What kind of uncertainty quantification is useful for seismic imaging?

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Goals of this presentation

1. Explain a novel approach to uncertainty quantification (UQ) that avoids dubious assumptions about linearity and Gaussianity.
2. Illustrate how assumed non-Gaussian distributions of prior model properties and measurements propagate to travel time.
3. Stimulate your feedback about what UQ would be desirable to provide for seismic imaging.

A new approach to image UQ

In the WKBJ approximation, Claerbout's condition for image value $I[\mathbf{x}]$ at a 3D point $\mathbf{x}$ reduces to

$$I[\mathbf{x}] = R[\mathbf{x}]\delta_{\mathrm{b}}[\tau_{\mathrm{s}}[\mathbf{x}] - \tau_{\mathrm{r}}[\mathbf{x}]],$$

where $R[\mathbf{x}]$ is the reflection coefficient and τ_{s} and τ_{r} are the from-source and to-receiver travel times (Scales 1995 Ch. 7). (The pulse

$$\delta_{\mathrm{b}}[t] = \int \mathrm{d}\nu F[\nu] \exp[-2\pi \mathrm{i} \nu t]$$

describes the signal bandwidth and attenuation.)

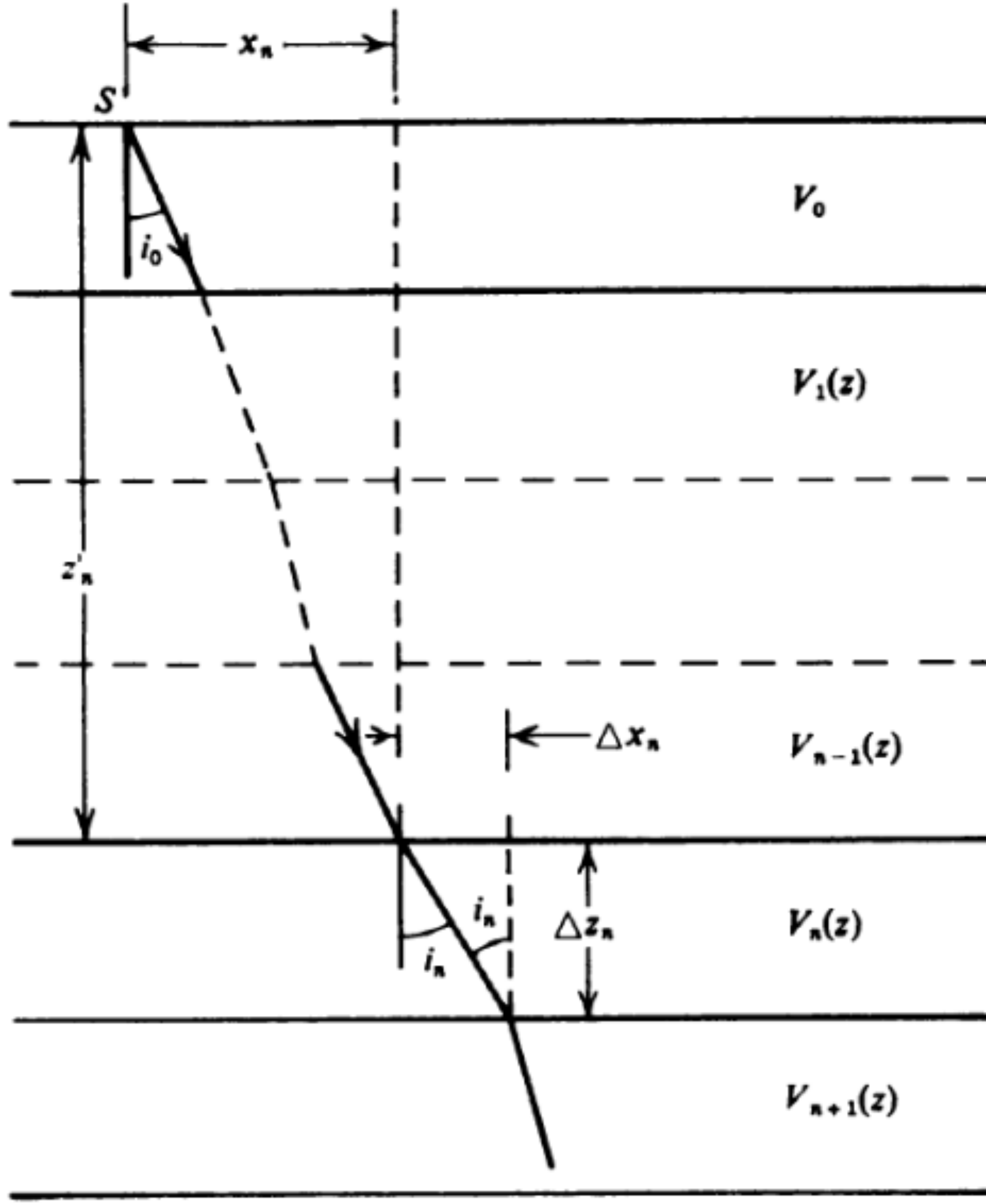
Any $R[\mathbf{x}]$ uncertainty appears proportionally in $I[\mathbf{x}]$.

