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# Extrapolated Full Waveform inversion with synthesized low frequencies

Yunyue Elita Li

Postdoctoral associate  
Department of Mathematics and Earth Resources Laboratory

In collaboration with **Laurent Demanet**

MIT Earth Resources Laboratory  
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Earth  
Resources  
Laboratory



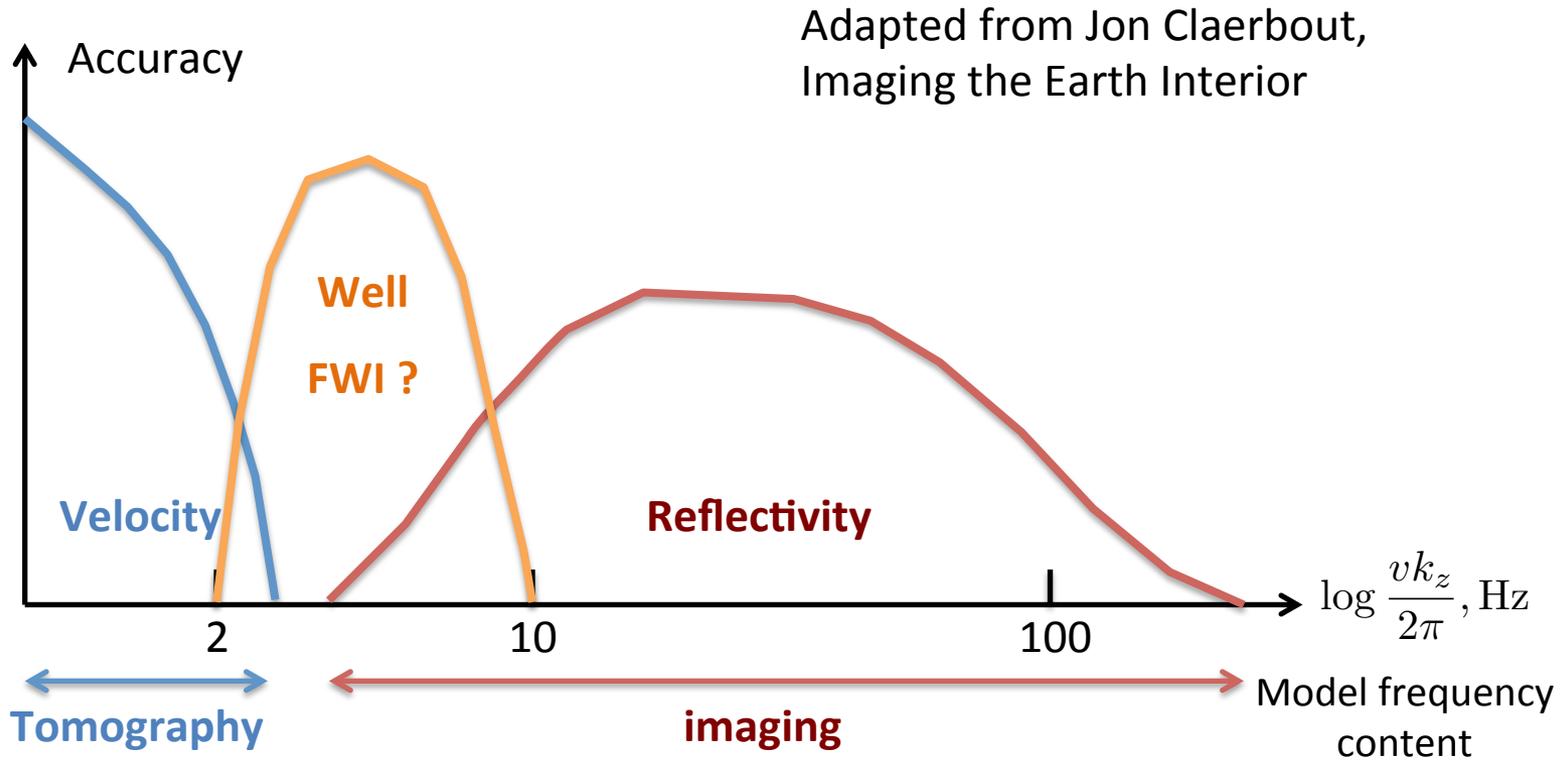
**Massachusetts  
Institute of  
Technology**

# Subsurface characterization by seismic imaging

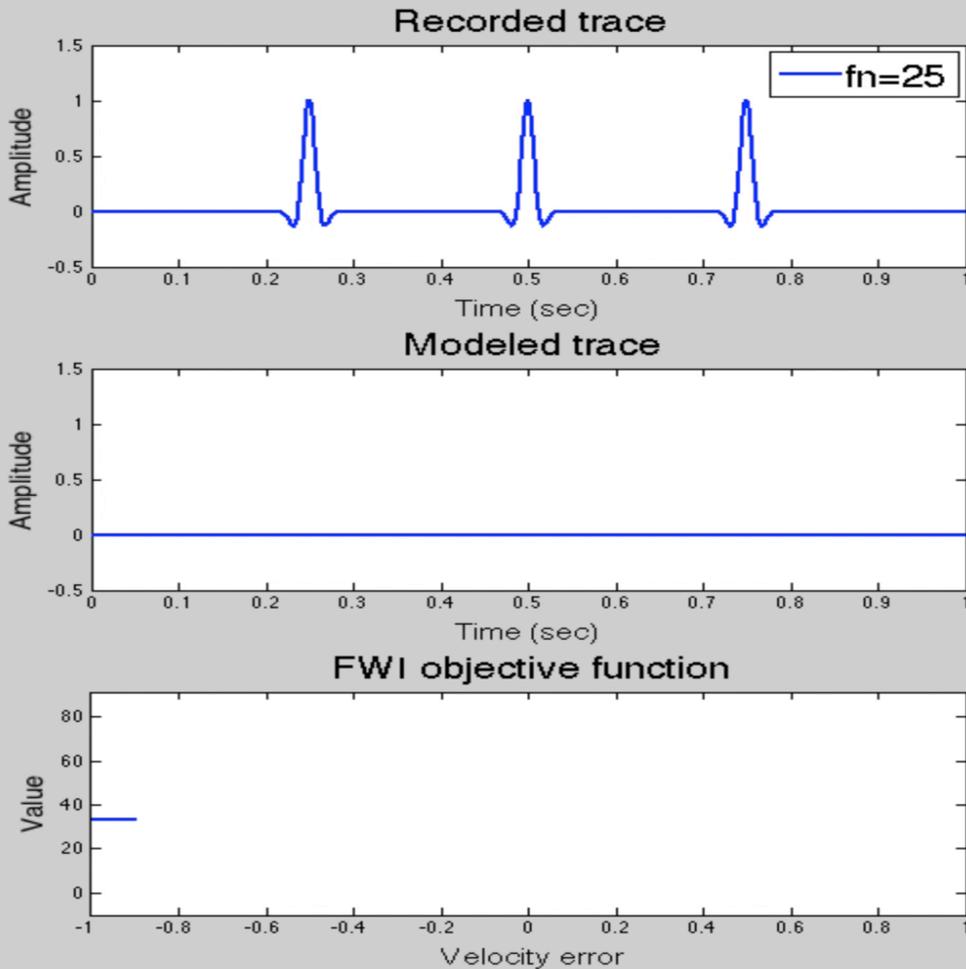
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- Incomplete dataset
  - Incomplete acquisition geometry
    - Surface acquisition, VSP
  - Incomplete bandwidth
    - Band-limited seismic data
- Incomplete resolvable wavenumbers

