MIT EARTH RESOURCES LABORATORY
ANNUAL FOUNDING MEMBERS MEETING 2018

Multicomponent elastic imaging: new insights from the old equations

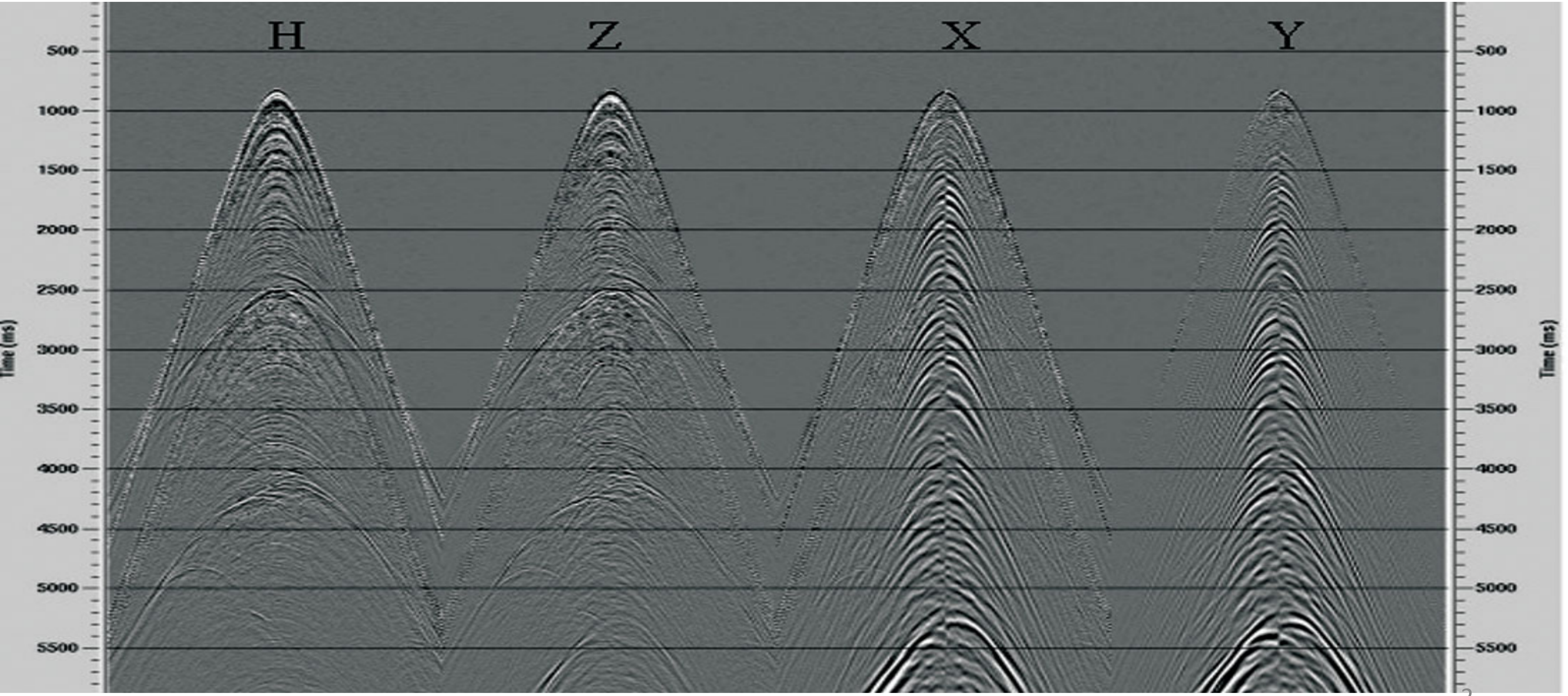
Yunyue Elita Li*, Yue Du, Jizhong Yang, and Arthur Cheng

