized. Our experience is that twenty million synthetic seismograms from different source mechanism and hypocenter combinations can be searched through on an 8-core workstation in about 20 minutes. Therefore, our algorithm can be easily extended to monitoring cases where many more stations and components are available.

Our methodology can also be applied to solve for the full moment tensor. In that case, we will have six independent moment tensor components $m_{jk}$ associated with the source mechanism in our objective function. The increase in the degree of freedom will require more search time. In addition, it is more challenging to resolve the six independent moment tensor components because velocity model error, anisotropy or even the inconsistency in the source time function in different moment tensor components become the hindrance.
2.7 Conclusions

In this study, we used our recently developed high-frequency waveform matching method to determine the microearthquakes in an oil field with the surface and borehole network data. This method is especially applicable to the study of microearthquakes recorded by a small number of stations, even when some first P arrival polarities are not identifiable due to noise contamination, or only the vertical components are usable. The objective function, formulated to include matching phase and amplitude information, first arrival P polarities and S/P amplitude ratios between the modeled and observed waveforms, yields reliable solutions. We also performed systematic synthetic tests to verify the stability of our method.

For the 40 studied events, we found that the hypocenters and strikes of the events are correlated with preexisting faults, indicating that the microearthquakes occur primarily by reactivation of the preexisting faults. We also found that the maximum horizontal stress derived from the source mechanisms trends in the NE or NNE direction; this is consistent with the direction of the maximum horizontal stress obtained from well breakout measurements and local tectonic stress analysis. Our investigation shows that the study of the source mechanisms of the induced microearthquakes can provide insights into the local stress heterogeneity and help to better understand the induced microearthquakes by oil or gas production.
Acknowledgement

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Figure 2-1. Distributions of near-surface stations and located events. a) Map view of the studied field. The blue hexagons (E1, E2 and E3) are the epicenters of synthetic events and the green triangles (VA11, VA21, VA31, VA41 and VA51) are the five near-surface stations. These stations are located in shallow boreholes 150 m below the surface to increase the signal-to-noise ratio (SNR). The black lines are the identified faults. b) Side view of the studied field. Most of the induced microearthquakes are localized around 1 km below the surface. A few shallow events have the largest travel time residues among all events.
Figure 2-2. a) Map view of the borehole network and the microearthquakes located by this network. The yellow diamonds (E4, E5) are the epicenters of synthetic events. The green circles are the surface locations of the five wellbores where receivers are installed. b) Side view of the borehole network and located microearthquakes. The green triangles indicate the borehole stations. The vertical distance between two consecutive receivers in a monitoring well ranges from ~20 m to ~70 m.
Figure 2-3. The vertical components of seismograms of a typical event recorded by the surface network and the corresponding spectrograms. The filtered seismograms (3~9 Hz) are in the left column; the original seismograms are in the middle; the spectrograms of the original seismograms are at the right. The zero time is the origin time of the event.
Figure 2-4. The vertical components of seismograms of a typical event recorded by the borehole network. The filtered seismograms (15~35 Hz) are in the left column; the original seismograms are in the middle; the spectrograms of the original seismograms are at the right. The zero time is the origin time of the event. It should be noted that the borehole data is dispersive, i.e., higher frequency contents arrive later as the energy is trapped within layers and propagates as guided waves.
Figure 2-5. Nine best solutions from contaminated synthetic data; the number before ‘str’ is the order; ‘1’ means the best solution and ‘9’ means the worst solution among these nine.
Figure 2-6. Comparison between modelled waveforms (red) and noisy synthetic data (blue) at five stations. From top to bottom waveforms from the vertical components at stations 1 to 5, respectively, are shown. The left column shows P-waves and right column shows S-waves. The green lines indicate the first P arrival times. For P-waves, zero time means the origin time, and for S-waves, zero time means the S-wave arrival time predicted by the calculated traveltime.
Figure 2-7. P- (right) and S-wave (left) velocity perturbations for the synthetic tests. The reference velocities, plotted with the bold black line, are used for calculating the Green’s functions. The perturbed velocities (colored lines) are used to generate the synthetic data for each station.
Figure 2-8. Moveouts of the P- and S-waves with distance. The source is at 900 m in depth, and the receivers (vertical components) are at 150 m in depth. The green lines indicate the first P- and S-wave arrivals obtained from finite-difference travel time calculation method based on the eikonal equation.
Figure 2-9. Comparisons between modeled waveforms (red) and synthetic data (blue) at 5 stations with perturbed velocity model. From top to bottom, waveforms from the vertical components at stations 1 through 5, respectively, are shown. The waveforms are filtered between 3 and 9 Hz. The left column shows P-waves and right column shows S-waves. The green lines indicate the first P arrival times. For P-waves, zero time means the origin time, and for S-waves, zero time means the S-wave arrival time predicted by the calculated traveltime. The “shift” in the title of each subplot indicates the time shifted in the data to align with the
synthetic waveforms. In the left column, the “+” or “-” signs indicate the first arrival polarities of P-waves in the synthetic data and those in the modeled data, respectively. In the right column, the number to the left of the slash denotes the S/P amplitude ratio for the synthetic data, and the number to the right of the slash denotes the ratio for the modeled waveform.
Figure 2-10. Comparisons between modeled waveforms (red) and synthetic data (blue) at nine borehole stations with the perturbed velocity model. In this test, nine vertical components in borehole YA, YB, YC and YD are used. The waveforms are filtered between 15 and 35 Hz. The true mechanism is (210°, 50°, -40°), and the best recovered one is (240°, 60°, -10°) in comparison.
Figure 2-11. Focal mechanism solutions for a typical event determined by the shallow network. The one at the bottom right (#1) is the best solution with maximum objective function value. The epicenter is shifted northward (Y) by about 750 m, eastward (X) by about 300 m and the depth is shifted 50 m deeper compared to the original hypocenter. The shift in epicenter may be biased by inaccuracy in the velocity model and by only using the vertical components. The shift can compensate the phase difference between the modeled seismograms and the real seismograms.
Figure 2-12. Comparison between the modeled waveforms (red) and the real data (blue) at 5 surface network stations for a typical event. For P-waves, zero time means the origin time, and for S-waves, zero time means the S-wave arrival time predicted by the calculated travel time.
Figure 2-13. Comparison between the modeled waveforms (red) and the real data (blue) from the borehole network. 11 stations and 5 first P-wave arrival polarities which can be clearly decided in the observed waveforms are used in this determination. For P-waves, zero time means the origin time, and for S-waves, zero time means the S-wave arrival time predicted by the calculated travel time.
Figure 2-14. a) Focal mechanisms of the 40 events inverted in this study from both the surface and borehole networks. The background color in the map indicates the local change in surface elevation with a maximum difference of about 10 m. Different focal mechanisms are grouped in several colors. The events and their focal mechanisms determined by the surface
network are plotted in the outer perimeter, while the ones by the borehole network are plotted in the inner ring. b) Side view of the depth distribution and focal mechanisms of the studied events. Because only vertical components are used in our focal mechanism determination, our results are not very sensitive to epicenter shifting. Therefore, the event epicenters shown in a) are from the travel time location and the event depths in b) are from the waveform matching process.
Table 2-1. Recovered focal mechanisms in the synthetic tests for different hypocenters and faulting types. The true focal mechanisms are listed in the row indicated by REF. Rows D1, D2 and D3 list the events at 1000 m, 1200 m, and 1700 m in depth, respectively.
Table 2-2. Recovered focal mechanisms in the synthetic tests for different faulting types using the deep borehole network. The true focal mechanisms are listed in the row indicated by REF. The synthetic events at two different hypocenters are tested (Figure 2-2).
<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$Q_p$</th>
<th>$Q_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 60</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>60 – 110</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>110 – 160</td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>160 – 264</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td>264 – 470</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>470 – 1090</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>1090 – bot.</td>
<td>300</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 2-3. One dimensional attenuation model used for the DWN waveform modeling.

The attenuation affects the waveform amplitudes and causes waveform dispersion.
Chapter 3 Joint Microseismic Location and Anisotropic Velocity Inversion using Differential Arrival Times and Differential Back Azimuths

Summary

We develop a new method to locate microseismic events induced by hydraulic fracturing with simultaneous anisotropic velocity inversion, using differential arrival times and differential back azimuths. Compared to the existing double-difference method, our method incorporates back azimuth information to better constrain microseismic locations in the case of downhole linear seismic arrays used for monitoring induced seismicity. The velocity inversion is constrained to a 1-D layered VTI (transversely isotropic structure with a vertical symmetry axis) structure to improve inversion stability given the limited passive seismic data. We derive analytical sensitivities for the elastic moduli ($C_{ij}$) and layer thickness $L$, and verify the analytical results with numerical calculations. The forward modeled travel times and sensitivities are all calculated analytically without weak anisotropy assumption. By incorporating the relative information among events, the extended double-difference method can provide better relative locations for events and, therefore, can characterize the fractures with higher accuracy. In the two tests with synthetic data, our method provides more accurate relative locations than the traditional methods, which only use absolute information. With fast speed and high accuracy, our inversion scheme is suitable for real-time microseismic monitoring of hydraulic fracturing.
3.1 Introduction

Microseismic monitoring is a commonly used and promising technique for characterizing the development of hydraulic fracturing in shale gas/oil play (Rutledge & Phillips, 2003; Maxwell et al., 2010; Zimmer, 2010; Li et al., 2011). However, there are several hindrances to the practical use of this technique: 1) the fracturing induced seismic events are generally weak and difficult to detect; 2) for the detected events, accurate picking of the first P- and S-wave arrival times and reliably determining their arriving back azimuths are sometimes difficult due to noise contamination; 3) even with good readings, the location of microseismic events is still prone to error due to the lack of an accurate velocity model, e.g., in gas/oil shale cases where strong VTI anisotropy (up to 30%) is commonly seen (Warpinski et al., 2009). Figure 1 shows some significant delay of the SV-wave compared to the SH-wave from a microseismic event due to anisotropy in shale during a hydraulic fracturing job. Perforation shots are often used to calibrate the velocity model for locating the microseismicity induced during the injection stages (Warpinski, 2005; van Dok et al., 2011). However, anisotropy and heterogeneity in the velocity structure can result in location errors if the induced events do not collocate with the perforation shots because the wave propagation paths differ. In many hydraulic fracturing treatments, fracturing often does not occur in the vicinity of the perforation shots and can be a few hundred meters away (e.g., Rutledge et al., 1998; Rutledge and Phillips, 2003). Therefore it is problematic to locate the microseismicity if only the velocity model constructed from the perforation shots is used. Furthermore, when events are located individually, the variability in location error among the events makes it difficult to delineate fractures (Eisner et al., 2010; Maxwell, 2010; Grechka et al., 2011).
For tackling the first issue, Song et al. (2010) used the waveform similarity for
neighboring induced microseismic events to detect weaker events by forming a library of
waveform templates from stronger events. In this study, we aim to deal with the second and
third issues. Taking advantage of the waveform similarity, we can obtain more accurate
differential arrival times for nearby microseismic events. Using the more accurate differential
times, relative seismic locations can be better determined with the double-difference location
method. It has the ability to remove common errors in both model and data that exist among
closely spaced microseismic events (Waldhauser and Ellsworth, 2000; Wolfe, 2002; Zhang and
Thurber, 2003).

For microseismic monitoring, however, in many cases linear seismic arrays deployed in
boreholes are used. To better constrain microseismic locations, back azimuth information of
seismic waves arriving at different sensors is needed. The back azimuths can be determined
either from P-waves or from SH-waves in a layered medium with VTI anisotropy. The SH-waves
usually have larger amplitudes but are also contaminated by P-wave coda. De Meersman et al.
(2009) improved the locations of induced microseismicity in North Sea Valhall oil field with
simultaneously estimated back azimuths for all stations within an array using a noise-weighted
SVD of the complex analytic signals, together with better travel time picks. Rutledge and Phillips
(2003) determined the induced event locations in Cotton Valley with both travel time and back
azimuth information. They improved relative event locations by extracting more consistent
arrival time picks from peaks and troughs of similar time-interpolated waveforms. It should be
noted that small unaccounted deviation of the borehole could result in considerable errors in
determining the azimuthal distribution of the fractures (Bulant et al., 2007). In our paper, we
extend the double-difference method to include back azimuths from P-waves in addition to arrival times. Similar to differential arrival times, we expect that differential back azimuths are also more accurate than the absolute values due to common errors. As a result, the new double-difference method using both differential times and back azimuths is able to better determine relative seismic locations.

When the event locations and origin times are known, the anisotropic velocity inversion can be greatly simplified. In ideal cases such as experiments, if compressional and shear waves that propagate horizontally and vertically in a homogeneous space are recorded with waves propagating in other directions, then the parameters of a VTI medium can be determined separately and the velocity inversion for a homogeneous space can be greatly simplified (Nihei et al., 2011). In real cases, however, it is very uncommon to have such an ideal source-receiver geometry. Mah and Schmitt (2003) used a global search method to simultaneously determine all elastic moduli of a homogenous composite material from travel times. However, the computer time required by their method increases very rapidly with the number of observations and unknown parameters. To obtain a velocity model that better reflects the structure between the actual microseismicity and the receivers, Grechka et al. (2011) simultaneously estimated the general anisotropy of the medium while locating the microseismicity, assuming the medium is a homogeneous anisotropic space. However, the receiver array often spans a large depth range and is likely to be in a very different formation from where the microseismic events are located. As a result, it could be unrealistic to assume the medium to be homogeneous in these cases. Zhang et al. (2009) estimated the heterogeneous isotropic velocity structure in an oil/gas reservoir with the double-difference
tomography method of Zhang and Thurber (2003). This method simultaneously locates seismic events and determines the velocity model by using differential travel times and absolute picks from reservoir induced seismic events. However, their ideal azimuthal coverage with five monitoring wells is rarely seen in hydraulic fracturing cases, and the anisotropy in the reservoir was not addressed in their study. It should be noted that sometimes an artificial effect of anisotropy may appear in travel times observed in boreholes if the deviation of the borehole is not correctly taken into account (Bulant et al., 2007).

Considering the above problems, our study focuses on determining the anisotropic structure of the medium and locations of the microseismic events: 1) we develop a new anisotropy velocity inversion method to determine the anisotropic structure between the microseismic events and the receiver array using arrival time picks (qP, qSV and SH), with the structure assumed to be a 1-D layered VTI medium given the limited spatial coverage of passive microseismic observations; 2) we extend the double-difference location method to use more accurate differential arrival times (Waldhauser & Ellsworth, 2000; Zhang & Thurber, 2006; Foulger & Julian, 2011; Castellanos et al., 2012) and differential back azimuths to better constrain the relative event locations. The layer approximation of the velocity structure is considered to be reasonable in most downhole microseismic monitoring cases, especially in the hydraulic fracturing cases, where the medium mainly consists of flat sedimentary rocks (e.g., Warpinski, 2005; Rutledge et al., 1998), and the epicentral distance between events and sensors often varies from a few tens to a few hundreds of meters. The velocity inversion using absolute picks is important to determine the absolute locations of the events, while the
differential observations are critical to improve the relative locations and better delineate the fractures (Zhang et al, 2009).

In this study, we derive analytically sensitivities for the elastic moduli \((C_{ij})\) and layer thickness \((L)\) in our seismic location and velocity inversion algorithm without any weak anisotropy assumption. Utilizing a method for calculating the travel times of qP, qSV and SH waves analytically (Tang & Li, 2008), our inversion scheme is fast and accurate, and especially suitable for real time monitoring.

3.2 Methodology

3.2.1 Microseismic Location with Differential Arrival Times and Back Azimuths

Let us denote the observed arrival time from event \(i\) to station \(k\) as \(o_{i,k}\) and the corresponding modeled arrival time as \(m_{i,k}\) from initial event location and velocity model. The conventional seismic location method simply relates the arrival time residual to the perturbations in event location and origin time by assuming the velocity model is known:

\[
\tau_{i,k} = o_{i,k} - m_{i,k} = \sum_{l=1}^{3} \frac{\partial T_{i,k}}{\partial x_{l}} \Delta x_{l} + \Delta \tau_{i} \tag{3-1}
\]

where \(x_{l}(l=1,2,3)\) denotes seismic locations in three dimensions, \(T_{i,k}\) is the travel time, and \(\tau_{i}\) is the origin time. If we take the difference between the arrival time residuals from event pairs \(i\) and \(j\) to a common station \(k\), it becomes the double-difference location method first proposed by Waldhauser & Ellsworth (2000) as follows:
\[
\tau_{r_k}^i - \tau_{r_k}^j = \sum_{l=1}^{3} \frac{\partial T_k^l}{\partial x_l^i} \Delta x_l^i + \Delta \tau^i - \sum_{l=1}^{3} \frac{\partial T_k^j}{\partial x_l^j} \Delta x_l^j - \Delta \tau^j
\]  

(3 - 2)

where the difference can also be defined as

\[
\tau_{r_k}^i - \tau_{r_k}^j = (t_k^i - t_k^j)^o - (t_k^i - t_k^j)^m
\]  

(3 - 3)

The double-difference method is capable of eliminating the unmodeled common errors existing along the ray paths between a closely spaced cluster of events and a receiver (Zhang & Thurber, 2006). In an anisotropic model, the sensitivity of the travel time with respect to the hypocenter is simply the phase slowness \(p_l\) at the source location,

\[
\frac{\partial T_k^i}{\partial x_l^i} = p_l^i
\]  

(3 - 4)

and the sensitivity of the travel time with respect to the origin time is unity (Equation 3-1). Note in isotropic media the phase slowness and group slowness are the same.

We extend the double-difference location method using differential arrival times (Waldhauser & Ellsworth, 2000) to include P-wave arrival back azimuths. The notations for back azimuth in the following are similar to those for arrival time. First, the residual between the observed and modeled back azimuths can be expressed as:

\[
\varphi r_k^i = \sum_{l=1}^{3} \frac{\partial \varphi_k^i}{\partial x_l^i} \Delta x_l^i
\]  

(3 - 5)

and the corresponding double-difference form is
$$\varphi_r^k - \varphi_r^l = \sum_{i=1}^{3} \frac{\partial \varphi_k^j}{\partial x_i^l} \Delta x_i^l - \sum_{i=1}^{3} \frac{\partial \varphi_k^j}{\partial x_i^l} \Delta x_i^j$$  \hspace{1cm} (3-6)$$

where $\varphi_r$ is the back azimuth residual. In ray approximation, the back azimuth angle can be expressed as:

$$\tan(\varphi_k^i) = \frac{\partial T_k^i/\partial y_r}{\partial T_k^i/\partial x_r}$$ \hspace{1cm} (3-7)$$

with horizontal slowness vector, where $\partial T_k^i/\partial x_r$ and $\partial T_k^i/\partial y_r$ are the travel time derivatives with respect to $x$ and $y$ coordinates at the receiver location, respectively. We define the positive $x$-axis as the zero back azimuth angle, and the angle increases counterclockwise. In a heterogeneous medium, the sensitivity of the back azimuth with respect to the hypocenter can be derived from equation (3-7):

$$\frac{\partial \varphi_k^i}{\partial x_i^l} = \alpha_1 \frac{\partial (\partial T_k^i/\partial y_r)}{\partial x_i^l} + \alpha_2 \frac{\partial (\partial T_k^i/\partial x_r)}{\partial x_i^l}$$ \hspace{1cm} (3-8)$$

where

$$\alpha_1 = \frac{1}{\partial T_k^i/\partial x_r} \frac{1}{1 + \left(\frac{\partial T_k^i/\partial y_r}{\partial T_k^i/\partial x_r}\right)^2}$$ \hspace{1cm} (3-9)$$

$$\alpha_2 = -\frac{1}{\left(\frac{\partial T_k^i/\partial x_r}{\partial T_k^i/\partial y_r}\right)^2} \frac{\partial T_k^i/\partial y_r}{1 + \left(\frac{\partial T_k^i/\partial y_r}{\partial T_k^i/\partial x_r}\right)^2}$$
In appendix A, we show how to approximate Equation 3-8 with the finite difference calculation. For a 1-D layered VTI velocity structure where the back azimuths can be directly given by the source-receiver geometries, the sensitivities with respect to the hypocenter can be expressed as:

\[
\frac{\partial \phi}{\partial x_s} = -\frac{y_s - y_r}{(x_s - x_r)^2 + (y_s - y_r)^2}
\]

\[
\frac{\partial \phi}{\partial y_s} = \frac{x_s - x_r}{(x_s - x_r)^2 + (y_s - y_r)^2}
\]

\[
\frac{\partial \phi}{\partial z_s} = 0
\]  \hspace{1cm} (3 - 10)

The differential back azimuth angles can be calculated by differentiating two angles obtained from the eigenvectors of the covariance matrix of the seismograms (Magotra et al., 1989), or can be calculated independently with the method described in Appendix B.