Uncertainty in $\tau_s[\mathbf{x}] - \tau_r[\mathbf{x}]$ affects $I[\mathbf{x}]$ very nonlinearly, potentially shifting or creating spurious features.

Classical UQ assumes all uncertainty is multivariate Gaussian ---mainly to make analysis and computation tractable; in the present work we tentatively explore non-Gaussian uncertainty, using simplified stochastic simulation of $\tau[\mathbf{x}]$.

A simple τ model

Consider a stack of $n \to \infty$ horizontal, piecewise-uniform layers between z_0 and $z_0 + \triangle z$ (following Telford et al. 1990 §4.3.2)…



A simple τ model...

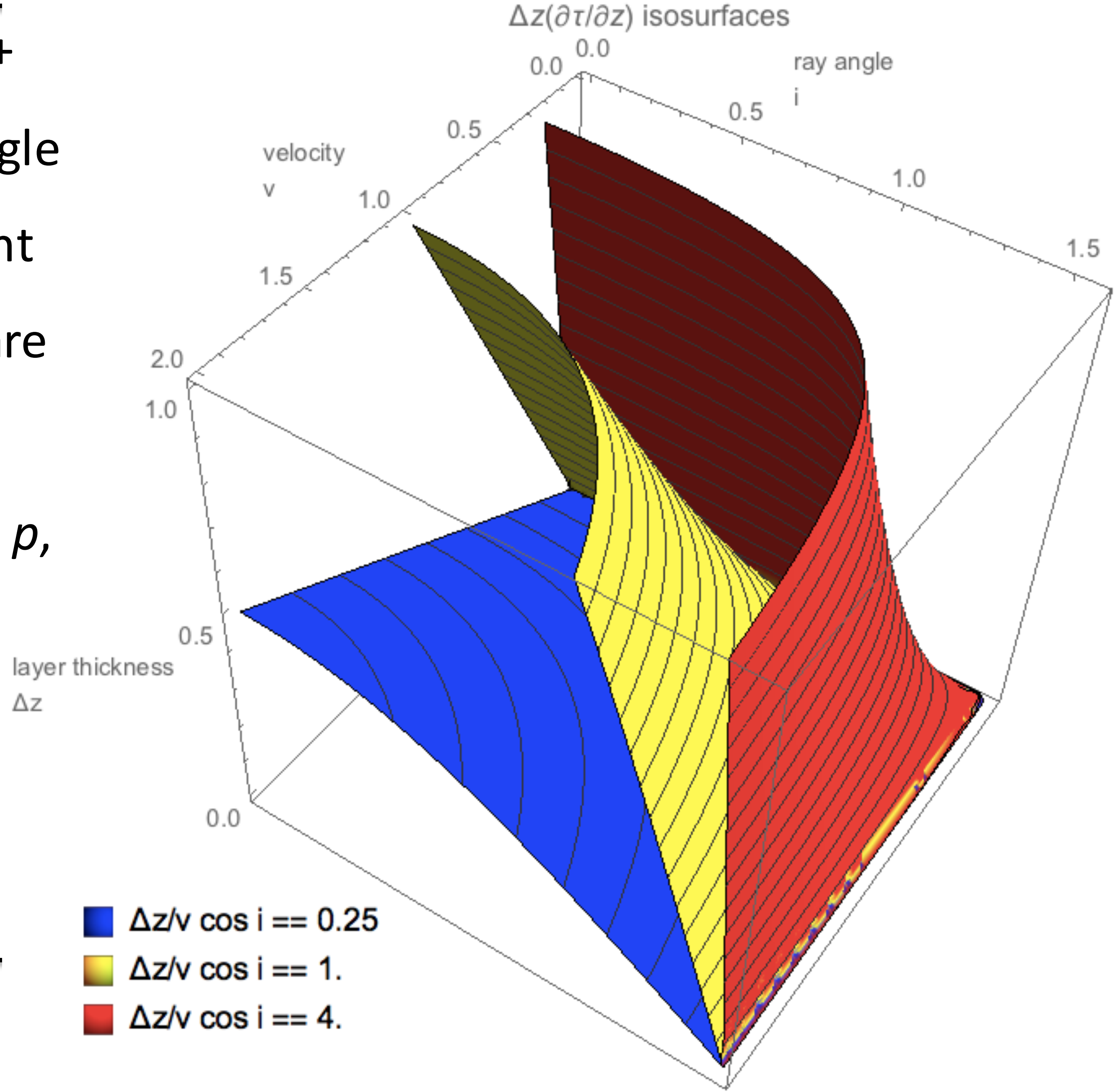
At depth $z_0 < z < z_0 + \triangle z$, $\tau[z]$, the ray angle $i[z]$, raypath constant p and velocity $v[z]$ are related by