# Seismic frequency gap



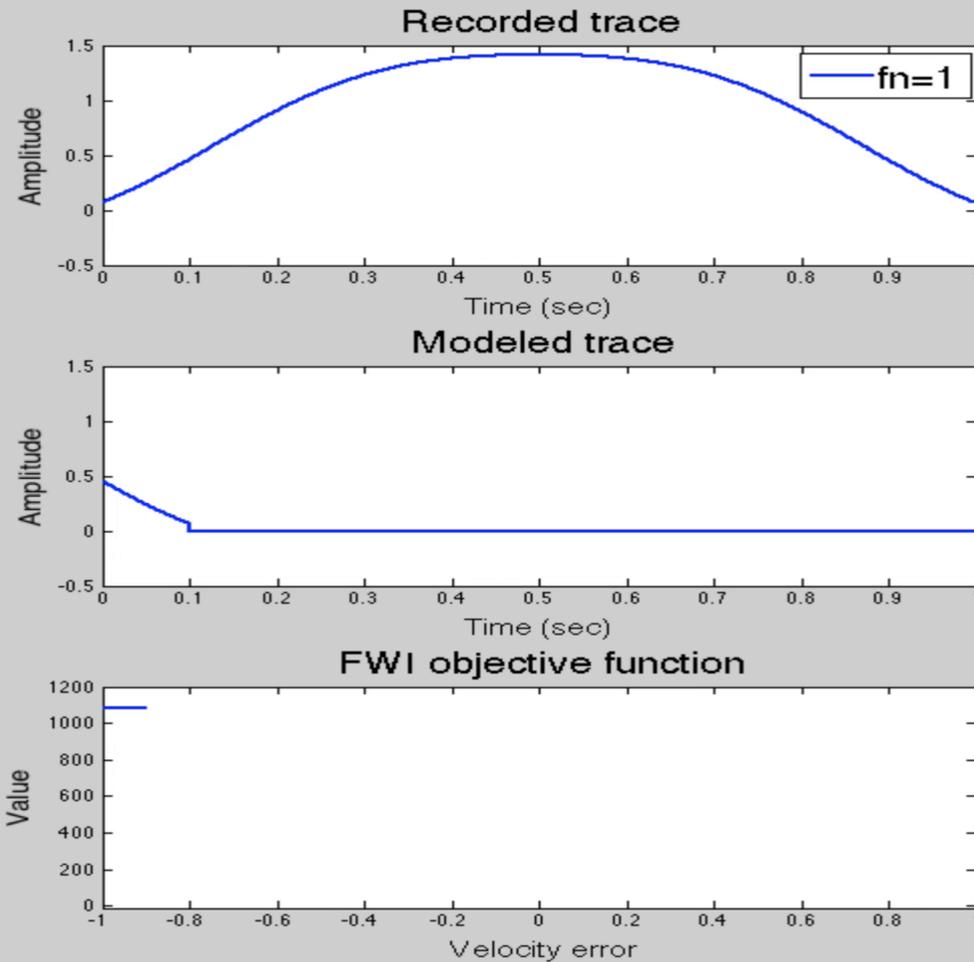
# Challenges of full waveform inversion



Nonconvex  
Local minimum

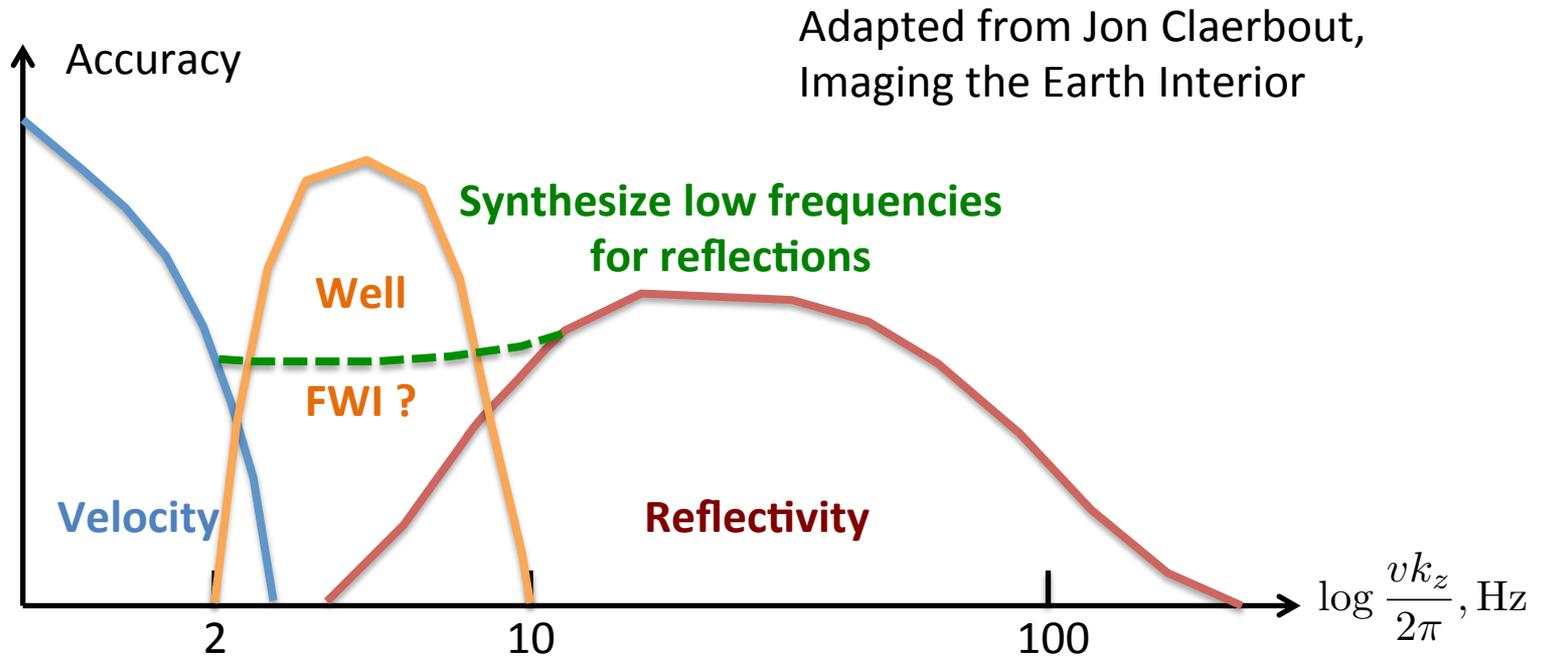
Nonlinear inversion  
Iterative methods

# Challenges of full waveform inversion



- Low frequency data
  - Missing from previous acquisitions
  - Expensive to acquire