To demonstrate how the extended double-difference method can reduce the common travel time and back azimuth errors for closely spaced events, we create an anisotropic heterogeneous model by adding strong random perturbation to a VTI layer model (Figure 3-2). For a VTI medium, the property of anisotropy can be characterized by five independent elastic moduli \( C_{11}, C_{13}, C_{33}, C_{55} \) and \( C_{66} \). Alternatively, the property can be characterized by the Thomsen parameters (Thomsen, 1986), which are the vertical P-wave velocity \( \alpha_0 \), the vertical S-wave velocity \( \beta_0 \) and three anisotropy parameters \( \epsilon, \delta, \gamma \). In the following discussions, we actually use the density-normalized elastic moduli \( \overline{C}_{ij} \), (defined as \( \overline{C}_{ij} = \frac{C_{ij}}{\rho} \)), but for simplification we still use \( C_{ij} \) to represent \( \overline{C}_{ij} \). The random perturbation is on the elastic moduli,
and for a certain location the perturbations on the five elastic moduli \((C_{11}, C_{13}, C_{33}, C_{55}, C_{66})\) are the same in percentage, i.e., the anisotropic parameters \(\epsilon, \delta\) and \(\gamma\) are invariant after the perturbation, while the vertical velocity \(\alpha_0\) and \(\beta_0\) are perturbed randomly (c.f. Equation 8, Thomsen, 1986). The random Gaussian heterogeneity \(\delta C(x, y, z)\) has correlation length about 10 m and peak amplitude about 0.15. The perturbation can be expressed as:

\[
C_{ij}(x, y, z) = C_{ij}^0(x, y, z) + \delta C(x, y, z) \cdot C_{ij}^0(x, y, z)
\]  

(3 - 11)

In this model, we create 8 closely spaced events in two neighboring parallel fractures (4 events on each fracture). We used an in-house finite-difference wave propagation code (fourth order in space, second order in time) to generate synthetic seismograms (Moczo et al., 2007), and we manually picked the arrival times for qP, qSV and SH in each seismogram at their first breaks. The events within a cluster are ordered (1 to 4) starting from the closer end to the receiver array to the distant end in one fracture, and then similarly for the other parallel fracture (5 to 8).

Figure 3-3 shows the travel times and back azimuths determined from the synthetic seismograms. For the travel times, a similar trend of variation can be found for all events, indicating a similar propagation path while occasional abrupt changes among the picks at a common receiver are due to picking errors and influences from small heterogeneities that only affect some events. The determined back azimuths from the waveforms are heavily biased by the heterogeneity, and vary in a quite wide range (\(~39^\circ\) to \(~55^\circ\)), compared to a much smaller theoretical range (\(~47.5^\circ\) to \(~49.5^\circ\), shaded box) in a case without random heterogeneity. The heterogeneities in this model have larger influence on the back azimuths in comparison with
the influence on the travel times, which are mostly perturbed by less than a few percents (not plotted for clarity). However, at a certain receiver the variation of the back azimuths among different events is small, e.g., usually less than a few degrees. This synthetic data indicates when the differential back azimuth information is included in the inversion, the events should be located more accurately in a tighter azimuthal range.

3.2.2 Strong Anisotropic Velocity Inversion for 1-D Layered VTI Structure

In our study, we parameterize the velocity structure as 1-D layers with different elastic modulus $C_{ij}$ ($ij = 11, 13, 33, 55, 66$) and thickness $L$ for each layer. First we study the sensitivity with respect to elastic modulus. For a VTI medium, the phase velocity $v$ for qP, qSV and SH waves with phase angle $\theta$ are given by (e.g., Thomsen, 1986; Tang & Li, 2008):

$$
\rho v_{p}^2(\theta) = \frac{1}{2} \left[ C_{33} + C_{55} + (C_{11} - C_{33}) \sin^2(\theta) + D(\theta) \right]
$$

$$
\rho v_{SV}^2(\theta) = \frac{1}{2} \left[ C_{33} + C_{55} + (C_{11} - C_{33}) \sin^2(\theta) - D(\theta) \right]
$$

$$
\rho v_{SH}^2(\theta) = C_{66} \sin^2(\theta) + C_{55} \cos^2(\theta)
$$

(3-12)

where

$$
D(\theta) = \left\{ (C_{33} - C_{55})^2 + 2[2(C_{13} + C_{55})^2 - (C_{33} - C_{55})(C_{11} + C_{33} - 2C_{44})] \sin^2(\theta) \right. \\
+ \left. [(C_{11} + C_{33} - 2C_{55})^2 - 4(C_{13} + C_{44})^2] \sin^4(\theta) \right\}^{1/2}
$$

(3-13)
3.2.2.1 Sensitivity with respect to Elastic Moduli

The sensitivity of travel time $T$ with respect to the elastic modulus $C_{ij}$ in layer $k$ is:

$$\frac{\partial T^k}{\partial C_{ij}^k} = \frac{\partial \left( \frac{l^k}{v_g^k} \right)}{\partial C_{ij}^k} = - \frac{l^k}{(v_g^k)^2} \frac{\partial v_g^k}{\partial C_{ij}^k}$$

(3 – 14)

where $l^k$ is the ray path within layer $k$ and $v_g^k$ is the group velocity in layer $k$. Note the derivative depends on the group velocity, thus the anisotropic velocity inversion problem becomes nonlinear. Using the relation between the phase and group velocities (Thomsen, 1986; Tang and Li, 2008), the sensitivity of the group velocity with respect to the elastic modulus is:

$$\frac{\partial v_g}{\partial C_{ij}} = \frac{1}{2v_g} \frac{\partial v_g^2}{\partial C_{ij}} = \frac{1}{2v_g} \frac{\partial}{\partial C_{ij}} \left( v^2 + \frac{1}{4v^2} \left( \frac{\partial v^2}{\partial \theta} \right)^2 \right)$$

$$= \frac{1}{2v_g} \left[ \frac{\partial v^2}{\partial C_{ij}} \left( 1 - \frac{1}{4v^4} \left( \frac{\partial v^2}{\partial \theta} \right)^2 \right) + \frac{1}{2v^2} \frac{\partial v^2}{\partial \theta} \frac{\partial \left( \frac{\partial v^2}{\partial \theta} \right)}{\partial C_{ij}} \right]$$

(3 – 15)

Equation 3-15 is a general expression for $qP$, $qSV$ and SH. However, the terms $\partial v^2 / \partial C_{ij}$ and $\partial \left( \frac{\partial v^2}{\partial \theta} \right) / \partial C_{ij}$ change for different types of waves. The derivation of these derivatives for $qP$, $qSV$ and SH is given in Appendix C. We notice Zhou and Greenhalgh (2005) derived analytical expressions of the sensitivities with respect to elastic moduli in VTI medium in different forms and with a quite different approach.
3.2.2.2 Sensitivity with respect to Layer Thickness

Here we study the sensitivity with respect to the layer thickness $L$. The perturbation of the ray path and travel time caused by the change of the layer thickness (or interface position) is illustrated in Figure 3-4.

Obeying Snell's law, we consider a virtual source $S_v$ emits a parallel ray (dashed) that hits the perturbed interface and is converted to the same ray (solid) in the second medium. Here we decompose the travel time perturbation into two parts: 1) the perturbation caused by changing the original source $S$ to the virtual source $S_v$; 2) the perturbation within $\Delta z$ caused by group velocity change ($V_g^1$ to $V_g^2$) and ray path change (solid to dashed). It should be emphasized that when the interface location is perturbed, the actual ray will change its paths in both layers and, in fact, does not coincide with either the solid or the dashed rays shown in Figure 3-4. The sensitivity decomposition above is just an exact alternative expression for the travel time perturbation. For the first part of the sensitivity, i.e., the travel time perturbation due to the source location changing from $S$ to $S_v$ as the result of the interface perturbation $\Delta z$, the sensitivity is

$$S_1 = \frac{\partial T}{\partial z} = \frac{\partial T}{\partial r} \frac{\partial r}{\partial z} = \frac{\partial T}{\partial r} \frac{-(\Delta z \tan(\Phi_1) - \Delta z \tan(\Phi_2))}{\Delta z} = \frac{\partial T}{\partial r} (\tan(\Phi_2) - \tan(\Phi_1)) \quad (3 - 16)$$

where the negative sign appears due to the coordinate definition (moving to the negative X-axis). $\partial T/\partial r$ is the horizontal (radial) phase slowness $p_r$.

For the sensitivity in the perturbation region $\Delta z$, we have
Then finally we can express the sensitivity of the travel time with respect to the layer thickness, or the interface position as

\[
\frac{\partial T}{\partial L} = S_1 + S_2 = p_r (\tan(\phi_2) - \tan(\phi_1)) + \frac{1}{\cos(\phi_1) V_g^1} - \frac{1}{\cos(\phi_2) V_g^2} (3 - 18)
\]

Here we derived the exact sensitivity without the need to recalculate the ray path. It should be noted that although our derivation is based on a simple two-layer model, the sensitivity expression \( \frac{\partial T}{\partial L} \) is actually valid for any multi-layer case. For any intermediate layers between the source and the receiver in a multi-layer case, the source and receiver positions shown above are simply replaced by the intersection point of the rays with the layer interfaces. The sensitivity expression remains the same for each layer. Although we derived the sensitivity in a situation where the top layer has a faster wave speed than the bottom layer, the sensitivity expression remains the same if the top layer has a slower wave speed. If the ray travels upwards, a negative sign should be added to the sensitivity \( \frac{\partial T}{\partial L} \).

### 3.2.3 Scheme for Location with Simultaneous Anisotropic Velocity Inversion

The inversion scheme for determining both the velocity structure and the hypocenters can be written in the following form:
\[
\begin{bmatrix}
Q_{DD}^t A_t \\
Q_{DD}^\phi A^\phi \\
w^t A_t \\
w^\phi A^\phi \\
w c P c
\end{bmatrix}
\begin{bmatrix}
\Delta C_{ij} \\
\Delta L \\
\Delta X
\end{bmatrix}
= 
\begin{bmatrix}
Q_{DD}^t \Delta T \\
Q_{DD}^\phi \Delta \phi \\
w^t \Delta T \\
w^\phi \Delta \phi \\
\ -w c P c C_{ij}
\end{bmatrix}
\] (3 − 19)

where \(Q_{DD}^t\) and \(Q_{DD}^\phi\) are the differential matrices for travel times (Wolfe, 2002; Zhang and Thurber, 2006) and back azimuths, respectively, and \(Q_{DD}^\phi\) is constructed similarly to \(Q_{DD}^t\); \(w^t\) and \(w^\phi\) are the relative weights for absolute travel times and back azimuths, respectively; \(A_t = [M_t \ H_t]\) is the sensitivity matrix of the travel time with respect to the velocity structure \((M_t, Equation 3-14 and 3-18), and the event hypocenters \((H_t, Equation 3-4); A^\phi = [0 \ H^\phi]\) is the sensitivity matrix of the back azimuth with respect to the hypocenters \(Equation 3-10); P^c\) is the constraint operator on \(C_{ij}\) that attempts to retain some well-determined anisotropic parameters \(\epsilon, \delta\) or \(\gamma\) from core sample measurements in the lab (Chang Li, personal comm.; Appendix D). \(\Delta C_{ij}\) is the perturbation on the elastic moduli, and \(\Delta L\) is the perturbation on the layer thickness; \(\Delta X\) is the perturbation on hypocenter and origin time of events; \(\Delta T\) is the travel time residual, and \(\Delta \phi\) is the back azimuth residual. Note since we assume 1-D layered VTI structure, the sensitivity of the back azimuth with respect to the velocity structure is null.

In our inversion, we parameterize the density normalized elastic moduli with the unit of \(Gpa \cdot cm^3/g\), as we found such parameterization would make the sensitivity more balanced for elastic moduli, layer thicknesses (meter) and source parameters (meter for hypocenter, second for origin time). The Levenberg-Marquardt algorithm (Levenberg, 1944) is used for the inversion. We iterate the inversion until the reduction in residuals becomes negligible. Also, as
local minima exist in this nonlinear inverse problem, different damping parameters were tried in the inversion to obtain the best results.

3.3 Numerical Examples and Discussions

In this section, we show two examples of joint microseismic location and anisotropic velocity inversion using differential data. In the first example, a model with strong anisotropic layers is used for the test (Figure 3-5). The model has four layers and three interfaces. There are two fracture systems in the model with two neighboring parallel fractures in each system and each fracture is associated with 4 events. The general trend of the fracture system is similar to that shown in Figure 3-2. In total there are 16 events used as passive sources for the anisotropic velocity inversion. The travel time data for this example are generated with the analytic ray shooting method by Tang & Li (2008), and the back azimuth data are determined from the source-receiver geometry. Note: Figure 3-5 shows the projections of events and ray paths in the radial planes for clarity, and the back azimuths of the events are different. We assume randomly half of the receivers have readings of the qP, qSV, SH and back azimuth \( \theta \) for each event to resemble a realistic noisy situation or variation in array sensitivity. The random receiver choice is different for different travel time phases as well as for back azimuths. The qP ray paths from one event in each cluster are illustrated with the dashed lines to show the ray angle coverage. The qSV and SH ray paths deviate from the qP ray paths due to different contrast between layers. However, the deviations are minor in this case and thus are not shown for avoiding redundancy.
We first examine the analytical derivation of the sensitivity with respect to the elastic modulus and layer thickness. We can use numerical second-order central differencing to approximate the derivative:

\[
\frac{\partial T}{\partial C_{ij}} \approx \frac{T(C_{ij} + \Delta C) - T(C_{ij} - \Delta C)}{2\Delta C}
\]

and

\[
\frac{\partial T}{\partial L_k} \approx \frac{T(L_k + \Delta L) - T(L_k - \Delta L)}{2\Delta L}
\]

For each perturbed parameter the travel time is computed accurately with an analytical method (Tang & Li, 2008). Therefore, the numerical difference can achieve great accuracy by using a very small increment. The numerical derivatives, though time-consuming to calculate, can be used as the reference. Note in the analytical calculation of the sensitivity with respect to the elastic modulus, the ray path is assumed to be stationary (not changed with small perturbation). However, in the numerical calculation, there is no such assumption of stationarity.

Using the model shown in Figure 3-5, we calculate and compare the analytical derivatives and numerical derivatives. In Figure 3-6, approximately the first one third of the rows are the sensitivities related to the qP waves, the second one third are the sensitivities related to the qSV waves and the last one third are the sensitivities related to the SH waves. Figure 3-6 shows that the differences between the analytical ones and the numerical ones are, in general, less than 0.5%. This comparison validates our derivations.
Figure 3-7 shows the comparison between the sensitivity matrix $A$ (including $A^t$ and $A^\phi$) and the differential sensitivity matrix $Q \cdot A$. As the back azimuths have null sensitivity with respect to the propagation medium in the 1-D layered anisotropic model, the last 130 rows related to the back azimuth sensitivities shown in the box in Figure 3-7(a) are all zeros. It can be found that the differential sensitivities $Q \cdot A$ with respect to the medium ($C_{ij}$ and $L$) have been significantly reduced compared to those in $A$ (boxed section), while the sensitivities with respect to the hypocenter and origin time have been mostly retained. This is mainly caused by the model parameterization (model represented as layers) and the limited space span of microseismic events. If the model is represented as smaller cells or grid nodes and microseismic events are more widely distributed, the differential model sensitivities will be more sensitive to the model parameters around the microseismic source region, as described in Zhang and Thurber (2006). For the example shown in Figure 3-5, the extended double-difference method in Equation 3-19 is most sensitive to event locations and origin times rather than the propagation medium properties. This is the essential reason why the extended double-difference method is capable of providing better constraints in event relative locations. This also means the model parameters in this case are determined mostly with absolute travel times but not differential travel times.

After validating analytic model sensitivities, we first check the influence of seismic anisotropy on microseismic locations. Assuming only the velocities of the layers in the horizontal direction are acquired through perforation shots or string shots (Warpinski et al., 2009), but the model anisotropy parameters are unknown and thus the model is considered isotropic, we locate the events with both the traditional method (Figure 3-8) and the extended
double-difference method (differential method, Figure 3-9), but with the velocity model fixed, using noise-free synthetic data. Note in this case the velocity inaccuracy does not affect the back azimuth. From Figure 3-8 and Figure 3-9, it can be clearly seen that neither method gives correct absolute locations, as the velocity in the horizontal direction is faster than the velocity in any other direction for each layer. Therefore, the hypocenters are relocated further away from the correct locations. However, it also shows that the extended double-difference method incorporating differential arrival times and back azimuths improves relative locations of the events. The parallel factures in the two fracture systems can be depicted correctly from the double-difference location result while the traditional method gives distorted fractures. The ability to recover fracture geometry more correctly by the extended double-difference method lies in its advantage of removing some common model errors along ray paths caused by the inaccurate velocity model.

We now consider locating the events with simultaneous anisotropic velocity inversion. To resemble realistic situations in our test, incoherent random noise for each travel time observation ($\sigma = 0.2$ ms) and coherent random noise at each station (or random station term, $\sigma = 0.6$ ms) are added to the observed travel times. For the observed back azimuths, incoherent random noise for each back azimuth observation ($\sigma = 1^\circ$) and coherent random noise at each station ($\sigma = 5^\circ$) are added to the observed data. This is to simulate the fact that the coherent noise is generally greater than the random noise and the noise level added to the data is similar to that observed by Grechka et al. (2010) for data with reasonable quality. Figure 3-10 shows the inverted elastic moduli and layer thicknesses starting with an isotropic model using the vertical P- and S-wave velocities and layer thicknesses that are different from the
correct values by about 10 m. Due to different ray angle coverage, the elastic moduli are recovered with varying degrees of success for different layers in the presence of noise. The second and third layers with rays sampling at more different angles are recovered relatively well. At a given group angle, the sensitivity with respect to different elastic moduli varies, and thus the recovery accuracy for different elastic moduli also changes, given noisy observations from the limited number of hypocenters. The variation of sensitivity with respect to the group angle has been discussed in detail by Chapman and Miller (1996). In general, the velocity inversion is ill-conditioned when the rays only sample the medium in limited directions, especially when the source locations need to be determined simultaneously, as there is a tradeoff between the velocity structure and the source locations (Zhang and Thurber, 2006). Because there exist uncertainties for the inverted elastic moduli for each layer, the thicknesses of the layers are also biased to some degree.

Figure 3-11 and Figure 3-12 show the location results for both the traditional method, which only uses the absolute arrival times and back azimuths, and the extended double-difference method. Comparing the absolute locations, the two methods produce similar results. However, the relative locations of the events given by the extended double-difference method are much better than the traditional method, and the fractures are successfully delineated with clear parallelism recovered.

In the second example, we use the anisotropic heterogeneous VTI model as shown in Figure 3-2 to test our method in the presence of strong model heterogeneities. The event-receiver geometry together with the perturbed $C_{11}$ model is shown in Figure 3-13. In this test,
three clusters of events associated with three fracture systems, each of which has 8 events in two neighboring parallel fractures (4 events on each fracture), are used as the passive sources for anisotropic velocity inversion. The receivers are located at the same depth as in the first example, but the horizontal locations are shifted to (50, 50) m. All these events in the fractures are assumed to have the same source property, and a double-couple mechanism is used in our finite-difference code to generate the synthetic waveforms for all events. Rodriguez et al. (2012) proposed a method to simultaneously invert for the source hypocenter and origin time, together with the source mechanism using a sparse representation theory.