$$v^{-1}\sin i = v_0^{-1}\sin i_0 = p,$$

$$v^{-1}\sec i = \partial\tau/\partial z,$$

implying ambiguity in the τ-increment

$$\triangle z\, \partial\tau/\partial z.$$



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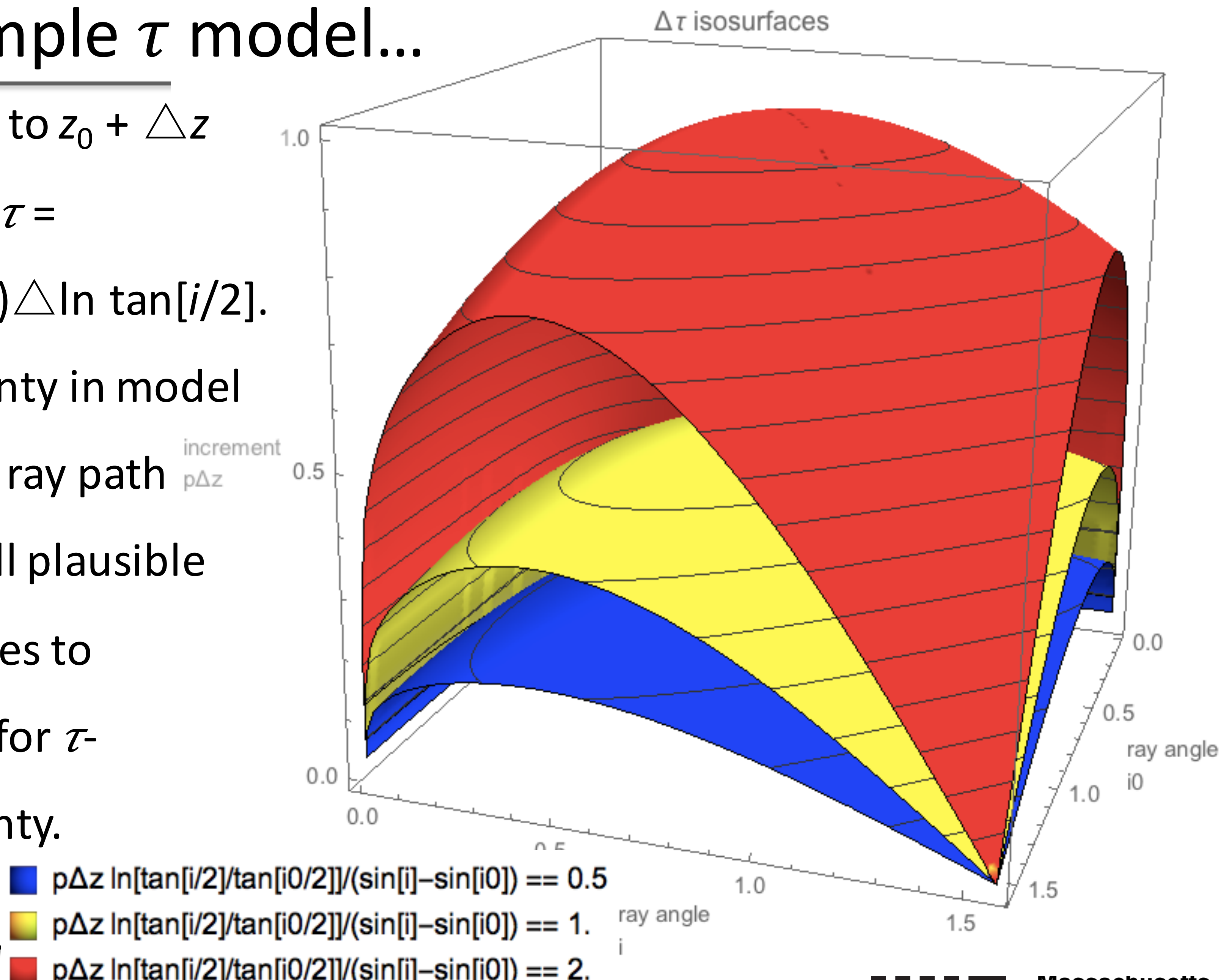
A simple τ model…