SINGAPORE GEOPHYSICS PROJECT

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

NATIONAL UNIVERSITY OF SINGAPORE

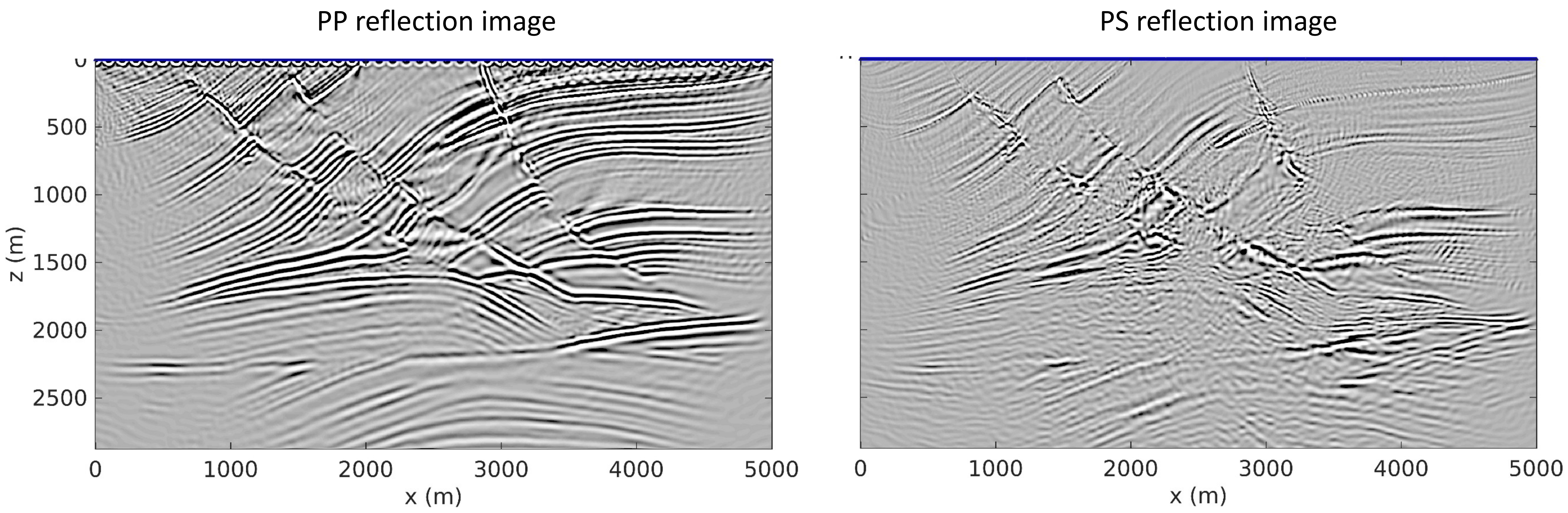
OBN acquisition: 4C data



Elastic imaging is not widely applied

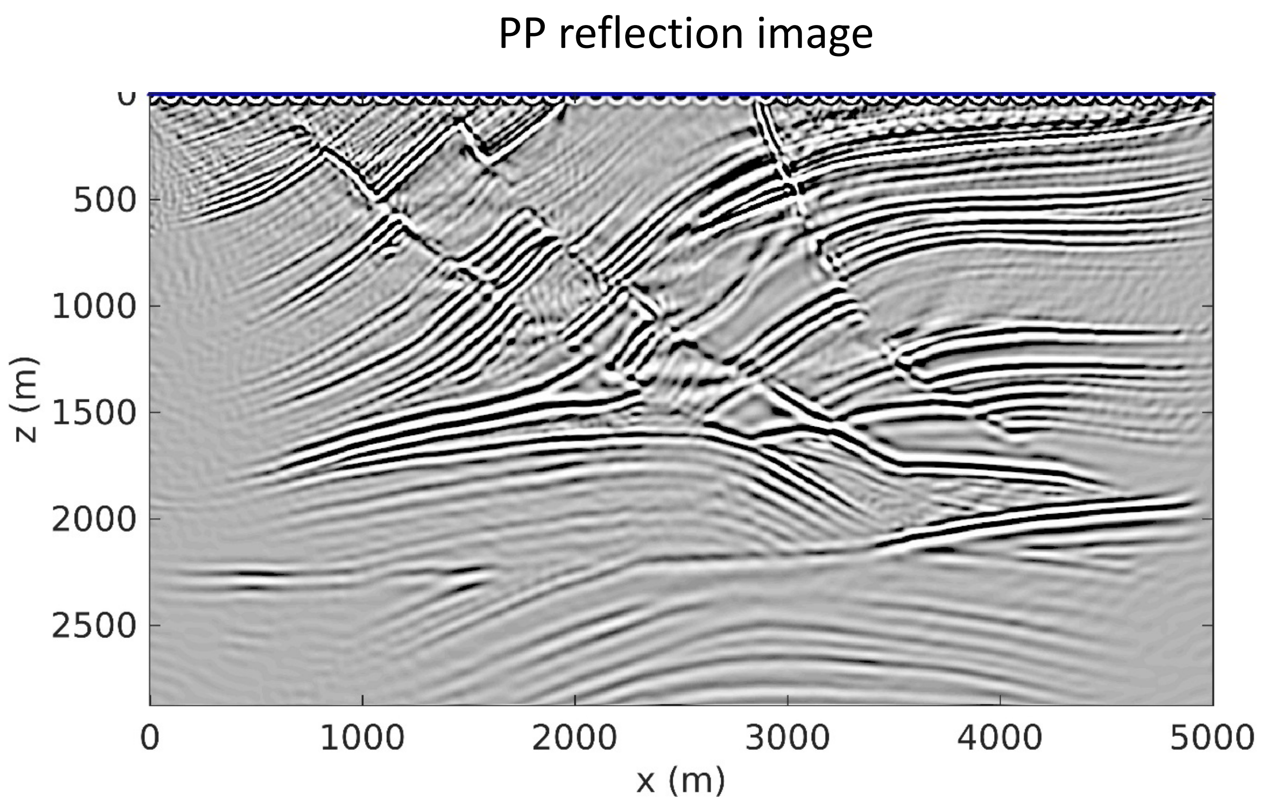
- Large computational cost compared with acoustic imaging
  - 5 times in runtime and memory in 2D
  - 9 times in runtime and memory in 3D
- Deteriorated image for converted waves
  - Polarity reversal at normal incidence
  - Complicated, cumbersome, and add hock