In this case, we only use phase picks of qP, qSV and SH from randomly selected one third of the receivers for each event, and for different phases the random choice is different. The sparse observation resembles the situation where microseismic events are weak and the identifiable phases change with the receivers as the radiated energy of different types of waves varies with direction. The qP, qSV and SH travel times are manually picked from the synthetic seismograms generated by our in-house finite-difference wave propagation code. Note in this test we did not add any noise, because 1) we have already tested the influence of coherent and incoherent noises in the first example and 2) our manual phase picking can introduce some errors in the observed arrival times. Therefore, in this example the differences between the observed and modeled travel times consist of three origins: 1) the random perturbation on the elastic moduli, which cannot be captured in our velocity inversion with layer VTI assumption, 2) picking errors and 3) errors introduced by high frequency ray approximation compared to finite frequency wave propagation. For this example we also start from an isotropic layered velocity model as we do in the first one.
In this example, we obtained the differential arrival times by cross-correlating the waveforms, while obtaining the differential back azimuths by the method described in Appendix B. The absolute back azimuths are determined from the eigenvectors of the covariance matrix of the seismograms (Magotra et al., 1989). In Figure 3-14, two events with a separation distance of 15 m in cluster 3 (the farthest cluster) are used as an example to show the similarity of waveforms after being shifted with the differential time given by waveform cross-correlation. The wiggles are aligned well after the shift, indicating correct differential arrival times have been determined. We found the differential back azimuths given by the method in Appendix B are similar to the differential back azimuths given by directly differentiating the absolute back azimuths at most receivers, as both approaches use waveform information automatically. In comparison, differential travel times obtained from waveform cross-correlation can often be more accurate than those from directly differentiating the manually picked absolute times (Waldhauser & Ellsworth, 2000). Note in our finite difference waveform modeling, we assume the events within one fracture system have the same mechanisms. This assumption should be reasonable considering the events occur at different segments of two parallel fractures and they are close in space.

Figure 3-15 shows the event locations determined by the traditional method with joint anisotropic velocity inversion. Due to the strong heterogeneity in the medium, the located events are shifted slightly away from the true locations, especially in the vertical direction. As different phases (qP, qSV and SH) are included for location, the radial distances are in general constrained well (Eisner et al., 2009). Still, we found the relative locations of the events are
poorly determined, e.g., the events on one fracture can be mistakenly located onto the other fracture, and the spacing between some events is also resolved with considerable errors.

Figure 3-16 shows the location result with the extended double-difference method with joint anisotropic velocity inversion. Comparing Figure 3-16 with Figure 3-15, it can be found that the relative locations of the events have been improved considerably. For instance, in the X-Y plot (map view) where the relative locations are sensitive to both back azimuths and travel times, we find including the differential back azimuth information can improve the relative location substantially, yielding satisfactory delineation of the parallelism of the fractures and recovery of the spacing between events. It should be noted that the events in different clusters are located with varying accuracy, as the random heterogeneities have diverse influences on the travel times and back azimuth of events at different locations. The side views also show improvement in relative location with the extended double-difference method.

Figure 3-17 shows the anisotropic velocity inversion result. Note in the inversion the VTI layer model with constant elastic moduli for each layer is only an approximation for the randomly perturbed layer model used in generating the synthetic waveform data. Similar to the previous example, elastic moduli in different layers and the layer thicknesses are recovered with varying degrees of closeness to the reference values.

3.4 Conclusions

In this research, we extend the double-difference location method to use both differential arrival times and differential back azimuths to locate the microseismic events. We
also develop an anisotropic velocity inversion method to determine elastic modulus and layer thickness of the VTI medium using the arrival times of microseismic events. The extended double-difference location system is combined together with the anisotropy velocity inversion system to simultaneously locate microseismic events and determine model parameters. The location improvement from velocity inversion and from double-difference constraints is complimentary, as the former improves the absolute locations and the latter improves the relative locations. The additional differential back azimuth information is extracted from available seismic records and thus this method does not require collecting any new data. We derived analytical sensitivities for elastic modulus and layer thickness for anisotropy velocity inversion without any weak anisotropy assumption. We also compared our analytical sensitivities with numerical sensitivities to validate our derivations. With inaccuracy in travel times and back azimuths either from noise or from model heterogeneities, the velocity structure (elastic moduli and layer thicknesses) can be recovered with varying degrees of success, depending on the ray coverage. It is shown that mostly the absolute travel times, but not the differential travel times, help to improve the velocity model due to the layer parameterization. From synthetic tests, it is shown that absolute event locations are significantly biased without considering anisotropy for the layered VTI model. Synthetic examples show that our new method with the differential information can produce better relative locations of the microseismic events and, therefore, delineate the fractures more clearly and resolve the spacing between events with better accuracy. Both characteristics are important in hydraulic fracturing monitoring, as the former one is critical for understanding the striking of fractures, while the latter one is critical for understanding pressure propagation from
the injection well. The pure analytical calculations involved in our inversion scheme make this method fast and accurate, and thus it is especially suitable for real time monitoring of the shale gas/oil production.

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Figure 3-1. Recorded three-component seismograms from a microseismic event induced by shale-gas hydraulic fracturing. qP, SH and qSV phase arrivals are marked by green, black and red lines. It clearly shows the SH wave arrives about 11 ms earlier than the qSV wave, indicating strong anisotropy in shale.
Figure 3-2. Randomly perturbed model for $C_{11} \ (Gpa \cdot cm^3/g)$. The model is constructed by adding spatially correlated ($L_c=10$ m) random Gaussian noise to a VTI layer model. One fracture system with two parallel fractures is shown here. The epicenter distance from the center of the fracture system to the receiver array is about 230 m. The parallel fractures trend in $45^\circ$ in the X-Y plane, and the dipping angles (away from the vertical direction) are about $55^\circ$. 
Figure 3-3. Travel times for qP, qSV, SH phases and back azimuths from eight events in a cluster. The shaded box indicates the theoretical back azimuths that would be observed if the medium is a VTI layer structure without random perturbations.
Figure 3-4. Ray path and travel time perturbations caused by layer interface change. $S$ denotes the source and $R$ denotes the receiver. The group angle and group velocity are $\Phi_1, V_g^1$ in the first medium, and $\Phi_2, V_g^2$ in the second medium, respectively. The original ray is denoted by the solid line and the perturbed interface and ray are denoted by the dashed lines.
Figure 3-5. 1D layered anisotropic model with receivers (black triangles) and events (red dots and red squares with white edges). Note: this is a side view of a 3-D model with each event projected onto its radial plane to clearly show that the ray angles, and the back azimuths of the events differ. The layers from top to bottom are numbered 1 to 4, respectively. The qP ray paths from selected events are illustrated with the dashed lines to show the ray angle coverage. In this synthetic model, there are 20 receivers numbered 1 to 20 from top to bottom. There are two fracture systems separated at about 50 meters in the horizontal direction. In each fracture system, there are two fractures with 4 events associated with each fracture.
Figure 3-6. Analytical sensitivities (a) and their differences from the numerical ones (b) for the elastic moduli ($C_{ij}^k$) and layer thickness ($L^k$), with the superscript being the layer index. The rows correspond to different observations from different events to different receivers for qP, qSV and SH waves. Note the thickness of the fourth layer ($L^4$) is a null parameter in our inversion and thus is not shown.
Figure 3-7. Comparison between the sensitivity matrix $\mathbf{A}$ (including $A^i$ and $A^p$) (a) and the differential sensitivity matrix $Q \cdot \mathbf{A}$ (b). The rows correspond to the observations from different events to different receivers for $qP$, $qSV$ and $SH$ and back azimuths. The boxed section (23 columns) marks sensitivities or differential sensitivities with respect to the medium properties ($C_{ij}$ and $L$), and the rest area indicates the sensitivities or differential sensitivities with respect to event hypocenters and origin times. Note Figure 3-6(a) shows the same first 470 rows of the boxed section shown in Figure 3-7(a) but in different color scales for clarity. Here column normalization in $\mathbf{A}$ has been applied (Zhang & Thurber, 2006) to balance different sensitivity
magnitudes for different parameters, and thus comparison of the sensitivity magnitudes among
different columns is not meaningful. $A$ and $Q \cdot A$ are calculated for the final results given in
Figure 3-10 and Figure 3-12.
Figure 3-8. Relocated microseismic events in X-Y, X-Z and Y-Z planes using the absolute arrival times and back azimuths without inverting the anisotropy model parameters. Red dots and squares indicate the true event locations on the two fractures, respectively. Blue circles and green squares indicate the located events associated with the two fractures in each system, respectively.
Figure 3-9. Same as Figure 3-8 but for the extended double-difference method.
Figure 3-10. Comparison between true values of elastic moduli and layer thicknesses and inverted ones by the extended double-difference method. Red dots indicate the true parameter values and cyan dots indicate the inverted values.
Figure 3-11. Relocated microseismic events in X-Y, X-Z and Y-Z planes using the absolute arrival times and back azimuths with inverting anisotropy model parameters. Red dots and squares indicate the true event locations on the two fractures, respectively. Blue circles and green squares indicate the located events associated with the two fractures in each system, respectively.
Figure 3-12. Same as Figure 3-11 but for the extended double-difference method.
Figure 3-13. Randomly perturbed model for $C_{11} \ (Gpa \cdot cm^3/g)$. This velocity model is the same as the one shown in Figure 3-2, but there are three fracture systems.
Figure 3-14. Waveform alignment for qP, qSV and SH phases for two events by waveform cross-correlation. The gray windows indicate the time window used for cross-correlation. The source wavelet used in the synthetic modeling is a Ricker wavelet with the central frequency $f_0 = 200$ Hz.
Figure 3-15. Relocated microseismic events in X-Y, X-Z and Y-Z planes for the second example using the absolute arrival times and back azimuths with inverting anisotropy model parameters. Red dots and squares indicate the true event locations on the two fractures, respectively. Blue circles and green squares indicate the located events associated with the two fractures in each system, respectively. There are three fracture systems in this example.
Figure 3-16. Same as Figure 3-15 except for the extended double-difference method for the second example.
Figure 3-17. Comparison between true values of elastic moduli and layer thicknesses and inverted ones by the extended double-difference method. Red dots indicate the true parameter values and cyan dots indicate the inverted values. Note in the inversion the VTI layer model with constant elastic moduli for each layer is only an approximation for the randomly perturbed layer model used in generating the synthetic data.
Chapter 4 Microseismic Joint Location and Anisotropic Velocity Inversion for Hydraulic Fracturing in a Tight Bakken Reservoir

Summary

To improve the accuracy of microseismic event locations, we developed a new velocity inversion method with double-difference constraints for determining both the hypocenters and the anisotropic velocity model for unconventional reservoirs. We applied this method to a microseismic dataset monitoring a Middle Bakken completion in the Beaver Lodge area of North Dakota. Geophone arrays in four observation wells improved the ray coverage for the velocity inversion. Using an accurate anisotropic velocity model is important to correctly assess height growth of the hydraulically induced fractures in the Middle Bakken.

Our results show: 1) moderate-to-strong anisotropy exists in all studied sedimentary layers, especially in both the Upper Bakken and Lower Bakken shale formations, where the Thomsen parameters ($\epsilon$ and $\gamma$) can be over 40%; 2) all events selected for high signal-to-noise ratio and used for the joint velocity inversion are located in the Bakken and overlying Lodgepole formations, i.e., no events are detected in the Three Forks formation below the Bakken; 3) more than half of the strong events are in two clusters at about 100 and 150 meters above the Middle Bakken. Reoccurrence of strong, closely clustered events suggests activation of natural fractures or faults in the Lodgepole formation. The sensitivity analysis for the inversion results
shows that the relative uncertainty in Thomsen’s δ is larger than other anisotropic parameters.

The microseismic event locations and the anisotropic velocity model are validated by comparing synthetic and observed seismic waveforms.

4.1 Introduction

Given their extremely low matrix permeability, oil or gas tight sandstone require successfully engineered fracture networks that generate flow paths for economical oil and gas production. Depending on geology, local stress regime, petrology, heterogeneities, and pumping rates, the hydraulically fractured networks can vary significantly from one place to another. Therefore, mapping of the fractures is a crucial input for production and for designing fracturing jobs, especially in areas where few jobs have been performed before. Microseismic monitoring has been used for more than a decade to map hydro-fracture networks during well completions (e.g., Rutledge & Phillips, 2003; Maxwell et al., 2010). However, there are inherent uncertainties in locating microseismic events. These uncertainties come from several sources: 1) limited geometry of monitoring arrays; 2) phase picking errors, especially for events with low signal-to-noise ratio; 3) downhole geophone orientations that are often not well-constrained; 4) inaccuracy in velocity models constructed from well logs or perforation shots (e.g., Warpinski et al., 2005), especially when strong anisotropy exists (Warpinski et al., 2009). In addition, if the microseismic events occur far from the perforation shots, the calibrated velocity models from perforation shots may not be truly representative (e.g., Rutledge and Phillips, 2003; Warpinski et al., 2008). To deal with the anisotropy issue, Grechka et al. (2011) have proposed a method
to estimate effective anisotropy simultaneously with locations of microseismic events in a homogeneous medium.

In this study, we simultaneously use anisotropic velocity inversion with double-difference location (Li et al., 2012a, 2012b). We apply this method to a Bakken microseismic dataset monitored at four wells. Besides providing coverage for detecting microseismic events along a long lateral well, multiple monitoring wells help improve the ray coverage, providing better constraints for the velocity inversion. We validate the microseismic event locations and anisotropic velocity models by comparing synthetic and observed seismic waveforms. Observed shear wave splitting also supports the determined anisotropy values. By using an accurate anisotropic velocity model, one can better locate events, helping constrain the dimensions of the induced fractures, especially their height.

4.2 Geology of the Studied Formations

The Bakken formation was deposited during the lowermost Mississippian period and is a relatively thin unit limited in areal extent to the deeper part of the Williston Basin (Meissner, 1991). Organic-rich shales in the Bakken have been documented as excellent source rocks for the petroleum found in reservoirs located around the unit. Our monitoring site is located in the Beaver Lodge area of North Dakota. In this area, the Bakken is further divided into three members, namely an upper shale member, a middle siltstone member, and a lower shale member, with a total thickness of about 120 ft. Oil production comes primarily from the Middle Bakken. The Bakken formation is conformably overlain by the Lodgepole Formation deposited
During the Mississippian period, and the lowermost Lodgepole Formation, adjacent to the Bakken, consists primarily of interbedded lime mudstones and calcareous shales. The Bakken Formation unconformably overlies the Three Forks Formation deposited during the upper Devonian period, which consists primarily of interbedded, highly dolomitic, siltstones and shales.

4.3 Microseismic Dataset and Previous Results

In May 2010, Hess Corporation conducted a microseismic survey over a 2-day period in the Beaver Lodge area of North Dakota (Hayles et al., 2011). The treatment was in a 3050 m Bakken horizontal well. Two producing wells and two injection wells were used as monitoring wells with 17 or 18 three-component geophones in each well. The microseismic data were sampled at 0.25 milliseconds over the monitoring period. This is an entirely sliding-sleeve completion, so no perforation shot was performed, and the geophone orientations were calibrated with string shots in the four monitoring wells.

This microseismic dataset was processed by four different vendors (Hayles et al., 2011). Each vendor constructed its own velocity model calibrated by the string shots, ball setting events, well logs and VSP information. These vendors differ in terms of the information they used in processing the dataset. One vendor used the P- and S-wave arrivals and the hodograms from a single well, while another vendor used a diffraction stacking technique on P-waves recorded on all available monitoring wells. As a result of differences in phase picks, velocity models, location methods, etc., the microseismic locations provided by these vendors vary considerably. The inconsistency in event locations hindered estimates of Stimulated Reservoir
Volume (SRV) and evaluation of the fracture height growth. Li et al. (2012c) showed that one vendor’s results are more consistent with the raw microseismic data by using each vendor’s velocity model and event locations and comparing predicted arrival times with the actual picks. However, because there are no perforation shots in this survey, uncertainties in the calibrated velocity models are considerable even in the best vendor’s results. In the following, we will show an improvement in microseismic event location by joint anisotropic velocity inversion.

4.4 Preprocessing and Methodology

4.4.1 Raw Microseismic Data Processing

Over the monitoring period of two days, several hundred microseismic events were detected. In order to give more even distribution of events for velocity inversion and to avoid the greater picking errors for low signal-to-noise ratio events, we selected several strong events in each fracturing stage and created a database of 100 events from all sixteen completion stages. Most of these 100 events were recorded by more than one monitoring well. Due to strong anisotropy in this region, the SV-waves often arrive appreciably later than the SH-waves and are contaminated by SH coda and other converted waves. Therefore, due to larger uncertainties in picked SV-wave arrivals, we used only the P- and SH-wave arrivals for location and velocity inversion. After careful quality-control on arrival picks, we selected 68 events with the best signal-to-noise ratios for the anisotropic velocity inversion. Figure 4-1 shows the seismograms and the picks for a strong event that was observed by all four monitoring wells. For this event, only picks from wells G1, G2, and G3 were used for location and velocity
inversion due to exceedingly large epicentral distance (>1 km) and low signal-to-noise ratios for well G4 (distribution of wells shown in Figure 4-2).

Because the azimuthal coverage of a downhole monitoring survey is generally poor, the back azimuths for the P-waves are usually used to help locate the events. The back azimuths are determined by analyzing the eigenvalues of the seismic trace matrices or hodograms (Magotra et al., 1989). Dreger et al. (1998) found that including back azimuths can improve the event locations when the stations are deployed in a narrow azimuthal range related to the events. For this dataset, we noted that back azimuths may be subject to appreciable uncertainty because we only have string shots to constrain the geophone orientations. Therefore, we used the P-wave back azimuths to constrain the event locations but gave them less weight.

4.4.2 Initial VTI Velocity Model

To construct a starting model, we first divided the section into layers each with nearly homogeneous velocity, using well logs from the observation wells to determine thicknesses of the layers. Then we computed average properties for each stratigraphic layer using sonic logs from a vertical Bakken well in this area. The available sonic properties in that well were vertical \( V_p \) and \( V_s \), and Thomsen’s \( \gamma \) (Thomsen, 1986). Thomsen’s \( \gamma \) was estimated by combining shear velocities from dipole and Stoneley modes (e.g., Tang, 2003; Walsh et al. 2007). The vertically propagating shear waves showed negligible splitting, consistent with VTI symmetry. We then assumed \( \epsilon = \gamma \), roughly consistent with other observations (Horne, 2013). Finally, we assumed
δ = 0.5ε, which lies within the large scatter in observations (Horne, 2013; Vernik and Liu, 1997; Havens, 2012). In fact, Thomsen’s δ is poorly constrained by well data.

4.4.3 Inversion Method

In our study, we constrain the velocity structure as a 1-D layered VTI medium and invert for the anisotropic parameters $C_{ij}^k$ and thickness $L_k$ for each layer $k$, as well as the hypocenters and origin times of all events. The analytic sensitivities were derived for these parameters, and for brevity, the derivations are not repeated in this paper (Li et al., 2012a, 2012b). To forward model the travel times, we used the generalized Snell’s law for tracing rays in a VTI layered medium analytically (Tang and Li, 2008). Extensive tests using synthetic data validate our method.

To improve the relative locations of the events, we use a double-difference velocity inversion method where both the differential travel times and differential back azimuths are used. Let us denote the observed arrival time from event $i$ to station $k$ as $o_t^i_k$, and the modeled one as $m_t^i_k$. The arrival time data residual can be expressed as

$$t_{r_k}^i = o_t^i_k - m_t^i_k = \sum_{l=1}^{3} \frac{\partial T_k^i}{\partial x_l^i} \Delta x_l^i + \Delta \tau^i$$

(4 – 1)

where $x_l \in \{x_s, y_s, z_s\}$ is the hypocenter, $T$ is the travel time and $\tau$ is the origin time; $\Delta x$ and $\Delta \tau$ are corrections to the hypocenter and the origin time determined from data residuals $t_r$. We take the difference between the arrival time residuals from event pairs $i$ and $j$ to a common
station $k$, it becomes the double-difference location method first proposed by Waldhauser & Ellsworth (2000) as follows:

$$t_{r_k}^i - t_{r_k}^j = \sum_{l=1}^{3} \frac{\partial T^i_k}{\partial x_l^i} \Delta x_l^i + \Delta \tau^i - \sum_{l=1}^{3} \frac{\partial T^j_k}{\partial x_l^j} \Delta x_l^j - \Delta \tau^j$$  \hspace{1cm} (4-2)

where

$$t_{r_k}^i - t_{r_k}^j = (t_k^i - t_k^j)^o - (t_k^i - t_k^j)^m.$$  \hspace{1cm} (4-3)

The double-difference method is capable of eliminating the un-modeled common error existing on the closely spaced ray paths from a cluster of events to a receiver (Zhang and Thurber, 2006). In our inversion, the differential times are obtained from waveform cross-correlation.