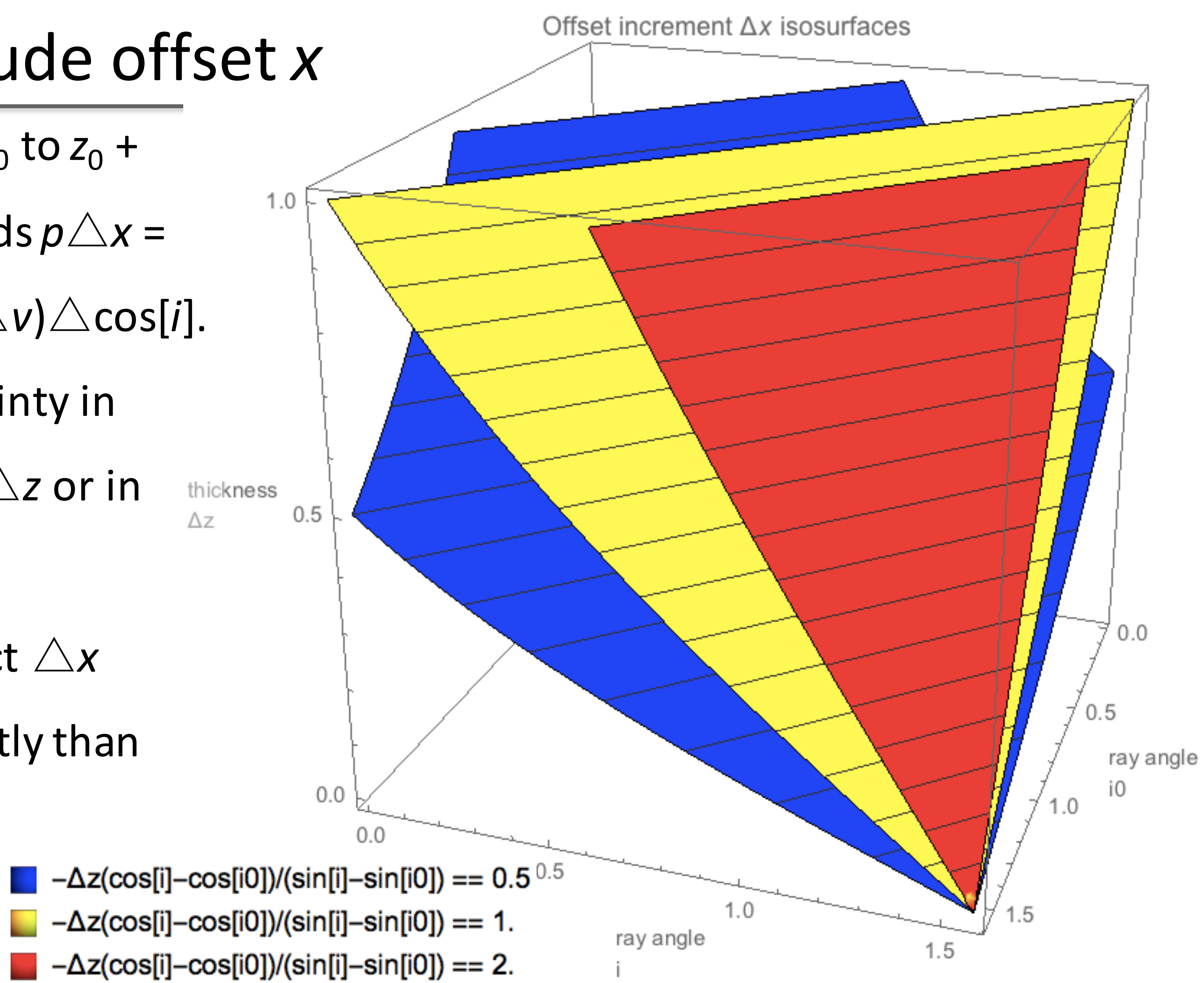
$\int$ from z_0 to $z_0 + \triangle z$ yields $\triangle \tau =$ $(\triangle z/\triangle v)\triangle \ln \tan[i/2]$. Uncertainty in model $\triangle z$ or in ray path p, i are all plausible candidates to account for τ-uncertainty.



Include offset *x*

∫ from z_0 to z_0 + $\triangle z$ yields $p\triangle x$ = $-(\triangle z/\triangle v)\triangle\cos[i]$.

Uncertainty in model $\triangle z$ or in ray path p, i affect $\triangle x$ differently than $\triangle\tau$.



“Why look at isosurfaces?”

The previous nonlinear relationships constitute a vector

$$(\triangle\tau/p, \triangle x, \triangle z) = (\triangle z/p\triangle v)\triangle\int(\csc[i], \sin[i], \cos[i])\,\mathrm{d}i.$$