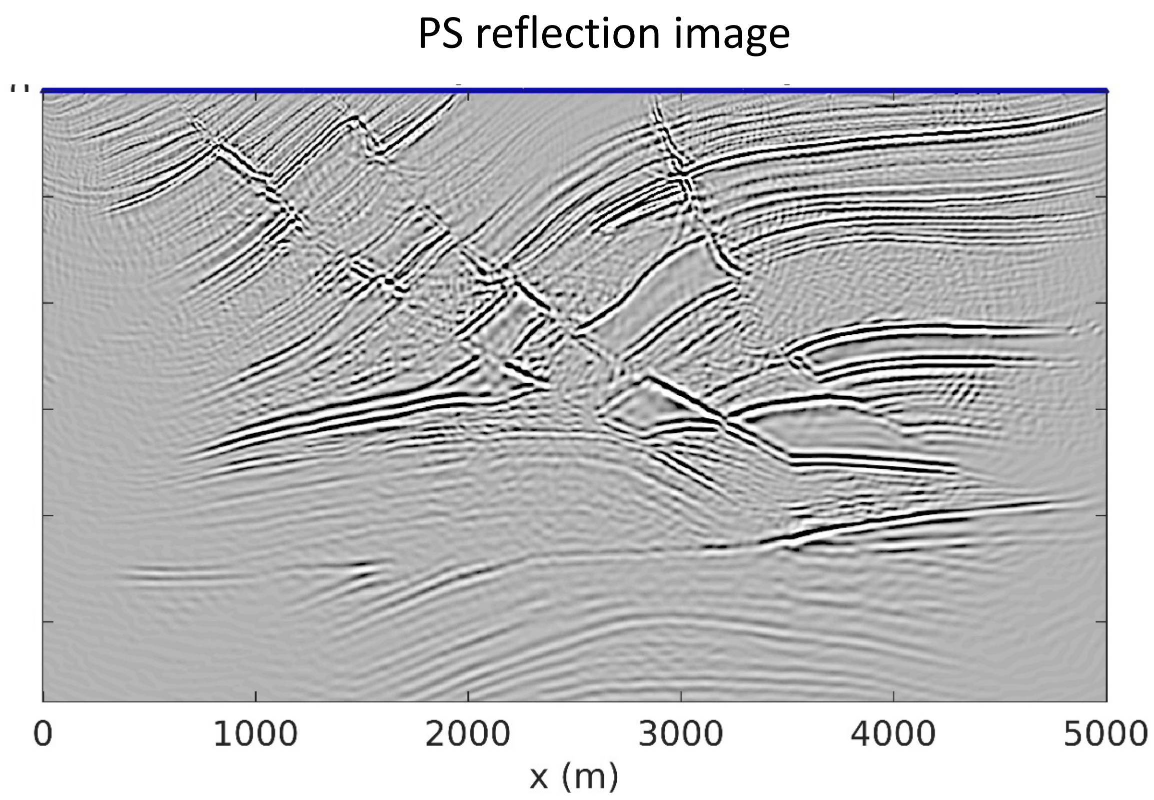
Industry standard imaging algorithm



- Converted wave imaging appears noisier, less coherent, and challenging for joint interpretation
- Images are obtained with 5 times the computation and memory cost of the acoustic images

Proposed imaging algorithm





- ✓ Converted wave imaging shows consistent geological features with higher resolution
- ✓ Imaging cost are reduced by 60% in computation and 80% in memory

Outline

- Elastic wave equations
  - Revisit of the elastic wave equations
  - A new set of separated P- and S-wave equations
- The elastic imaging condition
  - PP and PS images from inverse problem formulation
- Discussions and conclusions

Seismology 101: elastodynamic system

- Linear, isotropic, elastic medium (Aki and Richards, 1980)

Newton's Law:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \tau_{ij} + f_i$$

Hooke's Law:

$$\tau_{ij} = \lambda \delta_{ij} \partial_k u_k + \mu(\partial_i u_j + \partial_j u_i)$$