Similar to the double-difference for travel times, we can extend this method for back azimuths. First, the residual for a back azimuth observation can be expressed as:

$$\varphi_{r_k}^i = \sum_{l=1}^{3} \frac{\partial \varphi_k^i}{\partial x_l^i} \Delta x_l^i$$ \hspace{1cm} (4-4)

where $\varphi_{r}$ is the back azimuth residual. And the corresponding double-difference form is

$$\varphi_{r_k}^i - \varphi_{r_k}^j = \sum_{l=1}^{3} \frac{\partial \varphi_k^i}{\partial x_l^i} \Delta x_l^i - \sum_{l=1}^{3} \frac{\partial \varphi_k^j}{\partial x_l^j} \Delta x_l^j.$$ \hspace{1cm} (4-5)

The inversion scheme for determining both the velocity structure and the hypocenters can be written in the following form:
\[
\begin{bmatrix}
Q_{DD}^t A^t \\
Q_{DD}^\phi A^\phi \\
w^t A^t \\
w^\phi A^\phi \\
w^c P^c
\end{bmatrix}
\begin{bmatrix}
\Delta C_{ij} \\
\Delta L \\
\Delta X
\end{bmatrix} =
\begin{bmatrix}
Q_{DD}^t \Delta T \\
Q_{DD}^\phi \Delta \phi \\
w^t \Delta T \\
w^\phi \Delta \phi \\
-w^c P^c C_{ij}^0
\end{bmatrix}
\]  

(4 - 6)

where \(Q_{DD}^t\) and \(Q_{DD}^\phi\) are the differential matrices for travel times (Wolfe, 2002; Zhang and Thurber, 2006) and back azimuths, respectively; \(w^t\) and \(w^\phi\) are the relative weights for absolute travel times and back azimuths, respectively; \(A^t = [M^t \quad H^t]\) is the sensitivity matrix of the travel time with respect to the velocity structure \((M^t)\), and the event hypocenter \((H^t)\); \(A^\phi = [0 \quad A^\phi]\) is the sensitivity matrix of the back azimuth with respect to the hypocenter; \(P^c\) is the constraint operator on the elastic moduli \(C_{ij}\) that attempts to retain some predetermined anisotropic parameters \(\epsilon, \delta, \gamma\) estimated from the well logs. \(\Delta C_{ij}\) is the perturbation on the elastic moduli, and \(\Delta L\) is the perturbation on the layer thickness; \(\Delta X\) is the perturbation on hypocenter and origin time of events; \(\Delta T\) is the travel time residual, and \(\Delta \phi\) is the back azimuth residual.

In our inversion, we parameterize the density normalized elastic moduli with the unit of \(GPa \cdot cm^3/g\), as we found such parameterization would make the sensitivity more balanced for elastic moduli, layer thicknesses (meter) and source parameters (meter for hypocenter, second for origin time). The Levenberg-Marquardt algorithm (Levenberg, 1944) is used for the inversion. The nonlinear inverse problem involving the velocity structure and the event parameters is linearized and solved with iterations. In each iteration, the parameters are corrected with \(\Delta C_{ij}, \Delta L\) and \(\Delta X\), respectively. We iterate the inversion until the reduction in residuals becomes negligible.
4.5 Inverted Microseismic Event Locations and Anisotropic Parameters

We first used the best vendor’s layered isotropic velocity models, one for each well, to locate 100 selected events with the global search method for hypocenters and origin times that minimizes both the travel times and the back azimuth information. The determined values from the global search are then used as the initial guess for further study. Eventually, 68 out of the 100 selected events with most confident picks are used for the joint anisotropic velocity inversion and double-difference event location.

For the anisotropic velocity inversion, we start with the layered VTI velocity model constructed as described in section 4.2. Using the phase velocity relations in the VTI medium together with the generalized Snell’s law to account for refraction at the interfaces (Tang and Li, 2008), we traced rays in the layered VTI medium and calculated travel times and back azimuths, as well as all the sensitivities in Equation 3-5, analytically (Li et al., 2012a, 2012b). We then inverted for the anisotropic velocity and the thickness of each layer as well as for hypocenters of the 68 selected strong events. As the layer interfaces are well characterized by well-logs, we put heavy damping on the layer thicknesses in the inversion.

Figure 4-2 shows the map and side views of the determined locations of the 68 selected strong microseismic events. All these events are located within the Bakken and Lodgepole formations, and no strong events are located beneath the Bakken formation. Also, more than half of the selected events are located in two clusters at about 150 and 100 meters above the Bakken, respectively.
Figure 4-3 shows the final inverted six-layer anisotropic (VTI) model for the Lodgepole, the Bakken and the Three Forks formations. The anisotropy of the Upper and Lower Bakken shale is quite strong, with Thomsen’s $\epsilon$ and $\gamma$ over 40%. Table 4-1 shows the initial and final vertical $V_p$ and $V_s$ as well as the Thomsen’s parameters $\epsilon$, $\delta$ and $\gamma$, which are converted from the inverted elastic moduli $C_{ij}$. The changes in vertical velocities are less than 7% for all layers, and the changes for Thomsen’s parameters can be larger than 10%, especially for those parameters with very small values. As mentioned before, the initial values of the Thomsen’s parameters come with varying confidence from the well-logs, and therefore we allow some degree of alteration in the inversion with constraints. We address uncertainties and sensitivities due to varying ray coverage later in this paper. It also should be pointed out, as Li et al. (2012a) discussed, that the differential information does not improve the inversion for the layered VTI structure but only the relative locations of the events in our case. This is because we do not parameterize the source regions into small cells for 3-D heterogeneous tomography (Zhang and Thurber, 2006). In that case, the differential travel times would be sensitive to the structure close to the neighboring events.

Dense ray coverage is the key for a successful velocity inversion result. Figure 4-4 and Figure 4-5 show the P- and SH-ray paths used for the anisotropic velocity inversion, respectively. From the side views, we can see clearly that multiple monitoring wells at varying distances help to improve the ray angle coverage, thereby providing better constraints on the anisotropic parameters for the VTI velocity inversion.
Figure 4-6 shows the comparison between the synthetic and observed P-wave and SH-wave travel times (upper) and their residuals (lower) for the 68 selected strong events after the anisotropic velocity inversion with double-difference relocation. The travel time residual for P-waves and SH-waves has a standard deviation of 0.9 and 1.2 milliseconds, respectively; both have near zero mean. These standard deviations are much smaller than those of the previously reported best vendor result in this data set, which is 4.85 milliseconds (Li et al., 2012c). Figure 4-7 shows the comparison between the synthetic and observed back azimuths. The mean of the residual is 2.8°, and the standard deviation of the residual is 8°. One might expect a bit smaller residual considering most of the events are of good quality, but the lack of perforation shots makes the orientations of our geophones less certain.

4.6 Discussion

4.6.1 Clustered Events

Among the 68 strong events, over half are located about 100 to 150 m above the treatment well in the Bakken. We note that 29 events from several different completion stages are closely clustered within a tiny space – less than 20 meters in diameter (cluster 1 in Figure 4-3). These events are among the strongest selected events, and most are detected by three monitoring wells. Figure 4-8(a) shows P-waveform comparisons for the 29 events in cluster 1 at a receiver in well G1. Except for a few events, the cross-correlation coefficients for event pairs are greater than 0.7, indicating that the hypocenters and the source properties are quite similar. The reoccurrence of events in the same location is well known in earthquake seismology and
usually implies reactivation of a fault. Thus, the close clustering of these 29 big events from different stages at about 150 m above the treatment well suggests that a natural fracture swarm or a fault zone is being repeatedly activated due to injection of pressurized fluids during treatment. Figure 4-8(b) shows the P-waveform comparisons for the 7 relatively smaller events from cluster 2 at a receiver in well G3. The events in this cluster are spread more than the events in the cluster 1, so their waveform coherence is weaker.

We also found that all 68 selected strong events only come from the Bakken or the overlying Lodgepole formations. None of the events selected by signal-to-noise ratio come from the Three Forks formation under the Bakken. This is consistent with two other observations. One is that, in some cases, excess produced water in hydraulically stimulated Bakken wells seems to be coming from the water-bearing interval with high permeability in the overlying Lodgepole formation above the Bakken when aggressive treatment parameters are used or when the overlying shale formation fails to vertically contain the fracture propagation (Hassen et al., 2012). The other observation is that cores from the Lodgepole exhibit large vertical fractures (V. Grechka, 2013, personal communication). We infer that hydraulic stimulation has activated pre-existing fractures or faults, thereby creating a conduit up to the water-bearing upper interval and causing clustering of some of the larger microseisms along those fractures or faults. Given this explanation, we expect the occurrence of natural fractures or faults above the Middle Bakken affects the fracturing in the target interval.
4.6.2 Waveform Comparison

We also validate our location and velocity inversion results by generating synthetic waveforms and comparing them with observed ones. The synthetic waveforms are calculated by our in-house finite-difference code that can handle heterogeneous media with up to orthorhombic anisotropy. The source properties used in generating the synthetic waveforms are manually estimated from the observed P- and SH-wave amplitudes and polarities. Figure 4-9 shows the waveform comparison for a typical event from cluster 1 (See Figure 4-2 for geometry). With the final inverted anisotropic velocity model and determined hypocenters, we find not only good agreement between the modeled and observed travel times, but also between modeled and observed waveforms.

Figure 4-10 shows a typical event from Cluster 2, observed by the nearest well G3 (see Figure 4-2). This event is located in the Upper Lodgepole layer. The ray dip angles (measured from the vertical direction looking down) vary from about 40° to 140° for this event, and within this range the wave speed changes considerably due to the anisotropies of the Upper as well as Lower Lodgepole formations, which host this monitoring well. Still, we found that our two-layer anisotropic model for the Lodgepole Formation characterizes the arrival moveout well.

The SH- and SV-arrivals of this event from cluster 2 are also observed in the distant well G4. Figure 4-11 shows considerable observed shear wave splitting due to VTI anisotropy for this event in the well G4. The SV-arrivals in the vertical components, which are shown with black traces, are significantly delayed compared to the SH-arrivals in the horizontal components (marked with blue and green marks for observed and modeled picks, respectively). It is found that the delay times between SH- and SV-waves are about 10 ms, while the total travel times...
are about 180 ms for the first few receivers in the near horizontal direction in well G4. The magnitude of the travel time delay means the SH-waves are about 6% faster than the SV-waves in the horizontal direction, consistent with our determined S-wave anisotropy of about 7% for the Upper Lodgepole Formation from the velocity inversion.

Figure 4-12 and Figure 4-13 show the waveform and travel time comparisons for a typical event from the Bakken formation. Due to the fine layering, very strong anisotropy and high velocity contrast between the Bakken members, the observed waveforms become complex and thus it is hard to pick the travel times in distant wells. As a result, we only use travel time picks from the nearest well G2 for location and velocity inversion for this event. Still, with the final location and joint anisotropic velocity inversion results, we generate synthetic waveforms in wells G1, G2, and G3 and find good agreement between the synthetic and observed waveforms in all these wells. For instance, in the distant well G3 the SH-phases and their moveout across the receivers are well matched between the observed and synthetic ones (Figure 4-12); in well G1, the observed waveforms are matched well with synthetic waveforms, even for receivers in the complex Bakken formation (Figure 4-13). Note in Wells G1 and G3 the P-waves are difficult to observe, and the distinguishable matched phases are S-waves.

It should be noted that strong P- and S-coda waves are often observed in the field data for different events at different receivers, as shown in Figure 4-9 through Figure 4-13. The reverberating pattern of the coda waves, however, cannot be reproduced in the synthetic waveforms by our 1-D layered VTI velocity model, except for some wiggles which are related to interface reflections. As discussed above, the Lodgepole Formation is expected to contain ubiquitous natural fractures, thus the reverberating coda waves can be related to the wave
scattering of the induced microseismic events by the fractures. More detailed quantitative study of the coda waves needs rigid modeling of fracture scattering, and since the distribution of the fractures is unknown, many random cases need to be tried. The hybrid method presented in Chapter 5 can be used for such quantitative study.

4.7 Conclusions

In this paper, we present our results locating 68 relatively strong microseismic events from the Bakken and its adjacent formations in the Beaver Lodge area of North Dakota using double-difference constraints and simultaneous anisotropic velocity inversion. We found that a six-layer simple model with VTI anisotropy can characterize quite well the information observed by four monitoring wells separated by up to 1500 m. The simultaneous anisotropic velocity inversion significantly reduces the travel time residuals compared to standard location techniques. In general, we found very strong anisotropy in the shale members of the Bakken formation. The adjacent formations, consisting mainly of mudstones or siltstones, are also moderately anisotropic. The uncertainty analysis found that the travel time has relatively low sensitivities with respect to elastic moduli $C_{13}$ and $C_{33}$ in most layers due to limited ray coverage (Appendix A). Thus the relative uncertainty in Thomsen’s $\delta$, which involves $C_{13}$ and $C_{33}$, is larger than other anisotropic parameters. The anisotropic parameters inverted from joint velocity inversion can of course be used to locate other small events in this monitoring project.

Synthetic waveforms were also generated using the inverted hypocenters and the anisotropic velocity model. We compared the synthetic waveforms with the observed ones to verify our results based on the high-frequency ray approximation, finding satisfactory
agreement. Waveform matching of some complex phases beyond first arrivals further validates our results. The match is satisfactory even at receivers not used for locating these events.

Swarms of strong events are found to occur repeatedly in the Lodgepole formation, high above the treatment well. Waveforms from the clustered events show strong similarities, indicating that the locations and source properties of these events are very close and suggesting reactivation of preexisting Lodgepole fractures, likely through some water bearing fault or fracture conduits. It may be possible to optimize completion parameters in areas where these fractures exist if we can recognize them. Certainly, by properly locating events, microseismic monitoring is a valuable tool for detecting hydraulic fractures growing out of the designed zone and helps engineers properly evaluate fracture height growth and estimate the effectively stimulated reservoir volume.

Acknowledgements

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4.8 References


Kayles, K. et al., 2011. Comparison of microseismic results from the Bakken formation processed by three different companies: integration with surface seismic and pumping data, SEG extended abstract.


Figure 4-1. Seismic waveforms recorded by three component geophones in G1, G2, G3 and G4 from a strong event in cluster 2 (see Figure 4-3). The magenta lines represent the P- and SH-wave picks. Traces from two horizontal components and one vertical component are overlaid with different colors (black, blue and red). Due to large distance (>1 km) and low signal-noise ratios in array G4, only picks from G1, G2 and G3 were used for velocity inversion and the location of this event.
Figure 4-2. The microseismic event locations from the anisotropic velocity inversion in the map view (top) and side view (bottom). Blue triangles represent the four arrays (G1, G2, G3 and G4), and the blue numbers are the geophone ID’s used for later discussion. Note the geophone sequence starts from the top to the bottom of the well G4, then following by the well G3, G2, and G1. The black line indicates the portion of the horizontal well path in the Middle Bakken. The green shadow zone indicates the Bakken formation. Red dots represent the determined locations of the 68 selected strong events. Two blue boxes denote the locations of two groups of clustered events in the Lodgepole.
Figure 4-3. Final layered anisotropic (VTI) velocity model inverted from the joint anisotropic velocity inversion. The vertical axis is the depth relative to a special origin in meters. The blue and red lines are P-wave and S-wave velocities, respectively. The solid and dashed lines are the vertical and horizontal velocities, respectively. The green shadow zone indicates the Bakken formation.
Figure 4-4. The ray coverage of P-waves for the anisotropic velocity inversion in the map view (top) and side view (bottom). The ray paths from the located events (red stars) to each geophone (blue triangles) are shown with black lines. The bold red line shows the horizontal treatment well path in the Middle Bakken. The background colors indicate the vertical P-wave velocities.
Figure 4-5. Same with Figure 4-4, but for SH-waves. The background colors indicate the vertical S-wave velocities.
Figure 4-6. Comparison between the synthetic and observed P-wave and SH-wave arrivals (upper) and their residuals (lower). To the left of the bold black line are the P-wave travel times for the 68 events used for the anisotropic velocity inversion, and to the right are the SH-wave travel times. The travel times are first ordered according to the observation type (P or SH) and then according to the event ID; for each event, the travel times are ordered according to the receiver number (Figure 4-2). Most of the residuals are about 1% or less of the total travel times.
Figure 4-7. Comparison between the synthetic and observed back azimuths for the P-waves. The mean and standard deviation of the residuals are $2.8^\circ$ and $8^\circ$, respectively.
Figure 4-8. a) P-wave coda waveforms in the radial component of a geophone in well G1 for the 29 strong microseismic events in cluster 1 aligned by waveform cross-correlation, and b) aligned P-wave coda waveforms in the radial component at a geophone in well G3 for the 7 events in cluster 2. The blue line indicates the onsets of the P-arrivals. The geometry is shown in Figure 4-2.
Figure 4-9. Comparisons between modeled and observed waveforms (lower panel) and enlarged view of a few traces (upper panel) for a typical event from cluster 1. The black traces are the observed waveforms, and the colored traces (magenta) are the modeled ones; the blue
markers are the picked P- and SH-wave travel times, and the green ones are the modeled P- and SH-wave travel times. Traces from the North components in wells G3, G2 and G1 are shown from bottom up.
Figure 4-10. Comparisons between modeled and observed waveforms for a typical event from Cluster 2. The legends are the same as in Figure 4-9. Traces from the East components of well G3 are shown.
Figure 4-11. The same event from Cluster 2 as shown in Figure 4-10, observed in the vertical components of the distant well G4. The blue and green markers are the picked and modeled SH-wave travel times in the horizontal components, respectively. The SV-arrivals can be clearly observed about 10 ms delayed after the SH-wave arrivals, especially at the first few geophones, due to the shear wave splitting. Note in this well P-wave arrivals are not picked and not used due to low signal-to-noise ratio.
Figure 4-12. Comparisons between modeled and observed waveforms for a typical event from the Bakken. The markers are the same as in Figure 4-9. Traces in the East components of wells G1, G2 and G3 are shown.
Figure 4-13. Same as Figure 4-12, but traces in the North components of wells G1 and G2 are shown. The upper panel shows the close-up of a few typical traces from receivers within the Bakken formation in well G1 and the lower panel shows all traces in wells G1 and G2.
| VTI Parameters | Initial Model | | | Inverted Model | | | |
|---------------|---------------|-----|-----|---------------|-----|-----|
|               | \( V_p \) (m/s) | \( V_s \) (m/s) | \( \varepsilon \) | \( \delta \) | \( \gamma \) | \( V_p \) (m/s) | \( V_s \) (m/s) | \( \varepsilon \) | \( \delta \) | \( \gamma \) |
| Upper Lodgepole | 5776.1 | 3143.5 | 0.057 | 0.028 | 0.057 | 5738.8 | 3046.5 | 0.060 | 0.021 | 0.077 |
| Lower Lodgepole | 5214.7 | 2822.7 | 0.167 | 0.084 | 0.167 | 5051.5 | 2700.6 | 0.175 | 0.074 | 0.188 |
| Upper Bakken | 3156.8 | 1822.9 | 0.487 | 0.243 | 0.487 | 3337.5 | 1867.5 | 0.506 | 0.236 | 0.516 |
| Middle Bakken | 4765.8 | 2796.5 | 0.059 | 0.029 | 0.059 | 4710.4 | 2648.6 | 0.069 | 0.014 | 0.044 |
| Lower Bakken | 3303.1 | 1889.1 | 0.386 | 0.193 | 0.386 | 3381.2 | 1858.1 | 0.421 | 0.139 | 0.409 |
| Three Forks | 4666.5 | 2626.6 | 0.120 | 0.060 | 0.120 | 4492.8 | 2490.2 | 0.135 | 0.037 | 0.112 |

Table 4-1. Comparison between the initial and inverted anisotropic parameters.
Chapter 5  Detecting Fractures after Induced Microseismicity: A Hybrid Method for Modeling Scattered Waves from Fractures

Summary

A hybrid method to model the shear (SH) wave scattering from 2-D fractures embedded in a heterogeneous medium is developed by coupling the boundary element method (BEM) and the finite difference method (FDM) in the frequency domain. FDM is used to propagate an SH wave from a source through heterogeneities to fractures embedded in small local homogeneous domains surrounded by artificial boundaries. According to Huygens’ Principle, the points at these artificial boundaries can be regarded as ‘secondary’ sources and their amplitudes are calculated by FDM. Given the incident fields from these point sources, BEM is applied to model the scattering from fractures and propagate them back to the artificial boundaries. FDM then continues propagating the scattered field into the heterogeneous medium by taking the scattered field at the boundaries as ‘secondary’ sources. A numerical iterative scheme is also developed to account for the multiple scattering between different sets of fractures. The hybrid method can calculate scattering from different fractures very fast, thus Monte Carlo simulations for characterizing the statistics of fracture attributes can be performed efficiently. To verify the hybrid method, we compared scattering from multiple fractures embedded in a homogeneous space by our method and a pure BEM; also, we compared our method with the time-domain finite-difference method for vertical fractures in a layered
medium. Good agreements are found. The hybrid method is also applied to calculate the wave scattered from fractures embedded in complex media.