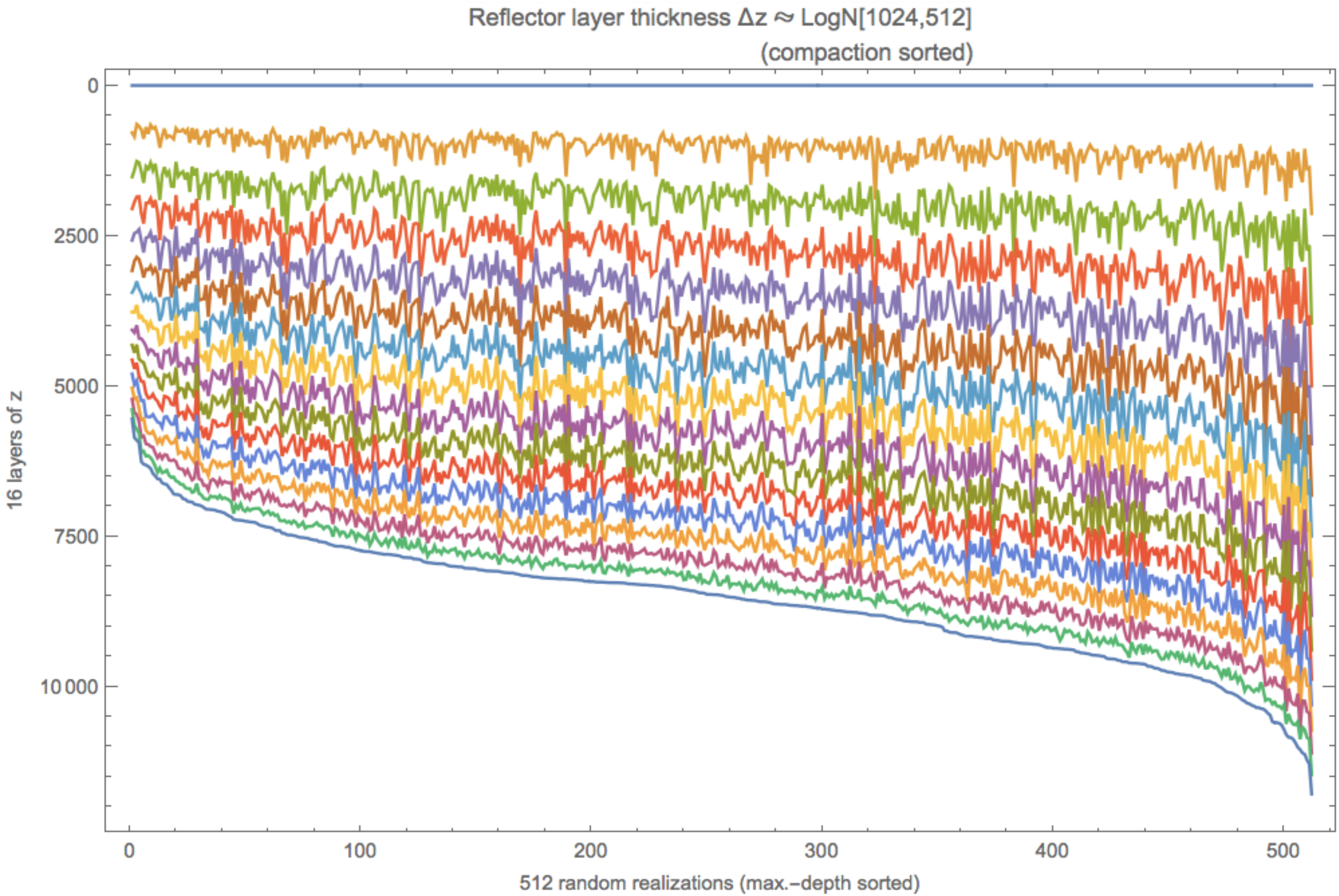
More generally, consider a multivariate random smooth nonlinear transformation and its covariance,

$$x_j = x_j[\mathbf{y}] = \langle x_j\rangle + J_{jk}(y_k - \langle y_k\rangle) + H_{jkm}(y_k - \langle y_k\rangle)(y_m - \langle y_m\rangle) + \ldots,$$