# Seismic frequency gap



# Outline

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- Challenges of Full Waveform Inversion
- **Synthesize low frequencies**
  - Data space: event separation and extrapolation
- Full waveform inversion
  - Initial solution: initialization using synthesized data
- Conclusions

# Seismic event separation

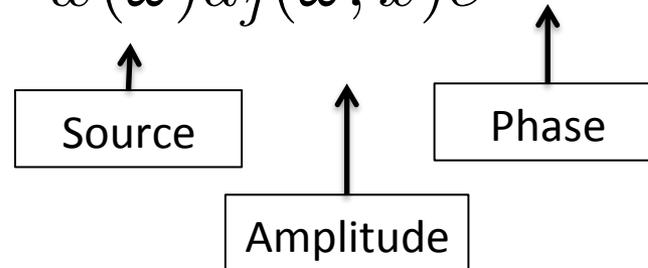
Li and Demanet, Geophysics, 2015

Data model:

$$\hat{d}(\omega, x) \approx \sum_{j=1}^r \hat{v}_j(\omega, x)$$

where atomic events

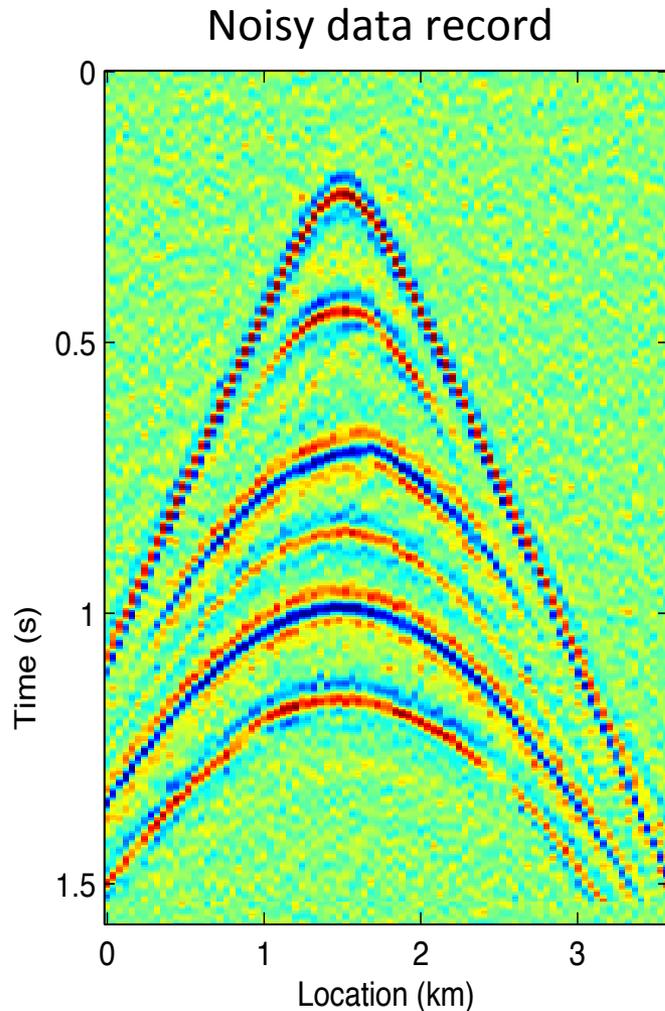
$$\hat{v}_j(\omega, x) = w(\omega) a_j(\omega, x) e^{ib_j(\omega, x)}$$



Separation is required for extrapolation (**assuming no attenuation**):

- \* Amplitude remains constant for all frequencies
- \* Phase is a linear function of frequencies

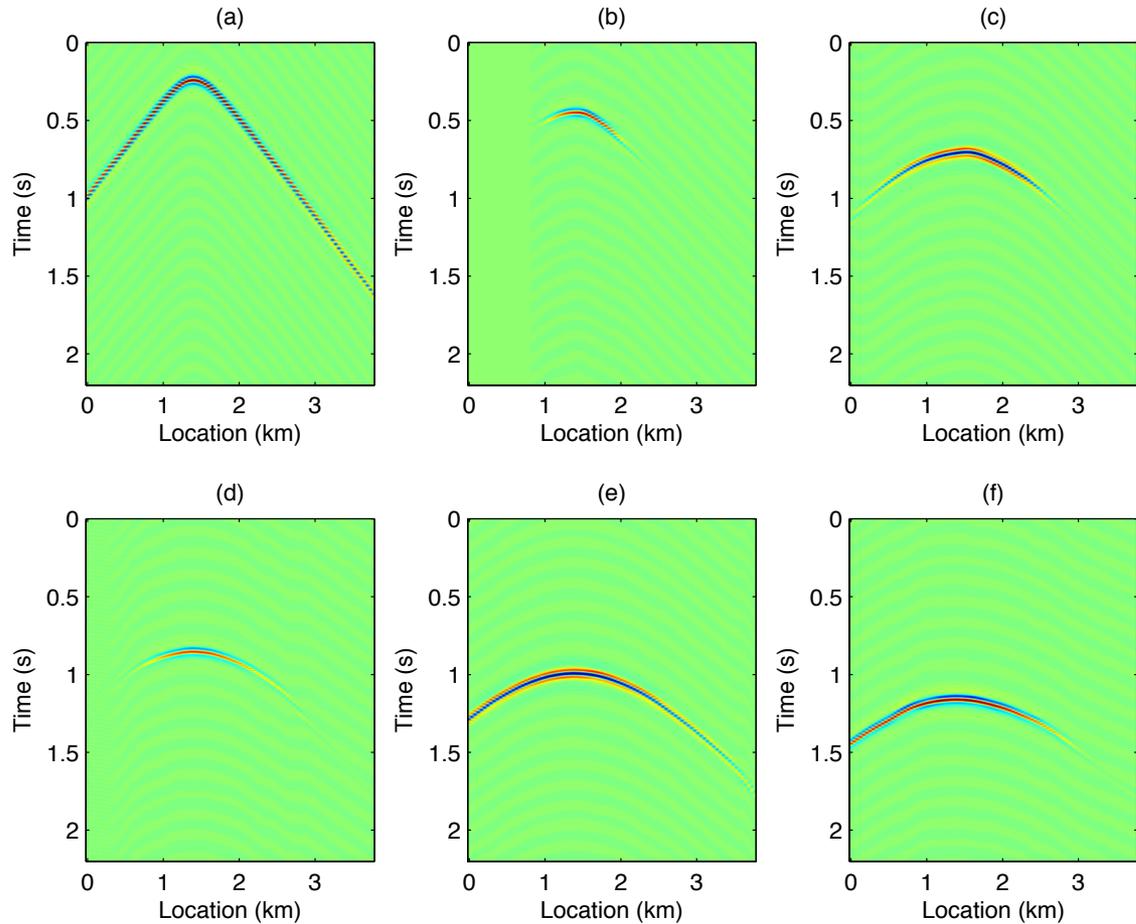
# Phase and amplitude tracking



Single shot gather:

- \* FDTD constant density acoustic
- \* Additive noise of 30%
- \* Band-limited [7-40] Hz

# Event separation

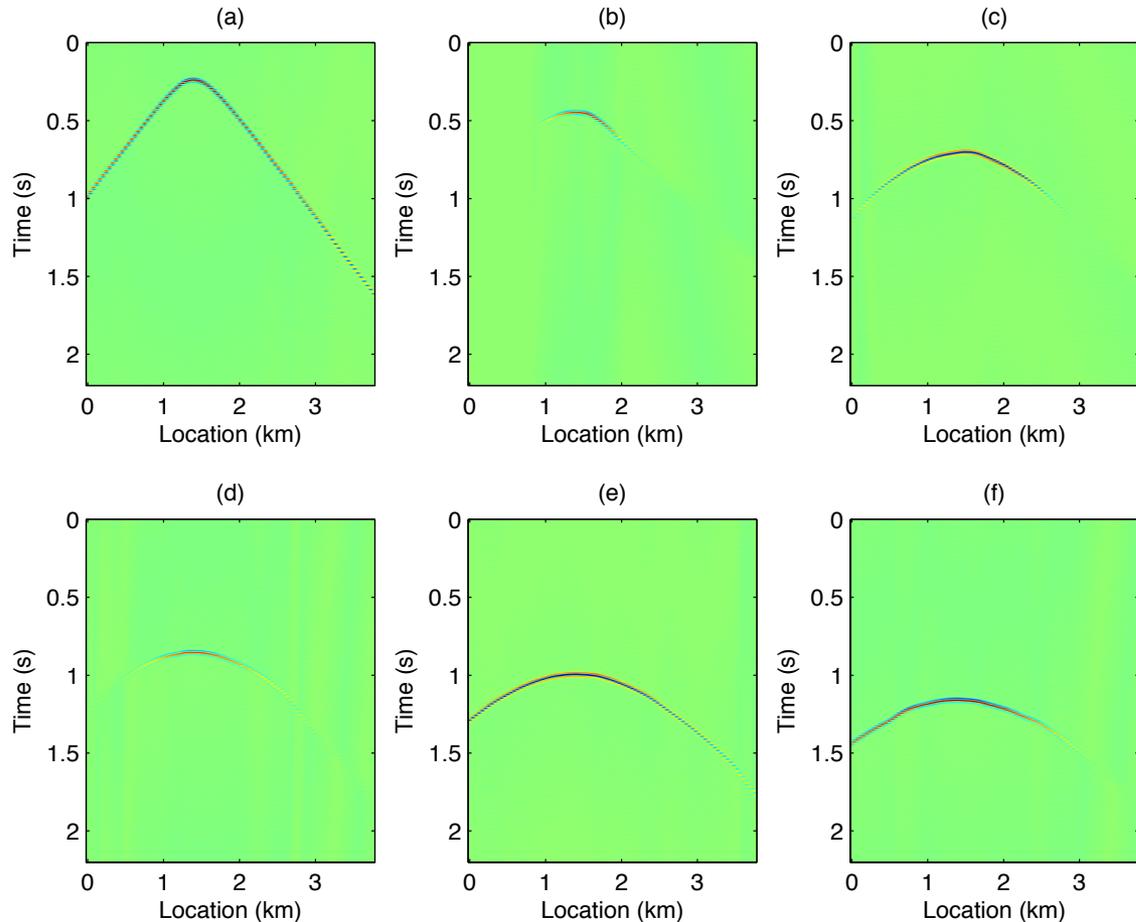


- Frequency bandwidth [7 40] Hz

$$\hat{d}(\omega) \simeq \sum_{j=1}^r a_j e^{i\omega\tau_j}$$

- Extrapolate to [0.5 70] Hz  
(Assuming no dispersion)