5.1 Introduction

Precise modelling and understanding of seismic wave scattering from subsurface fractures in a heterogeneous medium is essential for imaging the fractures from seismic survey data. This forward modelling problem has been extensively discussed in the literature using different physical models of fractures (Schoenberg 1980; Hudson et al. 1996; Liu et al. 2000), combined with analytical and numerical techniques, including finite difference method (FDM), boundary element method (BEM) and the finite element method (FEM). Liu et al. (1997) applied representation theorems to analytically calculate the scattered waves from fractures based on Kirchhoff approximation. Sánchez-Sesma & Iturrarán-Viveros (2001) applied an analytical approach to calculate scattering and diffraction from a crack with traction-free surface condition. Coates & Schoenberg (1995), Krüger et al. (2005), Groenenboom & Falk (2000) and Vlastos et al. (2003) used an effective medium theory and FDM to calculate seismic wave propagation through the fractures. When dealing with fractures of complex geometries, the effective medium method may have accuracy issues due to the limitation of grid meshing in the traditional FDM. Instead of calculating the effective elastic constant of each mesh grid, Slawinski & Krebes (2002), Zhang (2005) and Zhang & Gao (2009) directly impose the boundary conditions on auxiliary grid points surrounding the fractures. The complexity and computational cost of this method could be very high when non-planar fractures need to be considered, or
when the distances between fractures are smaller than the seismic wavelength, as dense meshing is required. Nakagawa et al. (2003) applied FEM to calculate 3-D elastic wave scattering from parallel fractures in a single horizontal layer. Pointer et al. (1998), Iturrarán-Viveros et al. (2005) and Iturrarán-Viveros et al. (2008) applied an indirect BEM to calculate the scattered field from fractures and cracks. Chen et al. (2011) applied BEM to model the scattering of shear (SH) waves from 2-D fractures with slip boundary conditions. Compared to the analytical, FD and FE methods, BEM is more flexible and accurate in implementing complicated geometries and boundary conditions. It is also more computationally efficient since one less computational dimension is needed compared to FDM or FEM. However, BEM requires the analytical expression of Green’s functions of the medium, which makes it only applicable for a few ideal scenarios, such as a homogeneous space or half-space. This requirement greatly restricts the practical applications of BEM to complicated geophysical problems. Bouchon & Coutant (1994) developed a boundary element—discrete wavenumber method where the Green’s functions are evaluated by wavenumber summation. Their method may only be suitable for propagating seismic waves in a layered homogeneous medium. Goto et al. (2010) presented a method for coupling the boundary integral equation method and the FEM. Their method is mostly focused on modelling spontaneous ruptures. In this paper, we present a hybrid method to model the scattering from fractures in heterogeneous media by coupling BEM and FDM. The complicated Green’s function in a heterogeneous medium is handled by FDM while the complex boundary conditions and geometry of fractures are modelled by BEM. The advantages of both FDM and BEM are effectively combined in our hybrid method. Since our hybrid method is developed in the frequency domain, it can calculate the
scattered field for different fracture distributions, source and receiver configurations, as well as source wavelets, very efficiently. When using Monte Carlo simulations to characterize the statistical properties of scattering from fractures, the efficiency issue becomes critical.

5.2 BEM and FDM in the Frequency Domain

In this section, we briefly introduce the BEM (Chen et al. 2011) to calculate SH wave scattering from fractures in a homogeneous free space, and the FDM adapted from Hustedt et al. (2004) to calculate the SH wave propagation in a heterogeneous medium.

5.2.1 BEM Modelling of Scattering from Fractures

The displacement of the scattered field (Aki & Richards 1980) from a fracture in the free space is

\[
    u^{\text{sc}}_k(x) = \int_s [u_k(\xi)] C_{kjq}(\xi) \frac{\partial G_p^i(x, \xi)}{\partial \xi_j} n_j(\xi) \, d\xi, 
\]

(5-1)

where \( \xi \) is a point on the 2-D fracture surface \( s \), as shown in Figure 5-1; \( C_{kjq}(\xi) \) is the elastic tensor; \( G_p^i(x, \xi) \) is the \( i \)th displacement component of the Green’s function at point \( x \) due to a unit force in the \( p \)th direction at point \( \xi \) on the fracture surface; \( n_j \) is the \( j \)th component of the normal vector \( n \) at the fracture surface \( s \); \( [u_k(\xi)] \) is the \( k \)th component of the displacement discontinuity.
where \( u_k^+(\xi) \) and \( u_k^-(\xi) \) are the total displacement on the upper and lower surfaces of fracture, respectively. The total displacement field \( u_k(\xi) \) is the sum of the incident \( u_k^i(\mathbf{x}) \) and scattered displacements. In this paper, the displacement discontinuity is determined from the linear slip condition (Schoenberg 1980), which assumes that the displacement discontinuity is linearly proportional to the traction on the fracture surface, and the traction is continuous across the fracture. For the SH wave, we have

\[
[u_k(\xi)] = u_k^+(\xi) - u_k^-(\xi),
\]

(5-2)

where \( Z_t \) is the fracture compliance. Inserting Equation 5-3 into Equation 5-1, the scattered field can be expressed as

\[
[u_2(\xi)] = Z_t(\xi)[\sigma_{21}(\xi)n_1(\xi) + \sigma_{23}(\xi)n_3(\xi)] \\
= Z_t(\xi) \left[ \mu \frac{\partial u_2(\xi)}{\partial \xi_1} n_1(\xi) + \mu \frac{\partial u_2(\xi)}{\partial \xi_3} n_3(\xi) \right].
\]

(5-3)

The displacement at any \( x \) is the sum of the incident and scattered displacements

\[
u_2(x) = u_2^i(x) + u_2^{\text{scn}}(x).
\]

(5-5)

For a point on the fracture surface \( x(x_1, x_3) \in s \), it should satisfy Equation 5-5.
where \( \mathbb{F} \) denotes the hypersingular integral equation or a Cauchy’s principal value. We take derivatives of Equation 5-7 over \( x_1 \) and \( x_3 \), respectively

\[
\frac{\partial u_2(x)}{\partial x_1} = \frac{\partial u_2(x)}{\partial x_1} + \int_s Z_s(\xi) \mu^2 \left[ \frac{\partial u_2(\xi)}{\partial \xi_1} n_1(\xi) + \frac{\partial u_2(\xi)}{\partial \xi_3} n_3(\xi) \right] \times \left[ \frac{\partial^2 G(x, \xi)}{\partial \xi_1 \partial x_1} n_1(\xi) + \frac{\partial^2 G(x, \xi)}{\partial \xi_3 \partial x_1} n_3(\xi) \right] d\xi,
\]

(5–7)

and

\[
\frac{\partial u_2(x)}{\partial x_3} = \frac{\partial u_2(x)}{\partial x_3} + \int_s Z_s(\xi) \mu^2 \left[ \frac{\partial u_2(\xi)}{\partial \xi_1} n_1(\xi) + \frac{\partial u_2(\xi)}{\partial \xi_3} n_3(\xi) \right] \times \left[ \frac{\partial^2 G(x, \xi)}{\partial \xi_1 \partial x_3} n_1(\xi) + \frac{\partial^2 G(x, \xi)}{\partial \xi_3 \partial x_3} n_3(\xi) \right] d\xi,
\]

(5–8)

by applying a theorem proved by Martin & Rizzo (1989). We now turn the displacement boundary integral Equation 5-1 into two traction-related boundary integral Equations 5-6 and 5-8. Solving these two equations provides values of the displacement derivative \( \frac{\partial u_2(x)}{\partial x_1} \) and \( \frac{\partial u_2(x)}{\partial x_3} \) across the fracture. By inserting these two derivatives into Equation 5-4, the displacement field scattered from the 2-D fracture can be finally calculated. Also, the derivative of the scattered field is
which is needed in the coupling of BEM with FDM.

5.2.2 Modelling Wave Propagation in a Heterogeneous Medium with FDM

In the frequency domain, the SH wave equation is

\[
\frac{\partial^2 u_{\text{sc}}(x)}{\partial x^2} = \int Z_i(\xi) \mu^2 \left\{ \frac{\partial u_2(\xi)}{\partial \xi_1} n_1(\xi) + \frac{\partial u_2(\xi)}{\partial \xi_3} n_3(\xi) \right\} \times \left[ \frac{\partial^2 G(x, \xi)}{\partial \xi_1 \partial x} n_1(\xi) + \frac{\partial^2 G(x, \xi)}{\partial \xi_3 \partial x} n_3(\xi) \right] d\xi,
\]

(5-9)

where \(\rho(x, z)\) is the density, \(u(x, z, \omega)\) is the displacement, \(\omega\) is the angular frequency, \(\mu(x, z)\) is the modulus of rigidity and \(s(x, z, \omega)\) is the source term. We discretize Equation 5-10 with a low dispersion fourth-order finite difference scheme in space by adapting Equation A3 in Hustedt et al. (2004)
\[-\omega^2 \rho_{i,j} u_{i,j} = \]
\[
\frac{1}{\Delta^2} \left\{ \frac{9}{8} \left[ \mu_{i+1/2,j} \left( \frac{9}{8} (u_{i+1,j} - u_{i,j}) - \frac{1}{24} (u_{i+2,j} - u_{i-1,j}) \right) - \mu_{i-1/2,j} \left( \frac{9}{8} (u_{i,j} - u_{i-1,j}) - \frac{1}{24} (u_{i+1,j} - u_{i-2,j}) \right) \right] - \mu_{i,3/2,j} \left( \frac{9}{8} (u_{i+2,j} - u_{i+1,j}) - \frac{1}{24} (u_{i+3,j} - u_{i,j}) \right) - \mu_{i,3/2,j} \left( \frac{9}{8} (u_{i-1,j} - u_{i-2,j}) - \frac{1}{24} (u_{i,j} - u_{i-3,j}) \right) \right] + \frac{1}{\Delta^2} \left\{ \frac{9}{8} \left[ \mu_{i,j+1/2} \left( \frac{9}{8} (u_{i,j+1} - u_{i,j}) - \frac{1}{24} (u_{i,j+2} - u_{i,j-1}) \right) - \mu_{i,j-1/2} \left( \frac{9}{8} (u_{i,j} - u_{i,j-1}) - \frac{1}{24} (u_{i,j+1} - u_{i,j-2}) \right) \right] - \mu_{i,j+3/2} \left( \frac{9}{8} (u_{i,j+2} - u_{i,j+1}) - \frac{1}{24} (u_{i,j+3} - u_{i,j}) \right) - \mu_{i,j-3/2} \left( \frac{9}{8} (u_{i,j-1} - u_{i,j-2}) - \frac{1}{24} (u_{i,j} - u_{i,j-3}) \right) \right] + S_{i,j}, \right\}

(5-11)

where \(\Delta\) is the spacing of the uniform discretization grid and \(S_{i,j}\) is the source term for the displacement field. We use a regular finite difference scheme instead of the mixed-grid scheme (Hustedt et al. 2004). This is because point source excitation in the mixed-grid scheme would result in numerical noises and obscure the weak scattered field. In general, Equation 5-11 can be written as

\[ A u = S, \]

(5-12)

where \(A\) is the impedance matrix constructed by the FD operators shown in Equation 5-11 and \(S\) is the source vector. To solve Equation 5-12, we can use the LU factorization (Operto et al. 2007) to decompose the matrix \(A\). We also implement Perfectly Matched Layers (PMLs) in the
surrounding areas to absorb outgoing waves by introducing a frequency-dependent complex part into the finite-difference coefficients within the PML region (Hustedt et al. 2004).

5.3 Hybrid Method

In this section, we first introduce the approach to coupling BEM with FDM for calculating the primary scattering from an individual set of fractures without considering the second or higher order interactions either between different sets of fractures or between fractures and heterogeneities. Then, we discuss an iterative method to include higher orders of scattering.

5.3.1 Coupling between BEM and FDM

The basic idea of the hybrid method comes from Huygens’ principle and the representation theorems (Aki & Richards 1980). Given a point source in a heterogeneous medium, as shown in Figure 5-2(a), the displacement at a certain point ‘within’ a local domain Σ, surrounded by boundary Γ, comes from the contribution of the displacement and its derivative along the boundary Γ, assuming that no body force exists within Σ. We can express the displacement Green’s function using a boundary integral equation

$$ G(x, x_0) = \oint_{\Gamma} \left[ \frac{\partial G(\xi, x_0)}{\partial n} G(x, \xi) - G(\xi, x_0) \frac{\partial G(x, \xi)}{\partial n} \right] d\xi, \quad (5-13) $$
where \( x_0 \) and \( x \) are the source and receiver positions, respectively; \( \xi \) is a point along boundary \( \Gamma \) and \( G(x, \xi) \) is the displacement Green’s function for SH wave. We assume that the local domain \( \Sigma \) is homogeneous such that the Green’s function \( G(x, \xi) \) can be analytically expressed as

\[
G(x, \xi) = \frac{i}{4\pi \mu} H_0^{(1)}(k|x - \xi|),
\]

where \( H_0^{(1)} \) is the zero order of the first kind Hankel function. The displacement Green’s function \( G(\xi, x_0) \) and its derivative \( \frac{\partial G(\xi, x_0)}{\partial n} \) along \( \Gamma \) in Equation 5-13, containing the effect from all heterogeneities outside of \( \Sigma \), can only be evaluated by FDM. They represent the amplitudes of a dipole source \( \frac{\partial G(x, \xi)}{\partial n} \) and a monopole source \( G(x, \xi) \), respectively. From Equation 5-13, the ‘incident field’ from the point source \( x_0 \) to a certain receiver point \( x \) within \( \Sigma \), is expressed in terms of the summation of the contributions from many monopole and dipole sources, which are treated as the ‘secondary virtual sources’ according to Huygens’ principal. Given these monopole and dipole virtual sources along \( \Gamma \), BEM can be applied to calculate the scattered field from fractures embedded within the small, local homogeneous domain \( \Sigma \). FDM is then applied to propagate the scattered displacement field \( u^{\text{sc}}(x) \) outward to any location \( x \) outside of \( \Sigma \). According to the representation theorems (Aki & Richards 1980)

\[
u^{\text{sc}}(x) = \oint_{\Gamma} \left[ \frac{\partial u_2^{\text{sc}}(\xi')}{\partial n} G(x, \xi') - u_2^{\text{sc}}(\xi') \frac{\partial G(x, \xi')}{\partial n} \right] d\xi',
\]

where \( \xi' \in \Gamma \). The displacement \( u_2^{\text{sc}}(\xi') \) and its derivative \( \frac{\partial u_2^{\text{sc}}(\xi')}{\partial n} \) represent the contributions of the scattered field from fractures and are calculated by BEM. They are the amplitudes of the
dipole \( \frac{\partial G(x,\xi)}{\partial n} \) and monopole \( G(x,\xi') \), respectively. From Equation 5-15, we decompose the ‘scattered field’ from fracture in terms of the summation of many monopole and dipole virtual sources. Given these sources, FDM is applied to calculate the displacement field outside of \( \Sigma \). We discuss the detailed implementation of the hybrid method in the following paragraphs. To couple FDM and BEM, we first need to discretize the boundary to FDM composed of regular finite difference grids, as shown in Figure 5-3. The reason for placing two boundaries for FDM and BEM in the coupling scheme will be elaborated in Section 3.2. Using the grid points as secondary sources, we can construct the incident field for fractures surrounded by FDM and discretize Equation 5-13 to

\[
G(x, x_0) \approx G^{\text{upper}} + G^{\text{down}} + G^{\text{left}} + G^{\text{right}},
\]

(5-16)

where

\[
G^{\text{upper}} = \sum_{i=2}^{M-1} \left[ - \frac{\partial u(\xi_i, x_0)}{\partial y} G(x, \xi_i) + u(\xi_i, x_0) \frac{\partial G(x, \xi_i)}{\partial y} \right] \Delta
\]

\[
+ \frac{1}{2} \sum_{i=1, i=M} \left[ - \frac{\partial u(\xi_i, x_0)}{\partial y} G(x, \xi_i) + u(\xi_i, x_0) \frac{\partial G(x, \xi_i)}{\partial y} \right] \Delta,
\]

\[
G^{\text{lower}} = \sum_{j=2}^{M-1} \left[ \frac{\partial u(\xi_j, x_0)}{\partial y} G(x, \xi_j) - u(\xi_j, x_0) \frac{\partial G(x, \xi_j)}{\partial y} \right] \Delta
\]

\[
+ \frac{1}{2} \sum_{j=1, j=M} \left[ \frac{\partial u(\xi_j, x_0)}{\partial y} G(x, \xi_j) - u(\xi_j, x_0) \frac{\partial G(x, \xi_j)}{\partial y} \right] \Delta,
\]
where \(M\) and \(N\) are the total grid points along horizontal and vertical directions. Since the corner points only span half-grid length compared to the rest of the points, a weight \(1/2\) is used in Equation 5-17 to account for this difference. The amplitudes of ‘dipole sources’, \(u(\xi_i, x_0), -u(\xi_j, x_0), u(\xi_k, x_0)\) and \(-u(\xi_l, x_0)\), can be obtained directly from FDM. The amplitudes of the ‘monopole sources’, \(-\frac{\partial u(\xi_i, x_0)}{\partial y}, \frac{\partial u(\xi_j, x_0)}{\partial y}, -\frac{\partial u(\xi_k, x_0)}{\partial x}\) and \(\frac{\partial u(\xi_l, x_0)}{\partial x}\), however, need to be evaluated via a fourth-order finite difference approximation, to be consistent with the global fourth-order accuracy of FDM scheme. For instance, the displacement gradient at point \(\xi_n\) (Figure 5-3) is

\[
\frac{\partial u(\xi_n, x_0)}{\partial x} = -\frac{2}{3}u(\xi_{n-1}, x_0) + \frac{2}{3}u(\xi_{n+1}, x_0) + \frac{1}{12}u(\xi_{n-2}, x_0) - \frac{1}{12}u(\xi_{n+2}, x_0)
\]

which requires the displacement values at four different grids, with two inside and two outside boundary \(\Gamma_{\text{FDM}}\).
After the amplitudes of surrounding monopole and dipole sources are determined, we can adopt BEM to calculate the displacement discontinuities across the upper and lower surfaces of the fractures subjected to the incidence from the surrounding sources. Given the displacement discontinuities, the scattered field $u_2^{\text{sca}}$ and the normal derivative of the scattered field $\frac{\partial u_2^{\text{sca}}}{\partial n}$ at another boundary $\Gamma_{\text{BEM}}$ are calculated analytically via eqs (4) and (9).