u_i particle displacement

τ_{ij} element of the stress tensor

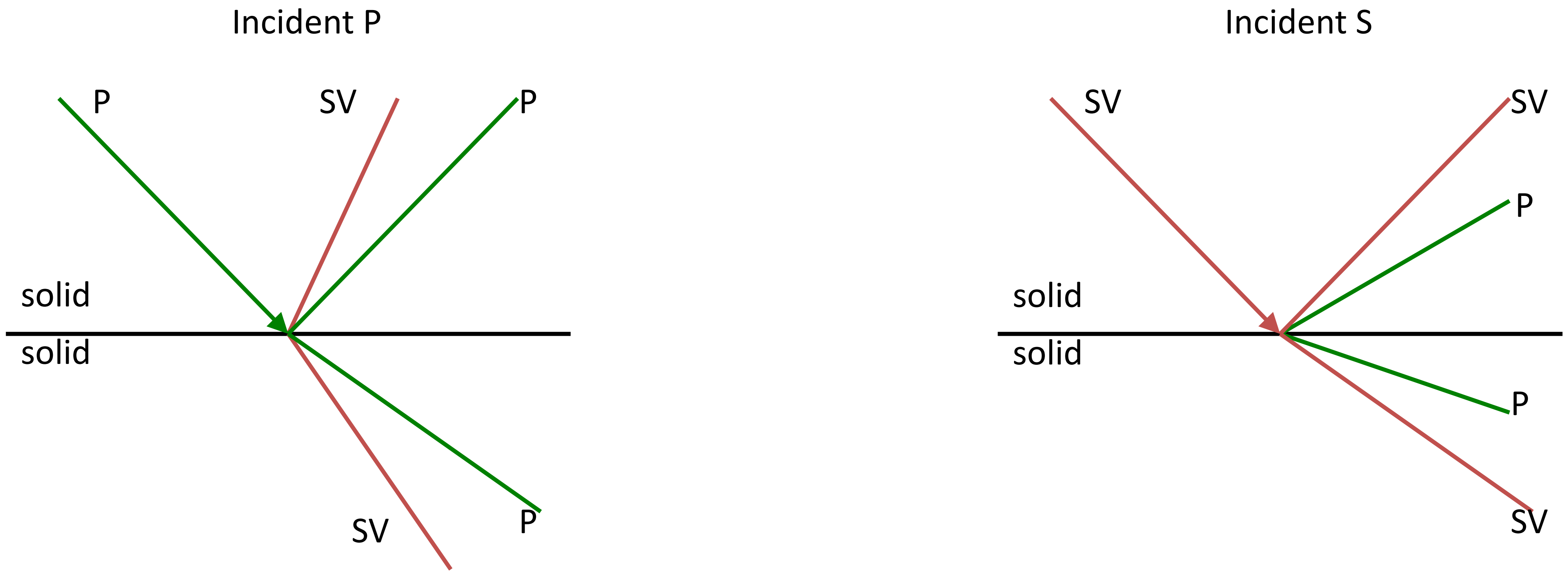
f_i force

ρ, λ, μ density and Lame constants

✧Need to propagate (and store) 5 fields in 2D, and 9 fields in 3D

✧Cannot interpret the P- and S-wave directly from the equations

Seismology 101: mode conversion

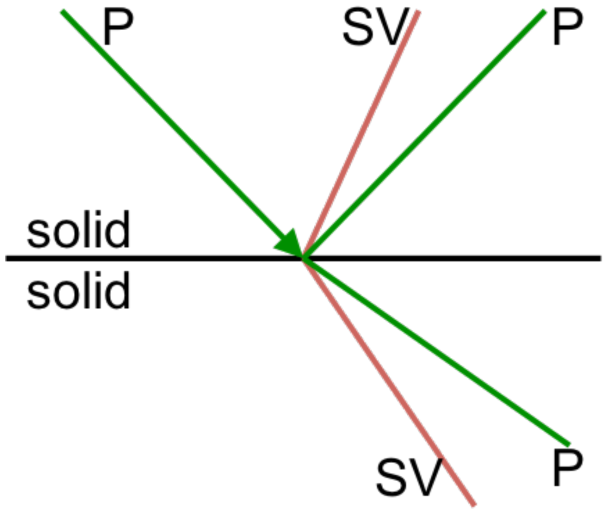


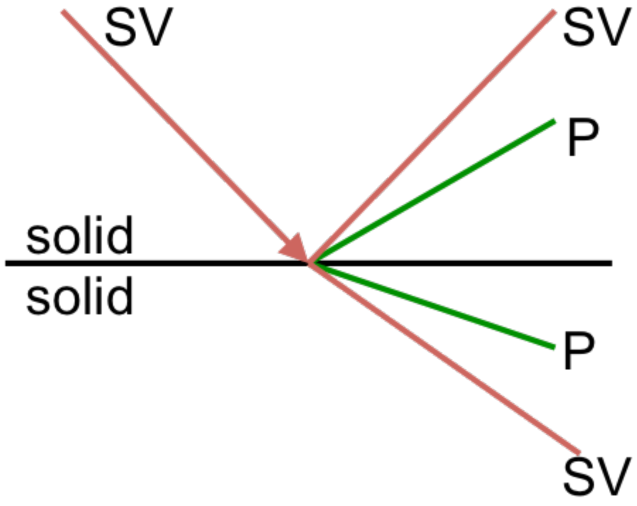
- ✧ Are these mode conversion types unconditional?
- ✓ New set of equations: clear mode conversion and its condition

New set of separated P- and S-wave equations

$$
\begin{aligned}
\boxed{\ddot{P} - \alpha\nabla^2 P} &= \nabla \cdot \mathbf{f} && \leftarrow \text{Source term} \\
&+ P\nabla^2\alpha + 2\nabla\alpha \cdot \nabla P && \leftarrow \text{P-wave interacts with } V_p \text{ boundary} \\
&- 2P\nabla^2\beta && \leftarrow \text{P-wave interacts with } V_s \text{ boundary} \\
&- 2\nabla\beta \cdot \nabla \times \mathbf{S} && \leftarrow \text{S-wave interacts with } V_s \text{ boundary}
\end{aligned}
$$