# Frequency extrapolation



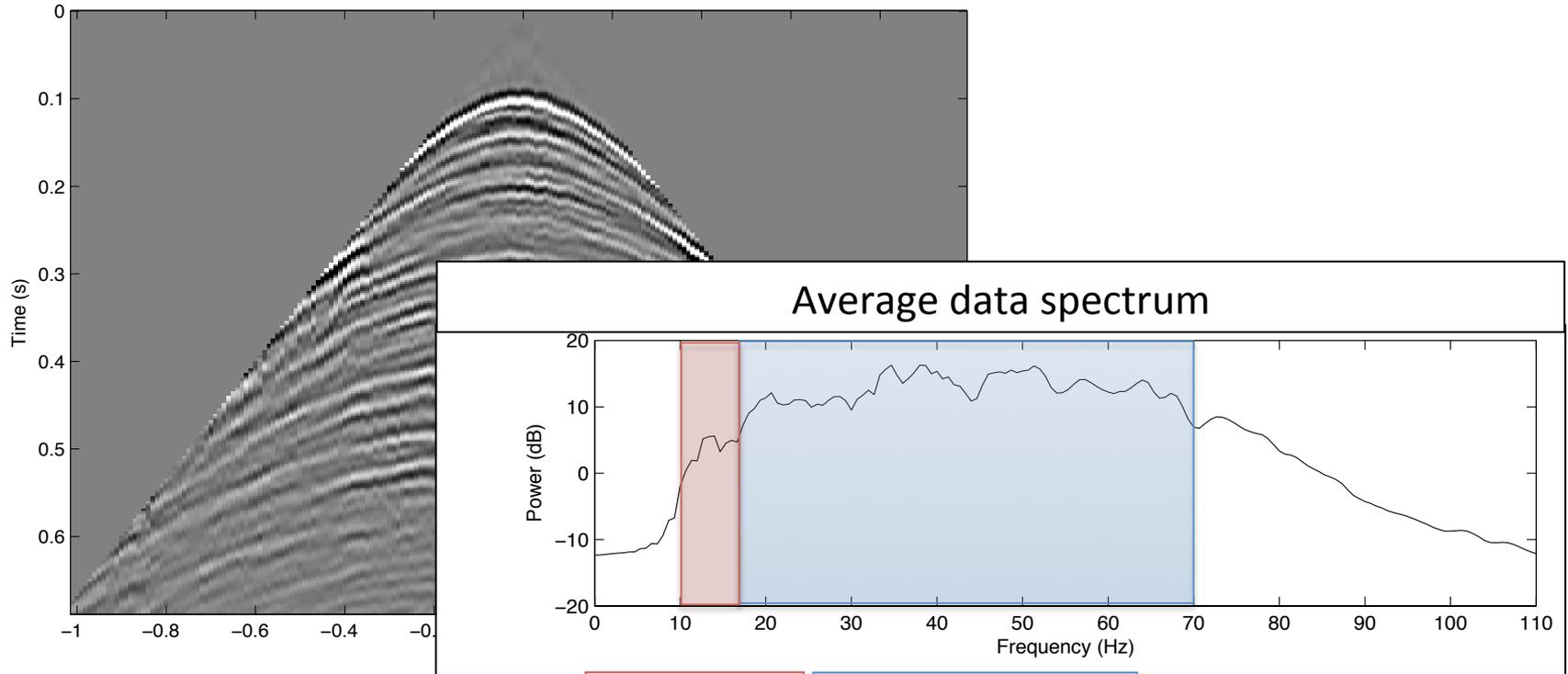
- Frequency bandwidth [7 40] Hz

$$\hat{d}(\omega) \simeq \sum_{j=1}^r a_j e^{i\omega\tau_j}$$

- Extrapolate to [0.5 70] Hz  
(Assuming no dispersion)