Finally, we rely on FDM to propagate the scattered field at $\Gamma_{\text{BEM}}$ outward to any outside location $x$. From Equation 5-15, the displacement at a certain point is

$$u_2^{\text{sca}}(x) = \int_{\Gamma_{\text{BEM}}} \left[ \frac{\partial u_2^{\text{sca}}(\xi')}{\partial n} G(x, \xi') - u_2^{\text{sca}}(\xi') \frac{\partial G(x, \xi')}{\partial n} \right] d\xi', \quad (5-19)$$

where $\xi' \in \Gamma_{\text{BEM}}$. The Green’s function $G(x, \xi')$ (monopole source) is directly implemented by FDM, while the dipole source at $\xi'_n$, for instance as shown in Figure 5-3, needs to be implemented by

$$u_2^{\text{sca}}(\xi'_n) \frac{\partial G(x_n, \xi'_n)}{\partial x} = u_2^{\text{sca}}(\xi'_n)$$

$$\times \frac{\frac{2}{3} G(x, \xi'_{n-1}) - \frac{2}{3} G(x, \xi'_{n+1}) + \frac{\Delta}{12} G(x, \xi'_{n-2}) - \frac{\Delta}{12} G(x, \xi'_{n+2})}{\Delta}, \quad (5-20)$$

which requires injecting the source excitations $2u_2^{\text{sca}}(\xi'_n)/3\Delta, -2u_2^{\text{sca}}(\xi'_n)/3\Delta, u_2^{\text{sca}}(\xi'_n)/12\Delta$ and $-u_2^{\text{sca}}(\xi'_n)/12\Delta$ at four positions at $\xi'_{n-2}, \xi'_{n-1}, \xi'_{n+1}$, and $\xi'_{n+2}$, in the right-hand side (RHS) of Equation 5-12, as shown in Figure 5-3; the numerical implementation of the monopole term in Equation 5-19 requires injecting the source excitation $\frac{\partial u_2^{\text{sca}}(\xi'_n)}{\partial x}$ in the RHS of Equation 5-12 at the corresponding position $\xi'_n$. 

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The aforementioned BEM can be applied to calculate multiple interactions among fractures embedded within the same local domain (Chen et al. 2011). When the separation between different fractures is larger than the wavelength, it is unrealistic to circumscribe all fractures into one big domain and assume the homogeneity of the whole domain. Therefore, we need an iterative method to include the multiple scattering, as discussed in Section 5.3.2.

5.3.2 Iterative Method for Multiple Scattering

In this section, we show how to apply an iterative method to calculate the multiple scattering between different sets of fractures or between fractures and heterogeneities. The idea of the iterative scheme is to calculate Born series to account for multiple scattering. In Appendix A, we briefly discuss the convergence conditions of the iterative scheme for the multiple scattering. To simplify the discussion, we assume that there are two sets of fractures A and B embedded in a heterogeneous medium. We first calculate the primary scattered field \( u_A^{sca} \) and \( u_B^{sca} \) from fracture set A and B individually, and then use FDM to propagate them outward simultaneously into the heterogeneous medium. After some interactions with heterogeneities, the primary scattering propagates onto both fracture sets A and B, hence resulting in second scattering. To calculate the second scattered field, we follow the same procedure described in Section 5.3.1. For fracture set A, the incident wave field along \( \Gamma_{FDM}^A \) is

\[
\begin{align*}
\mathbf{u}^{inc2}(\mathbf{x})_A &= \oint_{\Gamma_{FDM}^A} \left[ \frac{\partial u^{sca}(\xi)}{\partial n} G(\mathbf{x}, \xi) - u^{sca}(\xi) \frac{\partial G(\mathbf{x}, \xi)}{\partial n} \right] d\xi \tag{5-21}
\end{align*}
\]

where \( u^{sca}(\xi) \) is calculated by
which originates from the primary scattered fields \( u_{A}^{sca} \) and \( u_{B}^{sca} \), respectively, propagated numerically by FDM. The incident field on fractures set \( B \) is calculated with the same method.

BEM is applied again to calculate the second scattering \( u_{A}^{sca2} \) and \( u_{B}^{sca2} \) from fracture sets \( A \) and \( B \) due to the new incident field \( u_{inc2}^{A} \). Afterwards, FDM is used to propagate the second scattered fields \( u_{A}^{sca2} \) and \( u_{B}^{sca2} \) outward. We iterate the aforementioned steps to account for the multiple scattering until the energy of the higher scattering is negligible. The iterative scheme (eqs 21 and 22) assures the causality such that a lower order of scattering is the source for the next higher order of scattering. The temporal sequence of the different orders of the scattered field is determined implicitly by the phases.

We use boundary \( \Gamma_{FDM} \) when propagating the wave field by FDM into a local domain, and use the boundary \( \Gamma_{BEM} \) to propagate the scattered field out of the local domain, as shown in Figure 5-3. According to Huygens’ principle, \( \Gamma_{BEM} \) is an outward-radiation boundary. As a result, Equation 5-19 can only provide the correct primary scattered field \( u^{sca}(x) \) at \( x \) ‘outside’ the domain surrounded by \( \Gamma_{BEM} \), but not for any point ‘within’ \( \Gamma_{BEM} \). This causes issues for the calculation of secondary scattering from fractures, for example, the calculation of the displacement gradient \( \frac{\partial u^{sca}(\xi_{n})}{\partial x} \) for a grid point \( n \) at \( \Gamma_{FDM} \) requires displacement values \( u^{sca}(\xi_{n-1}), u^{sca}(\xi_{n-2}), u^{sca}(\xi_{n+1}) \) and \( u^{sca}(\xi_{n+2}) \) at four different grids according to Equation 5-18. If the separation between \( \Gamma_{FDM} \) and \( \Gamma_{BEM} \) is smaller than two grids, \( u^{sca}(\xi_{n+1}) \)
and $u^{\text{sca}}(\xi_{n+2})$ could fall inside the domain surrounded by $\Gamma_{\text{BEM}}$ and then their values are not calculated correctly. This leads to an incorrect computation of $\frac{\partial u^{\text{sca}}(\xi_n)}{\partial x}$. We can easily solve this issue by placing $\Gamma_{\text{FDM}}$ two grids bigger than $\Gamma_{\text{BEM}}$. It should be noted that these two boundaries can be collocated if we only consider the primary scattering from the fractures.

5.3.3 Monte Carlo Simulations

In order to perform Monte Carlo simulations to characterize the statistical properties of scattering from fractures, we need to calculate the scattering from random fracture realizations. Given that the background medium remains unchanged and does not depend on the geometries and properties of fractures, we only need to perform the LU factorization, which takes a major part of the computational time, on the impedance matrix $A$ ‘once’ for each frequency

$$L(\omega) \cdot U(\omega) = P(\omega) \cdot A(\omega) \cdot Q(\omega),$$  \hspace{1cm} (5-23)

where $L(\omega)$ and $U(\omega)$ are the lower and upper triangular matrices, respectively; $P(\omega)$ and $Q(\omega)$ are the row and column permutation matrices for numerical stability. To calculate the response of the medium to different source excitation $S(\omega)$, for example, point sources or secondary sources from fracture scattering, we only change the RHS of Equation 5-12 with the corresponding source term. Given $L$, $U$, $P$ and $Q$, solving Equation 5-12 takes negligible computational time. Therefore, Monte Carlo simulations can be implemented very efficiently.
5.4 Numerical Examples

In this section, we first provide two examples that compare the results from the hybrid method with the ones from the BEM in a homogeneous medium. We then show the simulations of the scattered waves from fractures embedded in a horizontally layered medium and a more complex Marmousi model. In Appendix B, we provide a benchmark model that calculates scattered field from 10 vertical fractures embedded in a three layered medium using our hybrid method and a time-domain finite difference method (TDFD, Coates & Schoenberg 1995). For the first example, a horizontal fracture is embedded in a homogeneous medium with $V_s = 2500 \text{ ms}^{-1}$ and $\rho = 2200 \text{ kgm}^{-3}$, as shown in Figure 5-4. The tangential compliance of the fracture is $10^{-9} \text{ mPa}^{-1}$. The fracture is 100 m in length. We denote the local domain with a white box surrounded by the artificial boundary $\Gamma_{\text{BEM}}$. The source has unit amplitude. The frequency of the incident wave is 20 Hz, corresponding to a wavelength of 125 m. The black dot represents the source location, and the white crosses represent the receiver locations. Figure 5-4 shows the amplitude of the scattered field, which exhibits strong scattering patterns in the forward and backward directions. The tip scattering from the fracture is relatively weak due to the direction of the incident wave. Figure 5-5 compares the amplitude and phase of the scattered field between the hybrid method and BEM. The maximum differences in both amplitude and phase are less than one percent.

In the second example shown in Figure 5-6, we have four inclined fractures embedded in the same background medium as in the previous example. The incident field is also the same as in the first example. The total scattered field shows a strong interference pattern, particularly between the inner pair of fractures. The scattering in the forward direction is much
stronger than in the backward direction. We use the iterative method to calculate the multiple scattering between different fractures (Figure 5-7). Since the first iteration only includes the single scattering from each fracture, there is some difference with the BEM solution. After five iterations, the result from the hybrid method converges to the result from BEM, with the difference smaller than one percent.

In most real cases, velocity heterogeneities exist and obscure the scattering signals from the fractures. In the next two examples, we show the scattering from fractures embedded in a layered model and a modified Marmousi model. For the layered model as shown in Figure 5-8, we placed 12 randomly distributed fractures, four of which cross each other within the same artificial boundaries. The densities for the three layers from top to bottom are 2000, 2200 and 2500 kg m$^{-3}$, respectively. The compliance of each fracture is $2.5 \times 10^{-10}$ m$^{-1}$ Pa$^{-1}$. Figure 5-9 shows the incident waves (red) and the scattered waves (blue) at the receivers. The incident waves refer to the response of the background medium to the source excitation. We see clear direct arrivals and primary reflections from the layer interfaces, while weak multiple reflections among the interfaces are difficult to observe. The scattered waves (amplitudes amplified by 10 times to show the details) arrive between the primary reflections from interfaces, as expected. The scattering from the fractures consists of single scattering from tips of fractures, multiple scattering among fractures, as well as multiple interactions between fractures and layer interfaces. The scattered waves show a coherent pattern, for example, similar waveforms are observed at different receivers with varying delay times.
We also placed 12 randomly distributed fractures in a modified Marmousi model (Figure 5-10). The fractures have varying lengths, inclinations and compliances ($5 \times 10^{-10} mPa^{-1}$). In an ideal case, our hybrid scheme requires homogeneity within the boundary FDM. Practically, if the medium heterogeneity is weak within $\Gamma_{FDM}$, our hybrid scheme can still be applicable. The incident waves show complex patterns due to the significant heterogeneities of the model, and the signals arriving after 1 s are relatively weak, as shown in Figure 5-11. In comparison, multiple scattering from fractures reverberate between 0.6 and 1.5 s. Though propagating through the complex model, coherent patterns can still be observed in the scattered waves. Figure 5-12 shows a typical incident waveform and a scattered waveform at a receiver (‘diamond’ in Figure 5-10). The maximum amplitude of the scattered wave is about 5 per cent of that of the incident wave. The amplitudes of the multiple scattering decay with the increasing scattering orders, and are negligible after the 5th iteration (Figure 5-13). The dominant energy spectrum shifts to a higher frequency band with the increasing scattering order, as higher frequency contents are scattered more strongly by the fractures. The first two orders of the scattered waves contain most of the total scattered energy, and we may only need two iterations to get acceptable results and, therefore, save computational time.

5.5 Conclusion

In this paper, we used a hybrid method in the frequency domain to model the SH scattered wave from fractures embedded in a 2-D heterogeneous medium by coupling BEM and FDM through two artificial coupling boundaries. We verified the method by comparing it with
the TDFD method in a case of multiple vertical fractures in a layered medium. The hybrid method is then applied to calculate scattering from non-vertical fractures embedded in a three-layer medium and in an even more complex Marmousi model, which could be quite challenging for BEM or FDM alone. The scattered fields from fractures in both examples show coherent patterns, even after propagating through a medium with considerable velocity variation. We also presented an iterative scheme to calculate multiple scattering between different sets of fractures as well as multiple interactions between fractures and surrounding medium heterogeneities. By isolating scattered waves from incident waves, as well as separating different orders of scattered waves, our method could provide a new approach to study coherence and patterns of scattering in seismic coda waves. The hybrid method is fast. It can be used to perform Monte Carlo simulations to characterize the statistics of scattering signals from random subsurface fracture networks due to its high efficiency of propagating scattered wave fields.

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5.6 References


Figure 5-1. Geometry of a fracture. The fracture has two surfaces $S^+$ and $S^-$. The local coordinate system can be different with the global one used by FDM.
Figure 5-2. Cartoon illustration of the hybrid method: using a boundary integral equation to represent the incident displacement field and the scattered displacement field from a fracture, which is embedded in a local homogeneous domain $\Sigma$ surrounded by an artificial boundary $\Gamma$ in a global heterogeneous medium. Black arrows represent incident field (a) and scattered field (b) propagated in the heterogeneous medium using FDM. Blue arrows represent incident field (a) and scattered field (b) propagated inside $\Sigma$ using Green’s function and BEM, respectively. Red dash arrows in (a) and (b) represent the displacement field we want to calculate in Equations 5-13 and Equation 5-15, respectively.
Figure 5-3. Two artificial boundaries $\Gamma_{FDM}$ and $\Gamma_{BEM}$ used in the hybrid method. Given the incident field calculated by FDM, boundary $\Gamma_{FDM}$ behaves as secondary sources for BEM. Given the scattered field calculated by BEM, boundary $\Gamma_{BEM}$ behaves as secondary sources for FDM. Calculating the amplitude $\frac{\partial u(\xi_n)}{\partial n}$ of a dipole source at $\xi_n$ on $\Gamma_{FDM}$ requires the displacement values at $\xi_{n-2}$, $\xi_{n-1}$, $\xi_{n+1}$ and $\xi_{n+2}$ provided by FDM to achieve fourth-order accuracy (Equation 5-18). The implementation of a dipole source $\frac{\partial G(\xi'_n)}{\partial n}$ at $\xi'_n$ to propagate the scattered field outward needs to implement four monopole sources at $\xi'_{n-2}$, $\xi'_{n-1}$, $\xi'_{n+1}$ and $\xi'_{n+2}$. The amplitudes of the four monopole sources are determined by the scattered field at $\xi'_n$ and the weight functions (Equation 5-20).
Figure 5-4. The primary scattering from a horizontal fracture. The black dot indicates the source with unit amplitude. The white crosses indicate the locations where the scattered fields are recorded for the comparison between the hybrid method and pure BEM shown in Figure 5-5. The white box indicates the local domain surrounded by $\Gamma_{\text{BEM}}$. 
Figure 5-5. Comparisons of amplitudes (a) and phases (b) of the scattered fields between the hybrid method and pure BEM. The results are essentially the same. The amplitude and phase differences are amplified by 10 times to show the details.
Figure 5-6. Similar to Figure 5-4, but for the total scattered field from 4 fractures. 5 iterations are performed to achieve convergence.
Figure 5-7. Comparisons of the amplitudes between the hybrid method and pure BEM. Five iterations are taken before higher orders of scattering become negligible. The difference is amplified by 10 times to show the details.
Figure 5-8. Velocity model for the layered medium containing 12 fractures. Some fractures cross each other in the same artificial boundaries. The white dot denotes the source. The white crosses denote the locations where the traces in Figure 5-9 are recorded.
Figure 5-9. Incident waves (red) and scattered waves (blue) from the fractures in the layered model. The incident wave refers to the response of the layered medium without the embedded fractures. The amplitudes of the scattered waves are amplified by 10 times to show the details. An inverse Fourier transform is used to synthesize the time-domain traces. The wave fields are sampled at 160 frequencies from 0 to 80 Hz. Ricker wavelet with a central frequency of 20 Hz is used. 5 iterations are used to achieve convergence.
Figure 5-10. The modified Marmousi velocity model. The white dot denotes the source. The white crosses denote the locations where the traces in Figure 5-11 are recorded. The lengths and inclinations of fractures vary but the compliance keeps the same \((5 \times 10^{-10} \text{ m/Pa})\).
Figure 5-11. Similar to Figure 5-9, but for the modified Marmousi model.
Figure 5-12. (a) Incident wave (red), (b) scattered wave (blue) and (c) the total wave (black) with scattered wave (blue) at the position denoted by a ‘diamond’ in Figure 5-10. The amplitudes are normalized to the maximum absolute value of the total wave at this position.
Figure 5-13. Time traces of the first five orders of the scattered wave and their spectrograms from the top to the bottom, respectively. The receiver position is denoted by a ‘diamond’ in Figure 5-10. The color in spectrograms is in the same logarithmic scale. The scattered wave exhibits frequency-dependent characteristics.
Chapter 6  Conclusions

The main objective of the thesis is to characterize the attributes of conventional and unconventional reservoirs through passive seismicity. Three new methods are developed in my research to determine the source mechanisms of reactivated fractures/faults in a conventional oil/gas field in Oman, to determine the distribution of induced microseismicity and the anisotropic velocity structure in an unconventional oil reservoir in North Dakota, U.S., and to model scattered waves from fractures.

In my thesis, first I consider the fractures/faults as seismic sources and analyze their attributes when they are reactivated seismically. The reservoir velocity structure is also determined with induced seismicity and in turn helps improve analysis of source properties. Then I consider fractures/faults as seismic scatterers. The two aspects are complementary in understanding the attributes of fractures/faults. For instance, reverberating coda waves, which are commonly observed in the Bakken microseismic dataset (Chapter 4), cannot be explained by the layered anisotropic velocity structure alone without extensive existence of fracture networks, as some core samples and the occurrence of distant events have suggested. The majority of these pre-existing fractures, however, were not reactivated by the hydraulic fracturing, or were reactivated aseismically, and, thus, stay invisible to the characterization of fractures as seismic sources. Therefore, one of the best means to provide supplementary information for the fracture network in the Bakken formation, in addition to the attributes determined as seismic sources, is to analyze the reverberating coda waves resulting from scattering of seismic energy from these sporadic induced events. This can be an important research topic in the future.
The major conclusions of my research are:

1) The objective function formulated to include matching phase and amplitude information, first arrival P polarities and S/P amplitude ratios between the modeled and observed waveforms, yields reliable source mechanism solutions through an optimized grid search. For different hypocenters and source types, comprehensive synthetic tests show that our method is robust to determine the focal mechanisms even when there is considerable velocity inaccuracy.

2) For the 40 studied events in a conventional oil/gas field in Oman, we found that the hypocenters and strikes of the events are correlated with pre-existing faults, indicating that the microearthquakes occurred primarily by reactivation of the preexisting faults. The majority of the events have a strike direction parallel with the major NE-SW faults in the region, and some events trend parallel with the NW-SE conjugate faults. We also found that the maximum horizontal stress derived from the source mechanisms trends in the NE or NNE direction. This is consistent with the direction of the maximum horizontal stress obtained from well breakout measurements and local tectonic stress analysis. We also observed that the faulting mechanism varies with depth, from strike-slip at shallower depth to normal faulting at greater depth.

3) A new method to locate microseismic events induced by hydraulic fracturing with simultaneous anisotropic velocity inversion by using differential arrival times and differential back azimuths is developed. The velocity inversion is constrained as 1-D layered VTI structure to improve stability given limited coverage from
microseismicity. We derived analytical sensitivities for the elastic moduli ($C_{ij}$) and layer thickness $L$. The forward modeled travel times and sensitivities are all calculated analytically without weak anisotropy assumption. The location improvements from anisotropic velocity inversion and from double-difference constraints are complimentary, as the former one improves the absolute locations and the latter one improves the relative locations.

4) The above method is used to locate 68 relatively strong microseismic events from the Bakken and its adjacent formations in the Beaver Lodge area of North Dakota. We found that a six-layer simple model with VTI anisotropy can characterize quite well the information observed by four monitoring wells separated by up to 1500 m. The simultaneous anisotropic velocity inversion significantly reduces the travel time residuals compared to standard location techniques. The microseismic event locations and the anisotropic velocity model are validated by comparing synthetic and observed seismic waveforms. The results show:

a. moderate-to-strong anisotropy exists in all studied sedimentary layers, especially in both the Upper Bakken and Lower Bakken shale formations, where the Thomsen parameters ($\epsilon$ and $\gamma$) can be over 40%;

b. high signal-to-noise ratio events used for the joint velocity inversion are located in the Bakken and overlying Lodgepole formations, i.e., no strong events are located in the Three Forks formation below the Bakken. More than half of the strong events are in the two clusters at about 100 and 150 meters above the Middle Bakken;
c. swarms of strong events are found to occur repeatedly in the Lodgepole formation, high above the treatment well. Waveforms showing strong similarities from the clustered events indicate the locations and source properties of these events are very close, and suggest reactivation of pre-existing Lodgepole fractures;

d. The uncertainty analysis found that the travel time has relatively low sensitivities with respect to elastic moduli $C_{13}$ and $C_{33}$ in most layers due to limited ray coverage. Thus the relative uncertainty in Thomsen’s $\delta$, which involves $C_{13}$ and $C_{33}$, is larger than other anisotropic parameters.