P-wave propagation





Li et. al., submitted to Geophysics, 2018

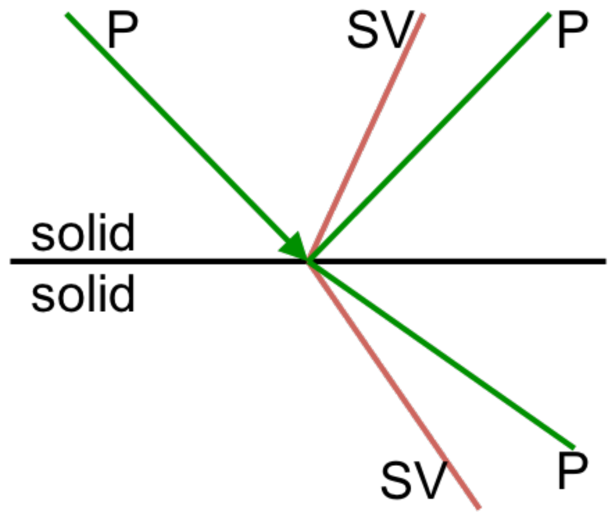
New set of separated P- and S-wave equations

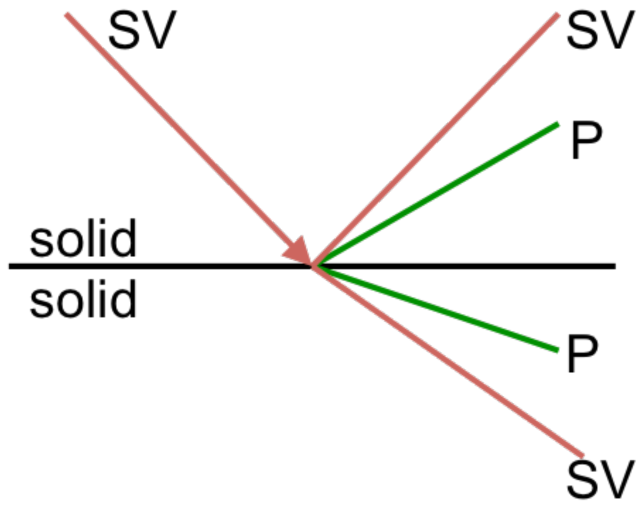
$$\boxed{\ddot{\mathbf{S}} - \beta\nabla^2\mathbf{S}} = \nabla\times\mathbf{f}$$ ← Source term

$$+\nabla\beta\cdot\nabla\mathbf{S} - (\nabla\beta)\times(\nabla\times\mathbf{S})$$ ← S-wave interacts with V_s boundary