# Low frequency extrapolation

Shot record on land



verification  
[10-16] Hz

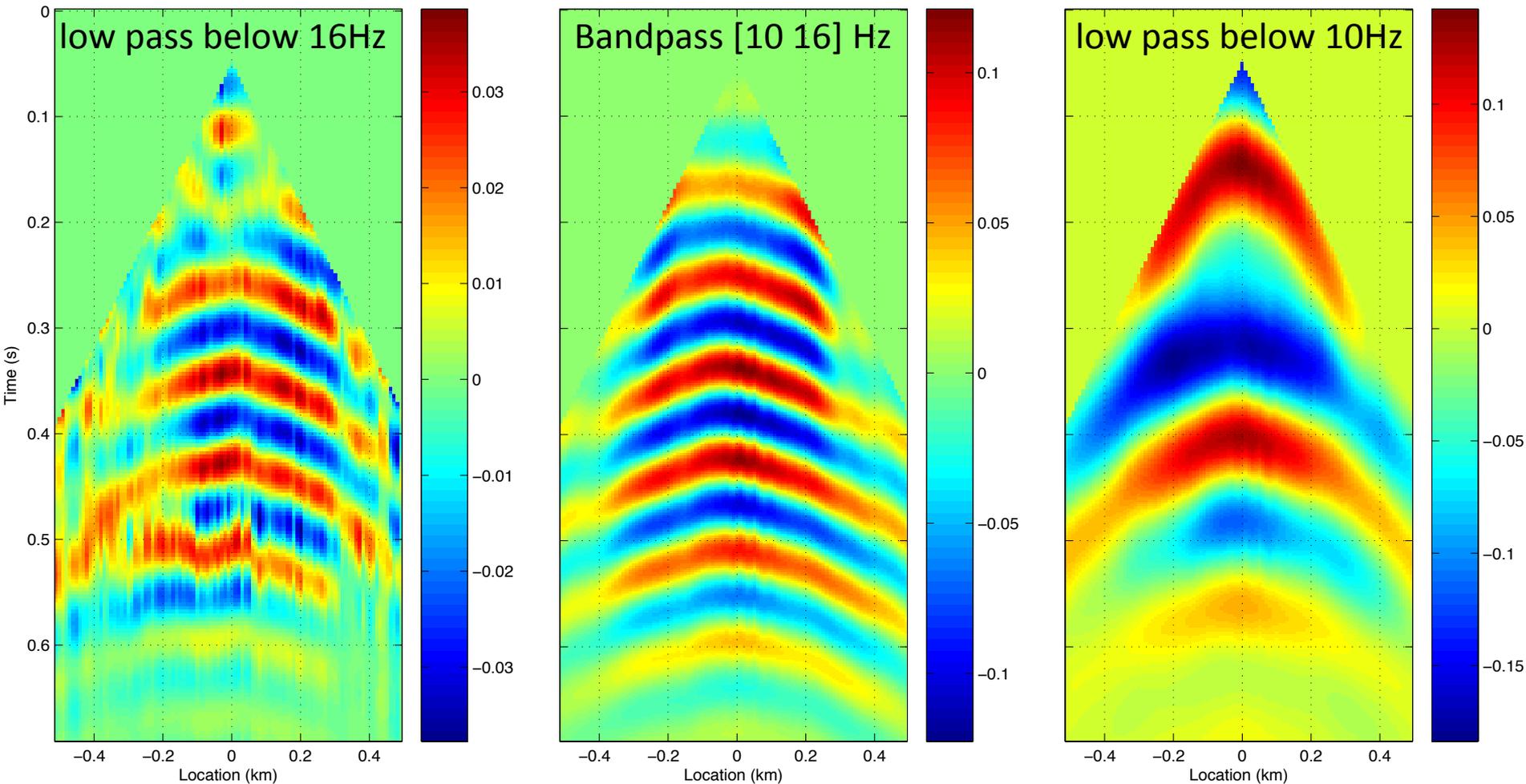
event separation  
[16 - 70] Hz

# Low frequency extrapolation

Recorded data

Extrapolated data

Extrapolated data

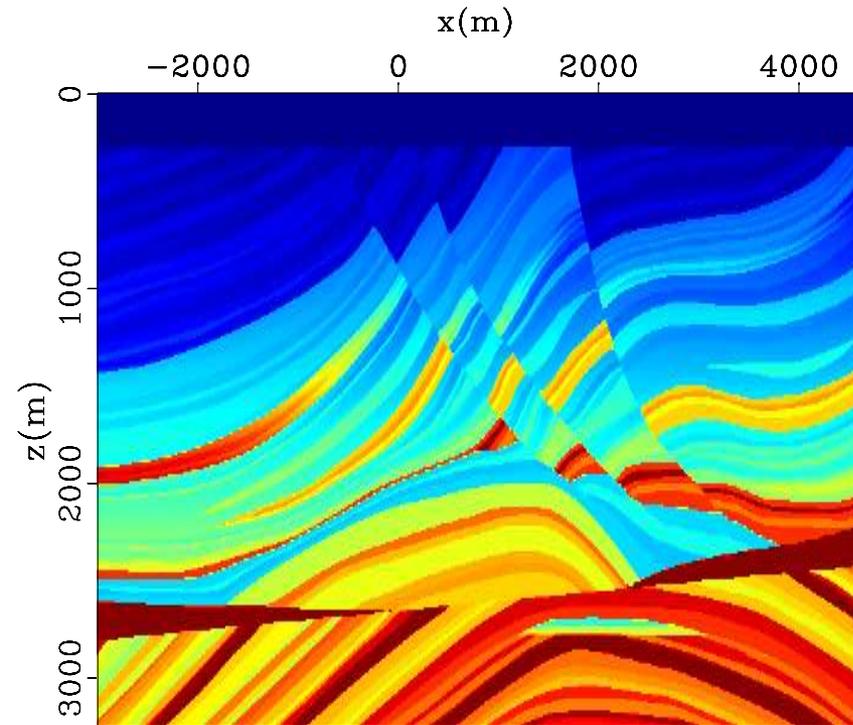


# Outline

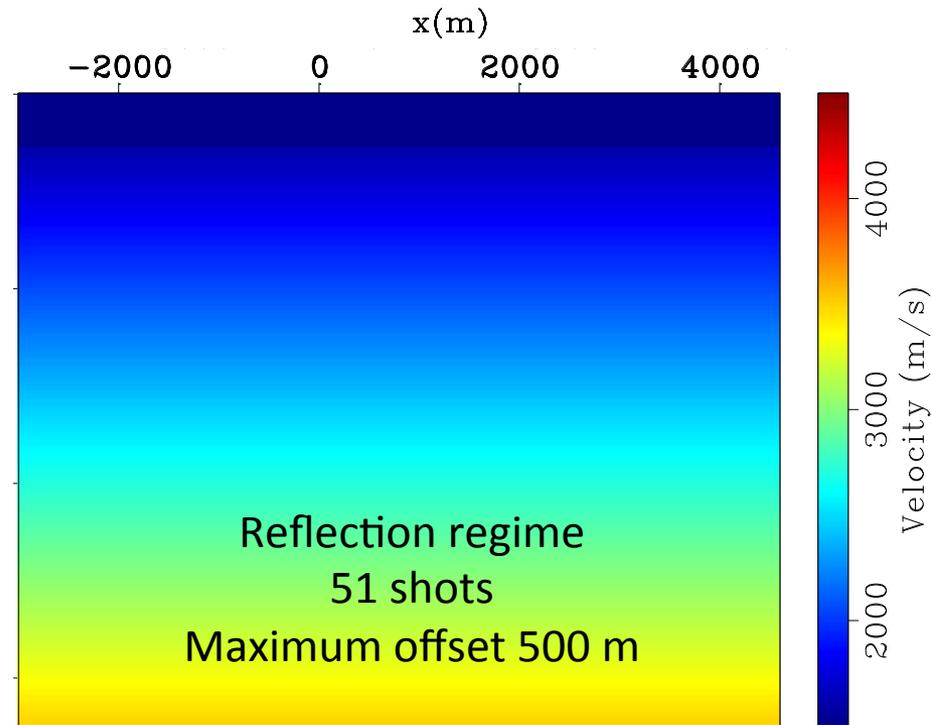
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- Challenges of Full Waveform Inversion
- Synthesize low frequencies
  - Data space: phase-tracking and extrapolation
- **Full waveform inversion**
  - Initial solution: initialization using synthesized data
- Conclusions

# Marmousi model



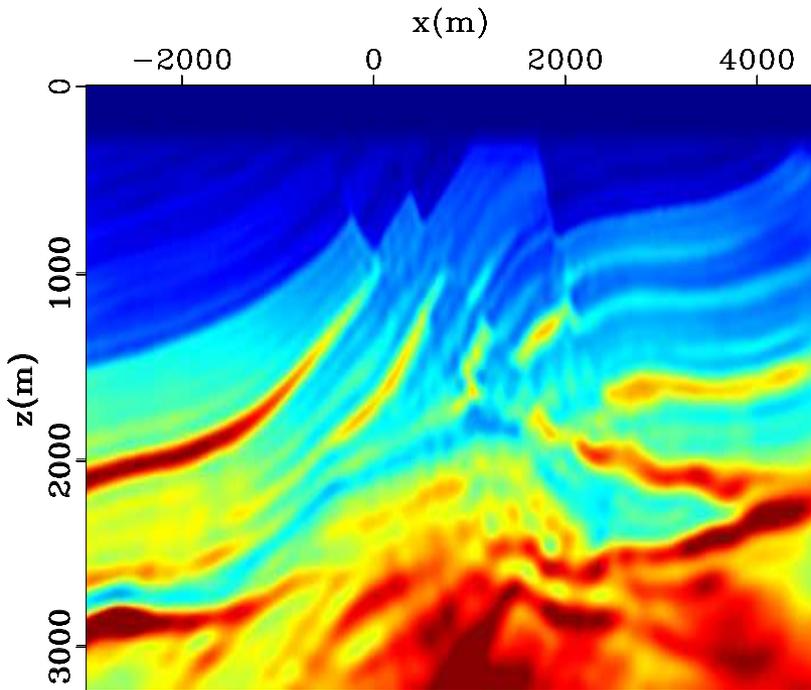
True model



Initial model

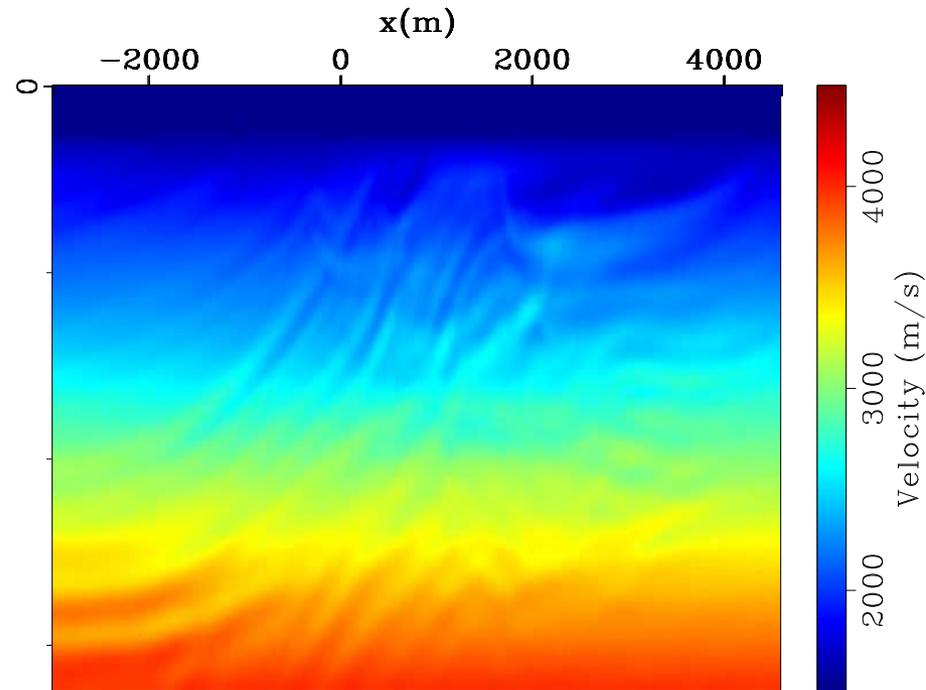
# Full waveform inversion at lowest available freq.