5) We proposed a hybrid method to model the shear (SH) wave scattering from 2-D fractures embedded in a heterogeneous medium by coupling the boundary element method (BEM) and the finite difference method (FDM) in the frequency domain through two artificial coupling boundaries. The hybrid method is fast. It can be used to perform Monte Carlo simulations to characterize the statistics of scattering signals from random subsurface fracture networks due to its high efficiency of propagating scattered wave fields.

6) The hybrid method is then used to calculate scattering from non-vertical fractures embedded in a three-layer medium and in an even more complex Marmousi model, which could be quite challenging for BEM or FDM alone. The scattered fields from fractures in both examples show coherent patterns, even after propagating through medium with considerable velocity variation. By isolating scattered waves from incident waves, as well as separating different
orders of scattered waves, our method can provide a new approach to study coherence and patterns of scattering in seismic coda waves.
Appendix A: Green’s Functions Calculation for the Deep Borehole Network

The reflectivity method used in the discrete wavenumber waveform modeling of Bouchon (2003) was originally developed in global seismology where sources are located underground and receivers are at the surface or near the surface. For the surveys using borehole receivers, however, the receivers can be located deeper than the source; thus the original reflectivity method needs to be revised and calculations in the reflectivity method need to be modified for this configuration. We followed the symbols and definitions used in the paper by Muller (1985) on the reflectivity method and only show the key modified equations. Figure A1 shows the diagram for borehole receiver configuration.

The source and receivers are required to be located at the interface between two identical layers in the implementation (Bouchon, 2003). The position of the source and receiver can be anywhere within a layer, however, an artificial splitting of the layer is applied at the depth of the receiver or the source, i.e., splitting the layer into two identical layers with an interface at the depth of the source or receiver. The reflectivity method is easier to apply in this way. After the splitting, the source is located at the bottom of layer $j$, and the receiver is located at the top of layer $m$ for the shallower-source-deeper-receiver situation.

In the following derivation, we use the P-SV system. For the SH system, the matrices and vectors are replaced with scalars. The overall amplitude vector $\mathbf{V}^{db}_{1,2}$ for the down-going waves at the source depth is:
where $R^+$ and $R^-$ are the reflectivities illustrated in Figure A1; $S^d_{1,2}$ and $S^u_{1,2}$ are the source amplitude vectors; $I$ is the identity matrix. $V_{1,2}^D$ takes all the reflections from the lower layers (first bracket) and the upper layers (second bracket) into consideration and, therefore is the amplitudes of the overall down-going P- and SV-waves at the source depth. After the overall down-going amplitudes are obtained at the source level, we need to propagate them down through the layers between the source and receiver by the overall down-going transmissivity matrix:

$$TT^D = F_{m-1}F_{m-2}...F_{j+1}F_j$$

(A2)

where $F_k$ characterizes the amplitude change through layer $k$ and through the bottom interface of layer $k$. Note that for layer $j$ there is no phase shifting through the phase matrix $E_j$ in $F_j$, as the source is already located at the bottom of layer $j$ after the artificial splitting. The overall down-going amplitudes at the receiver then are:

$$V_{1,2}^{D,R} = \begin{pmatrix} A^R_{1,2} \\ C^R_{1,2} \end{pmatrix} = TT^D V_{1,2}^D$$

(A3)

and the overall amplitudes of the up-going waves at the receiver are related to the amplitudes of the down-going waves by:
\[ V_{1,2}^{U,R} = \begin{pmatrix} B_{1,2}^R \\ D_{1,2}^R \end{pmatrix} = M T_m V_{1,2}^{D,R} \] (A4)

where \( M T_m \) is the local reflectivity matrix at the top of layer \( m \). Combining the amplitudes \( V_{1,2}^{U,R} \) and \( V_{1,2}^{D,R} \) with the Green’s functions calculated by the discrete wavenumber method (Bouchon, 2003) and integrating in the wavenumber and frequency domain, we can then obtain the analytic solution in a stratified medium where the receiver is deeper than the source.

References can be found in Chapter 2.
Figure A-1. Diagram of the reflectivity method for the deep borehole receiver configuration.
Appendix B: Weights in the Source Mechanism Inversion

In the source mechanism inversion, we choose four parameters, namely $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ to weigh the contributions from four terms: waveform cross-correlation, $L_2$ norm of the waveform difference, consistency in polarities and average S/P amplitude ratios for the synthetic and observed waveforms. Each weight is a positive scalar number and is optimally chosen in a way such that no single term will over-dominate the objective function. The weights $\alpha_1$ through $\alpha_4$ in the objective function are tried with different values, and we selected the ones that balance different terms. We used $\alpha_1 = 3$, $\alpha_2 = 3$, $\alpha_3 = 1$ and $\alpha_4 = 0.5$ for most of the synthetic tests and real events, and we found that the final solutions are not very sensitive to reasonable changes in the weights. There are three degrees of freedom for the choice of weighting parameters.

Both the first term and the second term evaluate the similarities between the synthetic and observed waveforms, and these terms are complementary. We need to perform waveform cross-correlation in the beginning to correct the waveform shift in time due to incorrect velocity models for P- and S-waves. The waveform cross-correlation mainly evaluates the phase information while the $L_2$ norm of the time-shifted waveforms evaluates more of the amplitude differences between the observed and modeled waveforms. The first two terms are not independent of each other, however, they have different sensitivities at different frequency bands and by combining them together the waveform similarity can be better characterized.

The third term evaluates whether the polarities of the first P-wave arrivals are consistent between the observed and synthetic seismic traces. Depending on our confidence of
identifying the polarities in the observed data, a weight $\beta$ is given to the polarities of the first P-wave arrivals. Larger $\beta$ indicates more confident picking. Zero (0) means undetermined polarity. Usually, we use $\beta = 3$ for confident pickings and $\beta = 0.5$ for pickings with less confidence. Polarity consistency at some stations can be violated if the polarity is not confidently identified (small $\beta$) and the other terms in the objective function favor a certain focal mechanism. For instance, when $\beta = 3$ is used for a first P-arrival pick at a station, the inversion would try to avoid finding a solution that does not yield a consistent polarity at this station, even if it means that the waveform fittings may be a bit less satisfactory. In comparison, if we use $\beta = 0.5$ for a first P-arrival pick at a station, the objective function will try to find a solution with better waveform and amplitude ratio matching, even if the consistency of the observed polarity is violated at this station. Therefore, the polarity information is integrated into our objective function in a flexible but somewhat subjective way.

The fourth term in the objective function is to evaluate the consistency of the average S/P amplitude ratios in the observed and modeled waveforms. A scalar number is calculated for the ratio of the observed trace, and a scalar number is also calculated for that of the synthetic trace at each station. Then the difference between the ratios is penalized. In our source mechanism inversion, we normalize the traces by energy, as the instrument responses are often not calibrated (aging, coupling, etc.), and the site effects are also not negligible. By normalization, we can mostly remove the issues associated with calibration and site effects, and the earthquake magnitude needs not to be determined. However, amplitude information of the P- and S-waves, which reflects the source radiation pattern and thus contributes to the
determination of source properties, is lost in the trace normalization. Thus, we include the fourth term to better constrain the inversion.

Depending on the data quality, velocity model and station coverage, each term may have varying contributions in inversions. For instance, if the velocity model is known with reasonable accuracy, and an event is recorded with good signal-to-noise ratios at stations deployed at favorable locations (e.g., stations are distributed at various azimuthal angles), the waveform matching from the first and second term may be enough to determine the source mechanism. In this case, the solution determined with only the first two terms in the objective function can still yield consistent polarities and S/P amplitude ratios between the synthetic and observed data, even if the latter two terms in the objective function are not used in the inversion. We tested events in such cases, and found changing the weights of (or even including/excluding) the third and fourth terms in the objective function only lead to small variations in strike, dip and rake, about 10° to 20°. However, data in most cases are of average quality, limited information of the velocity structure is known and stations are not distributed ideally, thus waveform matching may have very similar quantified number by the objective function from different source mechanisms, if only evaluated by the first two terms. That is, there are some matched and some mismatched cycles in the waveform comparisons from various mechanisms, rendering it difficult to distinguish which mechanism is correct. In such situations, the inclusion of the polarities and S/P amplitude ratio can help further distinguish which source mechanism is more reliable and trustworthy as it provides more consistent information in different aspects of the observed data. We tested events in less favorable situations and found sometimes many source mechanisms may be found from the inversion if
we only use the first two terms of the objective function. With the inclusion of the polarity and amplitude ratio information, the range of the possible solutions can be narrowed down tremendously.

In addition to weights \( \alpha_1 \) through \( \alpha_4 \), station weights can also be added to adjust the contribution from each individual station. For instance, a station at a further distance from an event oftentimes has less satisfactory matching between the synthetic and observed waveforms due to propagation effects, such as the actual velocity deviating from the reference 1-D velocity model used in calculating the synthetic data or lower signal-to-noise ratio, etc. The station is less weighted in such a case. Also, a station can be less weighted if we consistently find the waveform matching at this station is less satisfactory for many events due to poorer knowledge of the velocity model near the station or bad coupling. The choice of the station weights is based on station-event geometry and data quality and therefore is partly empirical. Sometimes, we can also adjust polarity weights and station weights and analyze the inversion results from different combinations of weights by visually inspecting the matching of each individual term in the objective function, in addition to the value of the objective function. In summary, the principles we follow in choosing the weighting parameters are:

1) try to balance the contribution from each term in the objective function, i.e., a satisfactory and reliable source mechanism should generate synthetic data that are consistent with the observed ones in waveform phases and amplitudes, polarities, as well as average S/P amplitude ratios;
2) try to use the same $\alpha_1$ through $\alpha_4$ for all events in a dataset, but the weights may vary from one dataset to another, based on the consistency rule described in principle 1);

3) in addition to the mostly-fixed weighting parameters $\alpha_1$ through $\alpha_4$, the polarity weights $\beta$ are empirically chosen for each pick based on the picking confidence;

4) each station is also weighted depending on the epicentral distance, seismic record quality, noise level, etc.

Subjectivity inevitably plays a role in our inversion, thus our method is not completely objective. All in all, we can weigh different terms in our inversion scheme based on balance of the contribution from each term, data quality, source-receiver geometry, inspection of the matching and, some experiences. In the future, automatic inversion for the best weights can also be included as part of the procedure in determining the source mechanisms.
Appendix C: Approximation of the Back Azimuth Sensitivity and Calculation of the Differential Back Azimuth

In any heterogeneous anisotropic medium, the derivatives in Equation 3-8 can be approximated with the following finite difference schemes. For the first derivatives at the receiver location, they are the phase slowness, and can be approximated using second order finite-difference, e.g.:

$$\frac{\partial T}{\partial x_r} \approx \frac{T(x_r + \Delta x, y_r, z_r, x_s, y_s, z_s) - T(x_r - \Delta x, y_r, z_r; x_s, y_s, z_s)}{2\Delta x} \quad (C1)$$

For the second derivatives, they can be approximated, e.g., as:

$$\frac{\partial}{\partial x_s} \left( \frac{\partial T}{\partial x_r} \right) \approx \frac{T(x_r + \Delta x, y_r, z_r; x_s + \Delta x, y_s, z_s) - T(x_r + \Delta x, y_r, z_r; x_s - \Delta x, y_s, z_s)}{4\Delta x^2} - \frac{T(x_r - \Delta x, y_r, z_r; x_s + \Delta x, y_s, z_s) - T(x_r - \Delta x, y_r, z_r; x_s - \Delta x, y_s, z_s)}{4\Delta x^2} \quad (C2)$$

The other second derivatives can also be calculated numerically by adding the finite increment $\pm \Delta l$ to different coordinate variables.

Here we also describe a method that can determine the differential back-azimuth ($\Delta \varphi$) from the observed waveforms without the need to solve the eigenvalue problems. Let us denote the signals from the first event as $p_1(t_n)$ and the signals from the second event as $p_2(t_n)$, which are column vectors. Then the observed seismograms in the north and east components are:
\[ n_1(t_n) = p_1(t_n) \sin(\varphi_1) \]
\[ e_1(t_n) = p_1(t_n) \cos(\varphi_1) \]  
(C3)

and

\[ n_2(t_n) = p_2(t_n) \sin(\varphi_2) \]
\[ e_2(t_n) = p_2(t_n) \cos(\varphi_2) \]  
(C4)

for the first and second events, respectively. Here \( \varphi_1 \) and \( \varphi_2 \) are the averaged back azimuths of the signals in the observation windows.

Then

\[ n_1^T n_2 + e_1^T e_2 = p_1^T p_2 \sin(\varphi_1) \sin(\varphi_2) + p_1^T p_2 \cos(\varphi_1) \cos(\varphi_2) = p_1^T p_2 \cos(\varphi_1 - \varphi_2) \]
\[ n_1^T e_2 - e_1^T n_2 = p_1^T p_2 \sin(\varphi_1) \cos(\varphi_2) - p_1^T p_2 \cos(\varphi_1) \sin(\varphi_2) = p_1^T p_2 \sin(\varphi_1 - \varphi_2) \]  
(C5)

The differential back-azimuth angle can be given by

\[ \tan(\Delta \varphi) = \tan(\varphi_1 - \varphi_2) = \frac{p_1^T p_2 \sin(\varphi_1 - \varphi_2)}{p_1^T p_2 \cos(\varphi_1 - \varphi_2)} = \frac{n_1^T e_2 - e_1^T n_2}{n_1^T n_2 + e_1^T e_2} \]  
(C6)

The derivations above do not assume any similarity between signals \( p_1(t_n) \) and \( p_2(t_n) \), i.e., they can be of different frequencies and amplitudes and Equation C6 is still valid. Nevertheless, windowing around the first arrivals of the seismograms is needed, otherwise the determined \( \Delta \varphi \) does not reflect the differential back-azimuth in the first arrivals but rather is an averaged result. Also, Equation C6 does not require \( p_1(t_n) \) and \( p_2(t_n) \) to be synchronized in theory. But we found the determined differential back azimuths are most accurate when two traces are first aligned by waveform cross-correlation (performed when determining the
differential travel times), as $p_1^T p_2$ is maximized and has the best signal-to-noise ratio when two signals are in phase.
Appendix D: Derivation of the Sensitivity Kernels for Anisotropic Velocity Inversion

To calculate the travel time along the ray path, we need to calculate the group velocity associated with the ray. For qP, qSV or SH, the group velocity $v_g$ and the phase velocity $v$ are related by

$$v_g^2(\Phi(\theta)) = v^2(\theta) + \left(\frac{dv}{d\theta}\right)^2 = v^2(\theta) + \frac{1}{4v^2} \left(\frac{dv^2}{d\theta}\right)^2 \quad (D1)$$

where the group angle $\Phi(\theta)$ and the phase angle $\theta$ differ by $\Delta\theta$:

$$\Phi(\theta) - \theta = \Delta\theta \quad (D2)$$

and $\Delta\theta$ can be found by

$$\tan(\Delta\theta) = \frac{1}{v} \frac{dv}{d\theta} = \frac{1}{2v^2} \frac{dv^2}{d\theta} \quad (D3)$$

The derivative of the phase velocity with respect to the phase angle is given by:

$$\frac{\partial v_{P,SV}^2}{\partial \theta} = \frac{1}{2} \frac{\partial}{\partial \theta} \left[ C_{33} + C_{55} + (C_{11} - C_{33}) \sin^2(\theta) \pm D(\theta) \right]$$

$$= (C_{11} - C_{33}) \sin(\theta) \cos(\theta) \pm \frac{1}{4} \left[ D_1 \sin(2\theta) + 4D_2 \sin^3(\theta) \cos(\theta) \right] \quad (D4)$$

where

$$D_1 = 2[2(C_{13} + C_{55})^2 - (C_{33} - C_{55})(C_{11} + C_{33} - 2C_{44})]$$

$$D_2 = (C_{11} + C_{13} - 2C_{44})^2 - 4(C_{13} + C_{44})^2$$
\[ D = \left[ (C_{33} - C_{55})^2 + D_1 \sin^2(\theta) + D_2 \sin^4(\theta) \right]^{\frac{1}{2}} \]  

\[ \text{(D5)} \]

And for SH wave

\[ \frac{\partial v_{SH}^2}{\partial \theta} = C_{66} \sin(2\theta) - C_{55} \sin(2\theta) \]  

\[ \text{(D6)} \]

\section*{qP, qSV}

The sensitivity of the phase velocity for qP and qSV waves with respect to the density-normalized elastic modulus \( C_{ij} \) is:

\[ \frac{\partial v^2_{P SV}}{\partial C_{ij}} = \frac{1}{2} \left( \frac{\partial (C_{33} + C_{55})}{\partial C_{ij}} + \frac{\partial (C_{11} - C_{33})}{\partial C_{ij}} \sin^2(\theta) + (C_{11} - C_{33}) \sin(2\theta) \frac{\partial \theta}{\partial C_{ij}} \pm \frac{\partial D(\theta)}{\partial C_{ij}} \right) \]  

\[ \text{(D7)} \]

where the plus sign is for qP wave, and the minus sign is for qSV wave. There are two derivatives we need to find, namely \( \partial \theta / \partial C_{ij} \) and \( \partial D(\theta) / \partial C_{ij} \). For the first term \( \partial \theta / \partial C_{ij} \),

\[ \frac{\partial \theta}{\partial C_{ij}} = \frac{\partial (\theta_g - \Delta \theta)}{\partial C_{ij}} = -\frac{\partial \Delta \theta}{\partial C_{ij}} = -\frac{\partial \tan \left( \frac{1}{2v^2} \frac{\partial v^2}{\partial \theta} \right)}{\partial C_{ij}} = \frac{-1}{1 + \left( \frac{1}{2v^2} \frac{\partial v^2}{\partial \theta} \right)^2} \frac{\partial \left( \frac{1}{2v^2} \frac{\partial v^2}{\partial \theta} \right)}{\partial C_{ij}} \]  

\[ \text{(D8)} \]

Here comes the only assumption in our derivation: \( \partial \theta_g / \partial C_{ij} = 0 \). The ray stationarity is valid as the ray path (group angle) perturbation is of higher order to the travel time perturbation, and is often used in isotropic travel time tomography (Zhang & Toksoz, 1998).