S-wave propagation

$$+2(\nabla\beta)\times(\nabla P)$$ ← P-wave interacts with V_s boundary





Li et. al., submitted to Geophysics, 2018

Insights from the equations

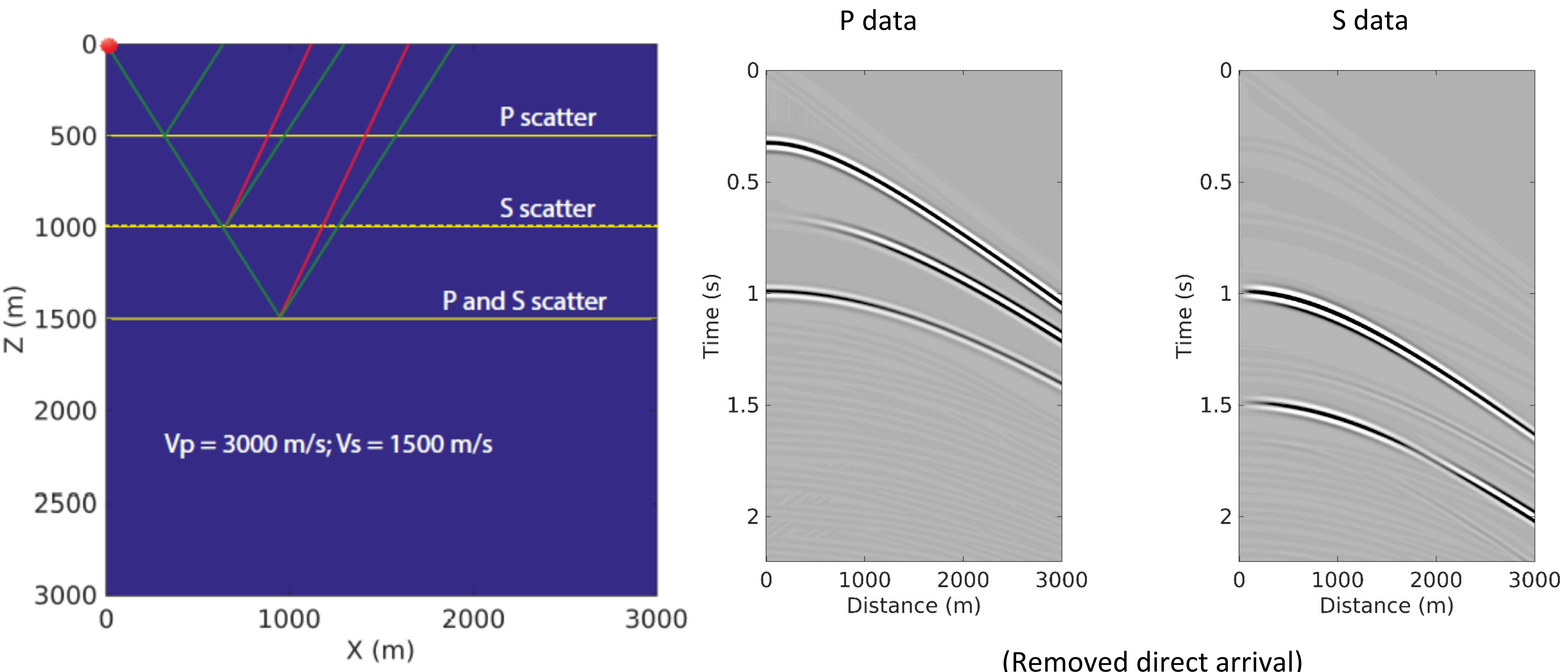
$$\ddot{P} - \alpha\nabla^2 P = P\nabla^2\alpha + 2\nabla\alpha\cdot\nabla P - 2P\nabla^2\beta \boxed{- 2\nabla\beta\cdot\nabla\times\mathbf{S}} + \nabla\cdot\mathbf{f}$$

$$\ddot{\mathbf{S}} - \beta\nabla^2\mathbf{S} = \nabla\beta\cdot\nabla\mathbf{S} - (\nabla\beta)\times(\nabla\times\mathbf{S}) + \boxed{2(\nabla\beta)\times(\nabla P)} + \nabla\times\mathbf{f}$$

- ✓ New set of equations: coupled but separated for P- and S-propagations in heterogeneous (Lamé) media (constant density)
- ✓ Wave-medium interactions can be directly interpreted
- ✓ Mode-conversion only happens at S-wave discontinuities!
- ✓ Discontinuities only in V_p are transparent to S-wave

Li et. al., submitted to Geophysics, 2018

Elastic simulations in heterogeneous media



(Removed direct arrival)

Outline

- Elastic wave equations
  - Revisit of the elastic wave equations
  - A new set of separated P- and S-wave equations
- The elastic imaging condition
  - PP and PS images from inverse problem formulation
- Discussions and conclusions

Imaging condition

image = source wavefield meets scattered wavefield

- ✧ Wavefields only recorded on the boundary
  - ✧ Source: source signature
  - ✧ Scattered: receiver recordings
- ✧ How does the wavefields meet?
  - ✧ P-wave: scalar
  - ✧ S-wave: vector