With recorded low frequency



Data [1-15] Hz

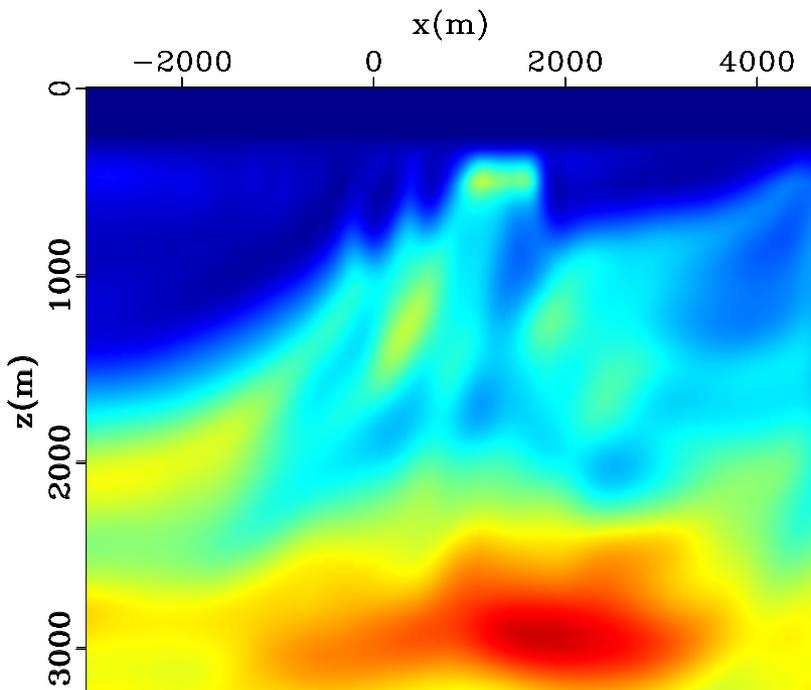
Without low frequency



Data [5-15] Hz

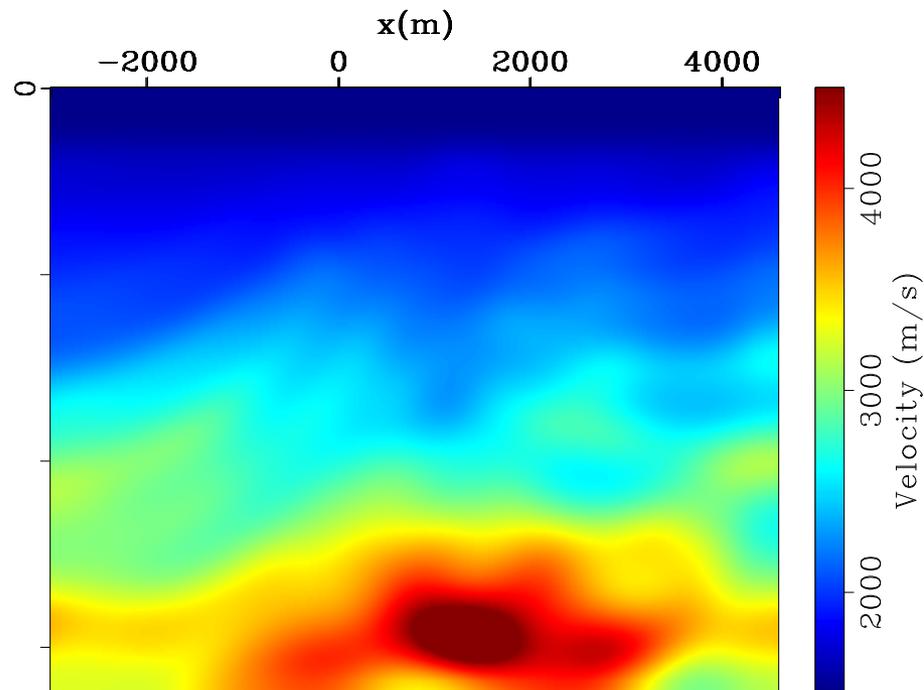
# Full waveform inversion with low frequency data