Define \( A_d = 1 / \left( 1 + \left( \frac{1}{2v^2} \frac{\partial v^2}{\partial \theta} \right)^2 \right) \), \( \partial \theta / \partial C_{ij} \) can be further simplified as:
\[
\frac{\partial \theta}{\partial C_{ij}} = -A_d \frac{\partial}{\partial C_{ij}} \left( \frac{1}{2v^2} \frac{\partial v^2}{\partial \theta} \right) = -A_d \left[ -\frac{1}{2v^2} \frac{\partial v^2}{\partial C_{ij}} \frac{\partial v^2}{\partial \theta} + \frac{1}{2} \frac{\partial}{\partial C_{ij}} \left( \frac{\partial v^2}{\partial \theta} \right) \right]
\]  

Equation D9 is a general expression for qP, qSV and SH waves. The term \(\frac{\partial}{\partial C_{ij}} \frac{\partial v^2}{\partial \theta} \) for qP and qSV waves can be derived from Equation D4:

\[
\frac{\partial}{\partial C_{ij}} \left( \frac{\partial v^2}{\partial \theta} \right) = \frac{\partial (C_{11} - C_{33})}{\partial C_{ij}} \sin(\theta) \cos(\theta) + (C_{11} - C_{33}) \cos(2\theta) \frac{\partial \theta}{\partial C_{ij}} \\
\pm \frac{1}{4} \left[ \frac{1}{D(\theta)} \left( \frac{\partial D_1}{\partial C_{ij}} \sin(2\theta) + 2D_1 \cos(2\theta) \frac{\partial \theta}{\partial C_{ij}} + 4 \frac{\partial D_2}{\partial C_{ij}} \sin^3(\theta) \cos(\theta) + 4D_2 (3 \sin^2(\theta) \cos^2(\theta) - \sin^4(\theta)) \frac{\partial \theta}{\partial C_{ij}} \right) \right] + (D_1 \sin(2\theta) + 4D_2 \sin^3(\theta) \cos(\theta)) \left( -\frac{1}{D^2(\theta)} \frac{\partial D(\theta)}{\partial C_{ij}} \right)
\]

(D10)

where

\[
\frac{\partial D_1}{\partial C_{ij}} = 2 \left[ 4(C_{13} + C_{55}) \frac{\partial (C_{13} + C_{55})}{\partial C_{ij}} - \frac{\partial (C_{33} - C_{55})}{\partial C_{ij}} (C_{11} + C_{33} - 2C_{55}) \right. \\
- \left. (C_{33} - C_{55}) \frac{\partial (C_{11} + C_{33} - 2C_{55})}{\partial C_{ij}} \right]
\]

\[
\frac{\partial D_2}{\partial C_{ij}} = 2(C_{11} + C_{33} - 2C_{55}) \frac{\partial (C_{11} + C_{33} - 2C_{55})}{\partial C_{ij}} - 8(C_{13} + C_{55}) \frac{\partial (C_{13} + C_{55})}{\partial C_{ij}}
\]

(D11)

And the derivative \(\partial D(\theta)/\partial C_{ij}\) is given by
\[
\frac{\partial D(\theta)}{\partial C_{ij}} = \frac{1}{2D(\theta)} \left( 2(C_{33} - C_{55}) \frac{\partial (C_{33} - C_{55})}{\partial C_{ij}} + \frac{\partial D_1}{\partial C_{ij}} \sin^2(\theta) + D_1 \sin(2\theta) \frac{\partial \theta}{\partial C_{ij}} + \frac{\partial D_2}{\partial C_{ij}} \sin^4(\theta) \right) \\
\quad + 4D_2 \sin^3(\theta) \cos(\theta) \frac{\partial \theta}{\partial C_{ij}} \right)
\]

(D12)

Substituting Equations D7, D10, D11 and D12 into Equation D9, we obtain an equation containing only one unknown term \( \partial \theta / \partial C_{ij} \). Combine the terms and solve for \( \partial \theta / \partial C_{ij} \):

\[
\left\{ 1 - \frac{A_d}{4v^4} \frac{\partial v^2}{\partial \theta} \left[ (C_{11} - C_{33}) \sin(2\theta) \pm \frac{1}{2D(\theta)} \left( D_1 \sin(2\theta) + \frac{\partial D_2}{\partial C_{ij}} \sin^4(\theta) + 4D_2 \sin^3(\theta) \cos(\theta) \right) \right] \\
\quad + \frac{A_d}{2v^2} \left[ (C_{11} - C_{33}) \cos(2\theta) \pm \frac{1}{4D(\theta)} \left( 2D_1 \cos(2\theta) + 4D_2 (3 \sin^2(\theta) \cos^2(\theta) - \sin^4(\theta)) \right) \right] \\
\quad \pm \frac{1}{4} (D_1 \sin(2\theta) + 4D_2 \sin^3(\theta) \cos(\theta)) \left( - \frac{1}{2D^3(\theta)} \right) (D_1 \sin(2\theta) + 4D_2 \sin^3(\theta) \cos(\theta)) \right\} \frac{\partial \theta}{\partial C_{ij}} \\
\quad = \frac{A_d}{4v^4} \frac{\partial v^2}{\partial \theta} \left[ \frac{\partial (C_{33} + C_{55})}{\partial C_{ij}} + \frac{\partial (C_{11} - C_{33})}{\partial C_{ij}} \sin^2(\theta) \right] \\
\quad \pm \frac{1}{2D(\theta)} \left( 2(C_{33} - C_{55}) \frac{\partial (C_{33} - C_{55})}{\partial C_{ij}} + \frac{\partial D_1}{\partial C_{ij}} \sin^2(\theta) + \frac{\partial D_2}{\partial C_{ij}} \sin^4(\theta) \right) \\
\quad - \frac{A_d}{2v^2} \left[ \frac{\partial (C_{11} - C_{33})}{\partial C_{ij}} \sin(\theta) \cos(\theta) \pm \frac{1}{4D(\theta)} \left( \frac{\partial D_1}{\partial C_{ij}} \sin(2\theta) + 4 \frac{\partial D_2}{\partial C_{ij}} \sin^3(\theta) \cos(\theta) \right) \right] \\
\quad \pm \frac{1}{4} (D_1 \sin(2\theta) + 4D_2 \sin^3(\theta) \cos(\theta)) \left( - \frac{1}{2D^3(\theta)} \right) \left( 2(C_{33} - C_{55}) \frac{\partial (C_{33} - C_{55})}{\partial C_{ij}} + \frac{\partial D_1}{\partial C_{ij}} \sin^2(\theta) \right) \\
\quad \quad + \frac{\partial D_2}{\partial C_{ij}} \sin^4(\theta) \right\} \] (D13)

After solving for \( \partial \theta / \partial C_{ij} \), we can then solve for \( \partial D(\theta) / \partial C_{ij} \) with Equation D12; then with \( \partial \theta / \partial C_{ij} \) and \( \partial D(\theta) / \partial C_{ij} \), we can solve for \( \partial v^2 / \partial C_{ij} \) with Equation D7 and for \( \frac{\partial (\partial v^2)}{\partial \theta} / \partial C_{ij} \) with Equation D10, respectively. With the latter two terms, we can finally obtain \( \partial v_g / \partial C_{ij} \) with Equation 3-15.
The derivation of the SH sensitivity is similar to that of qP and qSV, and we can follow a similar but simpler procedure. For SH wave,

\[
\frac{\partial v^2}{\partial C_{ij}} = \frac{\partial (C_{55} + (C_{66} - C_{55}) \sin^2(\theta))}{\partial C_{ij}}
\]

\[
\frac{\partial \left( \frac{\partial v^2}{\partial \theta} \right)}{\partial C_{ij}} = \frac{\partial ((C_{66} - C_{55}) \sin(2\theta))}{C_{ij}} \tag{D14}
\]

Substituting Equations D14 into Equation 3-15 for the sensitivity of group velocity:

\[
\frac{\partial v_g}{\partial C_{ij}} = \frac{1}{2v_g} \left[ \frac{\partial (C_{55} + (C_{66} - C_{55}) \sin^2(\theta))}{\partial C_{ij}} \left( 1 - \frac{1}{4v^4} \left( \frac{\partial v^2}{\partial \theta} \right)^2 \right) + \frac{1}{2v^2} \frac{\partial^2 ((C_{66} - C_{55}) \sin(2\theta))}{\partial C_{ij}} \right] \tag{D15}
\]

And then into Equation D9 for the sensitivity of phase angle:

\[
\frac{\partial \theta}{\partial C_{ij}} = A_d \left[ \frac{1}{2v^4} \frac{\partial^2}{\partial \theta^2} \left( \frac{\partial (C_{55} + (C_{66} - C_{55}) \sin^2(\theta))}{\partial C_{ij}} \right) - \frac{1}{2v^2} \frac{\partial ((C_{66} - C_{55}) \sin(2\theta))}{C_{ij}} \right] \tag{D16}
\]

For SH wave, we can write explicitly for \( \partial \theta / \partial C_{55} \) and \( \partial \theta / \partial C_{66} \):

\[
\left[ 1 - A_d \frac{1}{2v^4} \frac{\partial^2}{\partial \theta^2} (C_{66} - C_{55}) \sin(2\theta) + \frac{A_d}{v^2} (C_{66} - C_{55}) \cos(2\theta) \right] \frac{\partial \theta}{\partial C_{55}}
\]

\[
= A_d \left[ \frac{1}{2v^4} \frac{\partial^2}{\partial \theta^2} \cos^2(\theta) + \frac{1}{v^2} \sin(\theta) \cos(\theta) \right]
\]
\[
1 - A_d \frac{1}{2v^4} \frac{\partial v^2}{\partial \theta} (C_{66} - C_{55}) \sin(2\theta) + \frac{A_d}{v^2} (C_{66} - C_{55}) \cos(2\theta) \frac{\partial \theta}{\partial C_{66}}
\]

\[
= A_d \left[ \frac{1}{2v^4} \frac{\partial v^2}{\partial \theta} \sin^2(\theta) - \frac{1}{v^2} \sin(\theta) \cos(\theta) \right]
\quad (D17)
\]

Then following a similar procedure as for qP and qSV waves, we can solve for the sensitivity of SH waves with respect to the elastic moduli.
Appendix E: Constraint Operator $P^c$

In some cases, shale gas/oil drilling companies can make core samples of the subsurface structure and determined some of the Thomsen’s anisotropy parameters by measuring wave velocities at different directions. In our inversion, each $C_{ij}$ is inverted independently, but their certain combinations, which give the Thomsen’s anisotropy parameters, should be kept close to the values determined by the lab measurements. Therefore, we propose the following linear operators to constrain such combinations in Equation 3-19:

$$\frac{C_{11} - C_{33}}{2C_{33}} = \epsilon \rightarrow C_{11} - (2\epsilon + 1)C_{33} = 0 \quad (E1)$$

$$\frac{C_{13} + 2C_{55} - C_{33}}{C_{33}} \approx \delta \rightarrow C_{13} + 2C_{55} - (\delta + 1)C_{33} = 0 \quad (E2)$$

$$\frac{C_{66} - C_{55}}{2C_{55}} = \gamma \rightarrow C_{66} - (2\gamma + 1)C_{55} = 0 \quad (E3)$$

Note the expression for $\delta$ is approximate here as a linear relation between the elastic moduli $C_{ij}$ and the anisotropic parameters is required. The anisotropy parameters $\epsilon, \delta$ and $\gamma$ here are constants determined by lab measurements, if available. These additional constraints can also help to reduce the ill-condition and resulting non-uniqueness in the inversion for anisotropy.
Appendix F: Velocity Inversion Uncertainty Analysis

In this section, we present the velocity inversion uncertainty analysis in the inverted elastic moduli $C_{ij}$. We perform a bootstrap test for each $C_{ij}$ by randomly perturbing the determined value 100 times with a standard deviation equal to 5% of the value. Given a certain perturbation on an elastic modulus, the larger the resulting travel-time change, the larger the sensitivity of the travel-time with respect to that parameter. Thus, the parameter can be determined more reliably. In general, we find perturbations on $C_{11}$ and $C_{66}$ for any layer result in relatively larger changes in travel time than perturbations on other parameters do in the same layer, i.e., these two parameters for each layer are better determined relatively. In contrast, the travel time changes caused by perturbation on $C_{13}$ and $C_{33}$ are usually less than 0.5%, except $C_{13}$ in the Upper Lodgepole, suggesting these parameters are usually determined with appreciable uncertainty. The uncertainty for the parameter $C_{55}$ is moderate.

For each elastic modulus, the bar in Figure F1 shows the range of the average travel time residuals (RMSE) for the 100 random tests, and the blue dots show the mean values of these 100 average residuals. As most rays spend significant time traveling in the Upper Lodgepole formation, perturbations of elastic moduli of this layer cause larger changes in the average travel time residual, especially perturbations on $C_{11}$ and $C_{66}$, which characterize the horizontal P- and S-wave velocities, respectively (the ray angle coverages are shown in Figures F2 and F3). It should be noted sometimes the perturbation of certain elastic moduli can lead to small reduction in the travel time residuals. This is because constraints on Thomsen’s parameters are imposed in our inversion, thus elastic moduli attempting to retain
predetermined Thomsen’s parameters do not necessarily yield the minimum travel time residuals.

We further show why the travel time is more sensitive to some elastic moduli than others in a certain layer in Figures F2 and F3. Figure F2 shows the group velocity sensitivity \( \frac{\partial v_g}{\partial C_{ij}} \) (bolded lines) and ray count (gray bins) with group angles for P-waves in each layer. For a parameter in a certain layer, the more frequently the angles with higher sensitivity are sampled by rays, the less uncertainty the parameter has in the inversion. For instance, most P-rays travel beyond 60° in the Upper Lodgepole Formation, and \( C_{11} \) has sensitivity increasing with angle, larger than any other P-wave related parameter beyond 70°, thus \( C_{11} \) is relatively better constrained than the other parameters in this layer; also, \( C_{33} \) has sensitivity decreasing with angle, smaller than any other parameter beyond 50°, thus \( C_{33} \) is more poorly constrained than the other P-wave related parameters in this layer. It should be noted that both the P- and SH-waves are sensitive to \( C_{55} \), although for P-waves the largest sensitivity is at around 45°, and for SH-waves it is in the vertical direction (0°). In general, few rays are found at less than 40° in any layer, and therefore, the determination of \( C_{33} \) in all layers is subject to noticeable uncertainty. As a result, the inverted Thomsen’s \( \epsilon \) would come with unavoidable uncertainty and constraints on these parameters with predetermined values are necessary in the inversion. Also, we found the sensitivity with respect to \( C_{55} \) is always larger than \( C_{13} \), indicating the uncertainty in \( C_{55} \) should be smaller than in \( C_{13} \) comparatively.

Figure F3 shows the group velocity sensitivity and ray count with group angles for SH-waves in each layer. Similar to P-waves, most incident rays are at high angles. For SH-waves, when the group angle is larger than 50°, the group velocity sensitivity with respect to \( C_{66} \) is
larger than with respect to $C_{55}$ in all layers. Still, as mentioned above P-waves can also help to determine $C_{55}$. Judging from the sensitivities and the resulting travel time changes with perturbation on $C_{55}$ and $C_{66}$ (Figure F1), Thomsen’s $\gamma$ should be inverted with less uncertainty compared with $\epsilon$, which involves one of the least constrained parameter $C_{33}$. Also, the uncertainty in Thomsen’s $\delta$, which involves $C_{13}$ and $C_{33}$, is also determined with more uncertainty than with $\epsilon$. 
Figure F-1. Bootstrap test of the parameter sensitivities. ULP and LLP stand for the Upper and Lower Lodgepole formations, respectively; UB, MB and LB stand for the Upper, Middle and Lower Bakken formations, respectively; TF stands for the Three Forks formation. The parameters for each layer are $C_{11}, C_{13}, C_{33}, C_{55}, C_{66}$ in sequence. The red dots indicate the travel time residual (ms) of the reference solution with our regularized inversion. The bars indicate the range of the average travel time residuals (RMSE) for the 100 random tests. The blue dots indicate the mean values. Note the mean values for $C_{11}$ and $C_{66}$ in ULP are out of the range shown.
Figure F-2. Group velocity sensitivity (bold lines) and ray count (gray bins) with angle for P-waves in each layer. The sensitivities for each layer are normalized to the maximum sensitivity of all elastic moduli of that layer. 0° is the vertical direction, and 90° is the horizontal direction.
Figure F-3. Group velocity sensitivity (bold lines) and ray count (gray bins) with angle for SH-waves in each layer.
Appendix G: Convergence Conditions for Multiple Scattering

In this section, we briefly discuss the convergence condition of the multiple scattering between different fractures, that is, the Born series.

We start with a simple scenario of two fractures $A$ and $B$ embedded in a free homogeneous space. We can express eqs (7) and (8) as

$$
\begin{bmatrix}
A & A_b \\
B_a & B
\end{bmatrix}
\begin{bmatrix}
U_a^{\text{sca}} \\
U_b^{\text{sca}}
\end{bmatrix} =
\begin{bmatrix}
u_a^{\text{inc}} \\
u_b^{\text{inc}}
\end{bmatrix},
$$

where $A$ and $B$ are the matrices that characterize the response of fracture $A$ and $B$ to an external field, such as the incident field or the scattered field; $A_b$ is a propagator matrix (Green’s function) that propagates the response from fracture $B$ to fracture $A$ and vice versa for $B_a$; $U_{\text{sca}} = [u]$ is displacement discontinuity on the surfaces of the fractures due to the external field. Since $A_b$ and $B_a$ can only be calculated numerically for a heterogeneous medium, we can not solve Equation G1 directly and need the iteration scheme to account for the multiple interactions.

The first order Born scattered fields are

$$
U_a^{\text{sca}} = A^{-1}u_a^{\text{inc}}
$$

$$
U_b^{\text{sca}} = B^{-1}u_b^{\text{inc}}
$$

where $u_a^{\text{inc}}$ and $u_b^{\text{inc}}$ are the incident field on fracture $A$ and $B$, respectively. The second order Born scattered fields are
where $U_{sca}^{ab}$ originates from the scattering of fracture $B$ ($U_{sca}^b$), propagated through the medium ($A_b$) and causing the response on fracture $A$ ($A^{-1}$). Similarly, the third Born scattering is

$$
U_{sca}^{aba} = (A^{-1}A_b)U_{sca}^{ba} = (A^{-1}A_b)(B^{-1}B_a)A^{-1}U_{inc}^a,
$$

$$
U_{sca}^{bab} = (B^{-1}B_a)U_{sca}^{ba} = (B^{-1}B_a)(A^{-1}A_b)B^{-1}U_{inc}^b,
$$

by adding iteration factors $A^{-1}A_b$ and $B^{-1}B_a$ to the second order Born scattering. The next order scattered field can also be calculated by adding the iteration factors to the scattered field of the previous order. To guarantee the convergence of the multiple scattering series, the spectral radii of the iteration factors (maximum absolute eigenvalue) needs to be smaller than 1 (Strang 2007)

$$
\max |\lambda (A^{-1}A_b)| < 1
$$

$$
\max |\lambda (B^{-1}B_a)| < 1.
$$

For a scenario of $N$ fractures, we have

$$
\begin{bmatrix}
F_{11} & F_{12} & F_{13} & \cdots & F_{1n} \\
F_{21} & F_{22} & F_{23} & \cdots & F_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
F_{n1} & F_{n2} & F_{n3} & \cdots & F_{nn} \\
\end{bmatrix}
\begin{bmatrix}
U_{sca}^1 \\
U_{sca}^2 \\
U_{sca}^3 \\
\vdots \\
U_{sca}^n \\
\end{bmatrix}
= 
\begin{bmatrix}
u_{inc}^1 \\
u_{inc}^2 \\
u_{inc}^3 \\
\vdots \\
u_{inc}^n \\
\end{bmatrix}
$$

(G5)
where $F_{ii}$ is the matrix that characterizes the response of fracture $i$ to the external field and $F_{ij}(i \neq j)$ is the propagator that propagates the response from fracture $j$ to fracture $i$.

The first Born scattering for the $i$th fracture is

$$U_{sca}^{i1} = F_{ii}^{-1}U_{inc}^i.$$  \hspace{1cm} (G6)

The second Born scattering for the $i$th fracture is

$$U_{sca}^{i2} = \sum_{j=1, j \neq i}^{N} (F_{ii}^{-1}F_{ij})U_{sca}^{j1} = \sum_{j=1, j \neq i}^{N} (F_{ii}^{-1}F_{ij})F_{jj}^{-1}U_{inc}^j.$$  \hspace{1cm} (G7)

The third Born scattering for the $i$th fracture is

$$U_{sca}^{i3} = \sum_{j=1, j \neq i}^{N} (F_{ii}^{-1}F_{ij})U_{sca}^{j2}.$$  \hspace{1cm} (G8)

The higher order scattered fields can also be derived in the similar way.

To converge the multiple scattering, the maximum absolute eigenvalue of the iteration factors summation needs to satisfy

$$\max \left| \sum_{j=1, j \neq i}^{N} (F_{ii}^{-1}F_{ij}) \right| < 1.$$  \hspace{1cm} (G9)

assuming that the first scattered field $U_{sca}^{i1}$ from each fracture is approximately the same. For instance, $\max |\lambda(\sum_{j=1, j \neq i}^{N} (F_{ii}^{-1}F_{ij}))|$ is 0.46 in the second example with four inclined fractures at 20 Hz (Figure 5-6). The Born series are converged after five iterations.
Appendix H: Benchmark our Hybrid Method with the TDFD Method

In this section, we provide a benchmark model to compare our hybrid method with the TDFD (Coates & Schoenberg 1995). The model consists of a three-layer medium and 10 vertical fractures embedded in the middle layer, as shown in Figure H1. Figure H2 shows the scattered waveforms at nine receivers calculated by TDFD method and our hybrid method. We find the agreement between these two methods is very good.

Figure H-1. Velocity model for the layered medium containing 10 vertical fractures. The white dot denotes the source. The white crosses denote the receiver locations.
Figure H-2. Waveform comparison between the hybrid method and TDFD method at 9 receivers shown in Figure H-1, where the receiver 1 starts from the rightmost position.