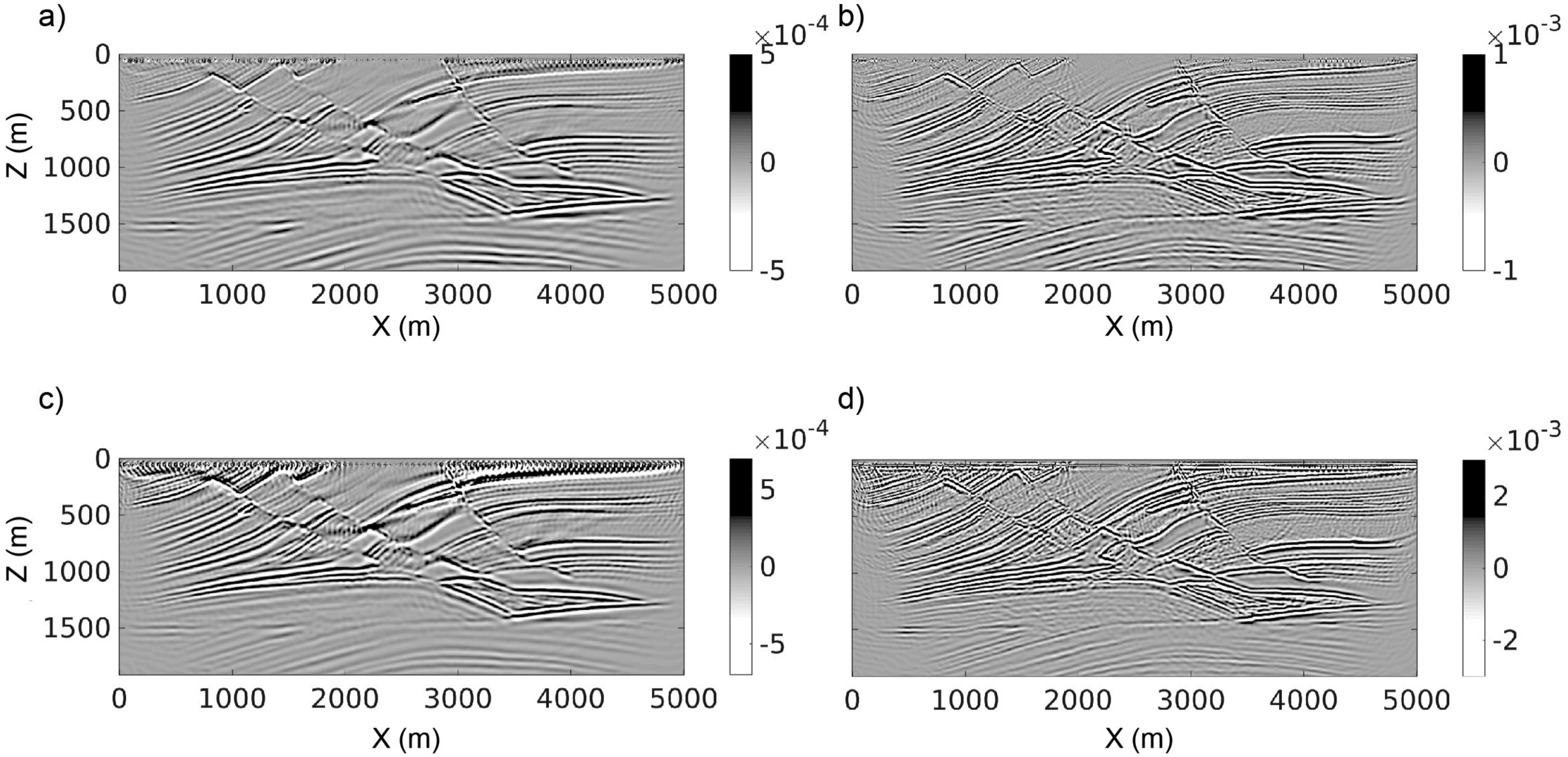
- ✓ Approximate wavefields by solving wave equations
  - ✓ Source: forward propagation
  - ✓ Scattered: backward propagation
- ✓ Formulate imaging problem as an inverse problem
  - ✓ P-wave: take a gradient
  - ✓ S-wave: take a curl

PP Image

PS Image

Using acoustic propagators

Using elastic propagators



Discussions and conclusions

- We derive a new set of coupled, but separated wave equations for P- and S-wave propagation
- This work provides a rigorous theoretical basis for the vector image conditions
- Better interpretation of the PP and PS images based on fundamental wave physics

Limitations

- Constant density assumption
  - P- and S-waves are fully coupled at all density discontinuities
  - Images are contaminated with density contrasts
- P- and S-data separation in the recorded data
  - Potential data are needed for this formulation
  - Inverse problem to solve for the separated fields

Acknowledgements

- Economic Development Board for supporting the Petroleum Engineering Program
- MOE Tier 1 Grants R-302-000-165-133 and R-302-000-182-114