With recorded [1 - 5] Hz



FWI using modeled data

With extrapolated [1 -5] Hz

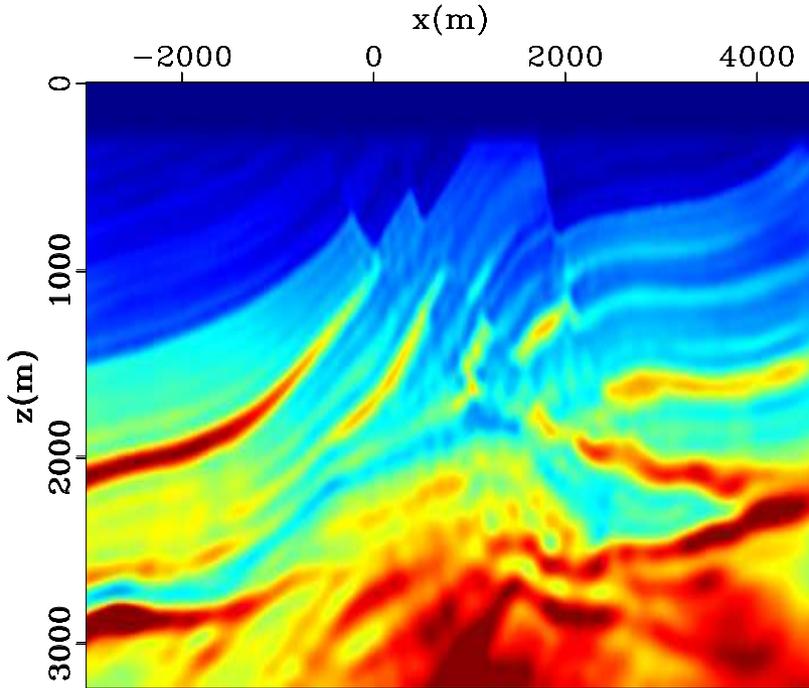


FWI using extrapolated data

Low wavenumber model to initialize FWI at higher frequencies

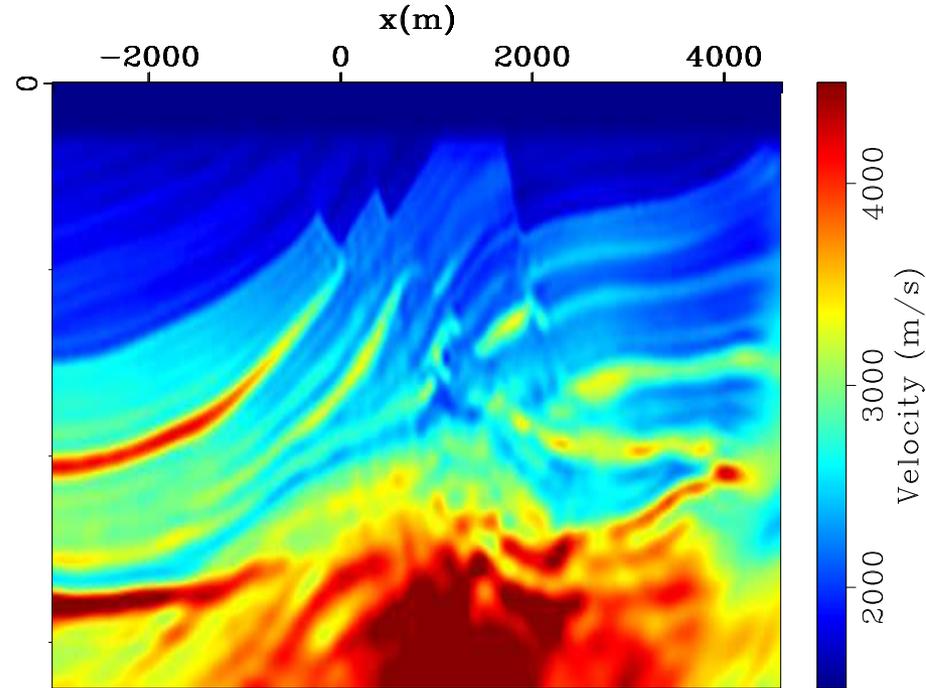
# Full waveform inversion with frequency sweep

With recorded low frequency



Modeled data [1-15] Hz

With extrapolated low frequency



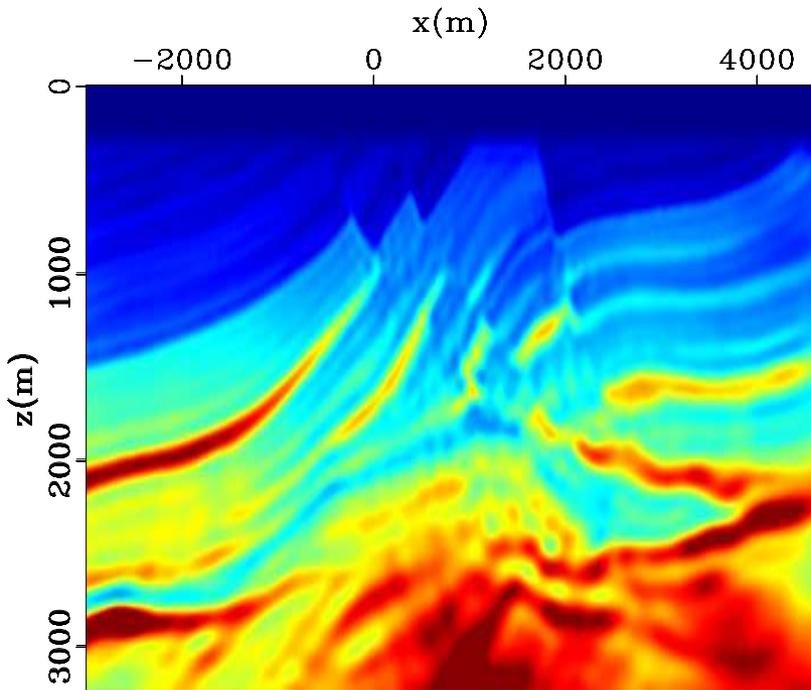
Extrapolated data [1-5] Hz

Modeled data [5-15] Hz

Li and Demanet, to appear in Geophysics, 2016

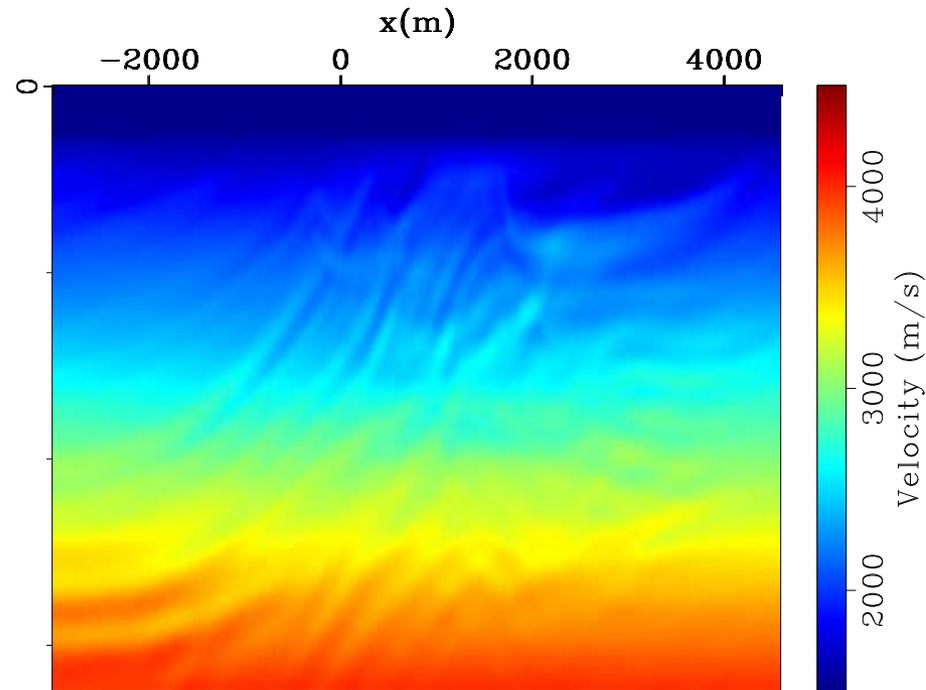
# Full waveform inversion at lowest available freq.

With recorded low frequency



Modeled data [1-15] Hz

Without low frequency



Modeled data [5-15] Hz

# Conclusions

- I. Low frequency data can be extrapolated from the band-limited field recordings
  - a. Event tracking → separation
  - b. Non-attenuative medium → frequency extrapolation
  
- II. Extrapolated low frequencies are imperfect, but reliable for initializing FWI
  - a. Amplitude information → normalized
  - b. Phase information → reliable for initializing FWI

# Thank you!

## Filling in the low frequency gap