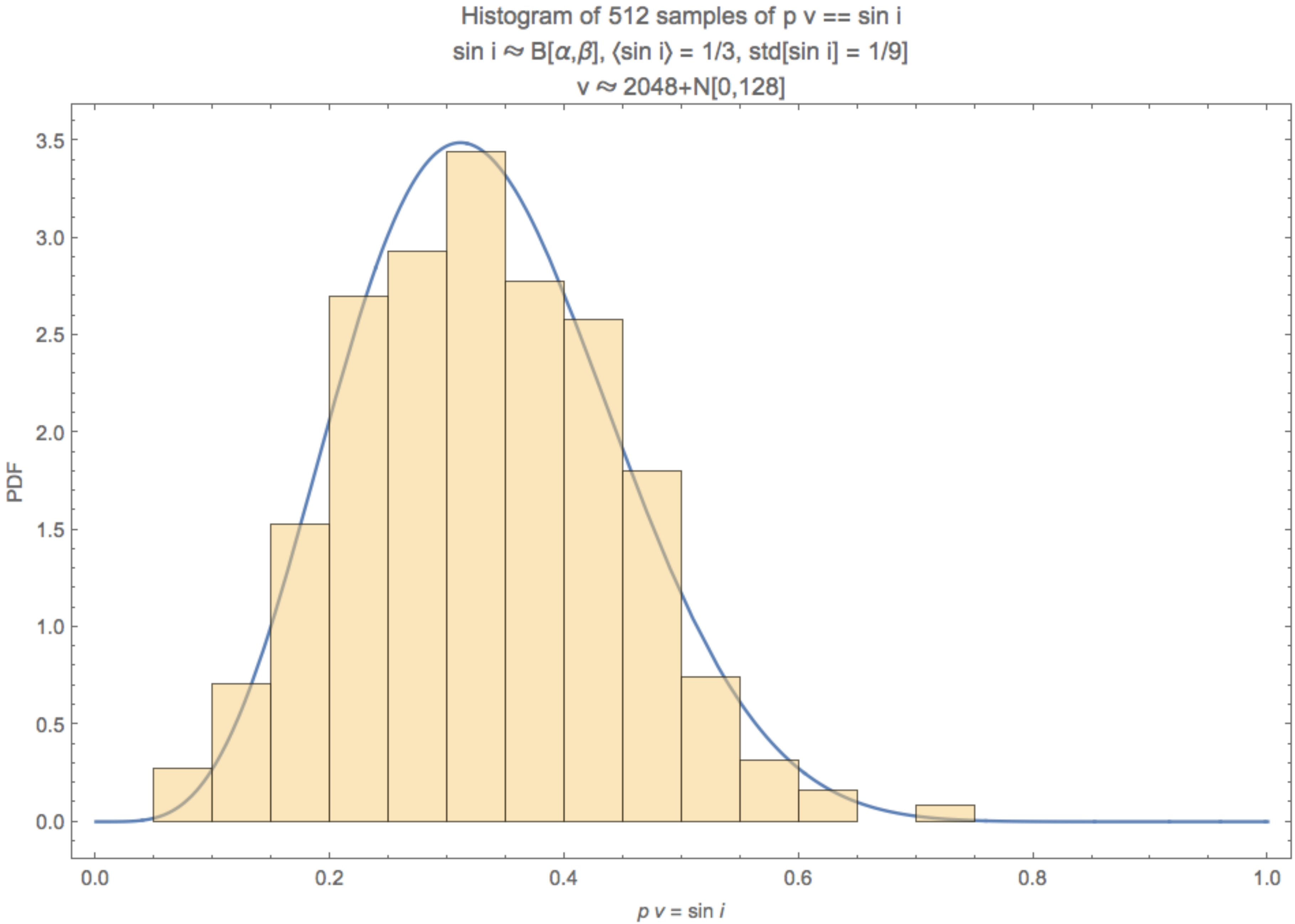
$$\mathrm{cov}_{ij}[\mathbf{x}] = J_{jk}\mathrm{cov}_{km}[\mathbf{y}]J_{im} + \ldots$$

A differential geometry analysis of curvilinear-coordinate isosurface intersections can reveal search trajectories to optimally update $(\triangle\tau/p, \triangle x, \triangle z)$ [Fournier et al. 2015: *Decision Guidance*. International Application No. PCT/US2015/016036].

Simulate random depth increments



Simulate random ray parameters



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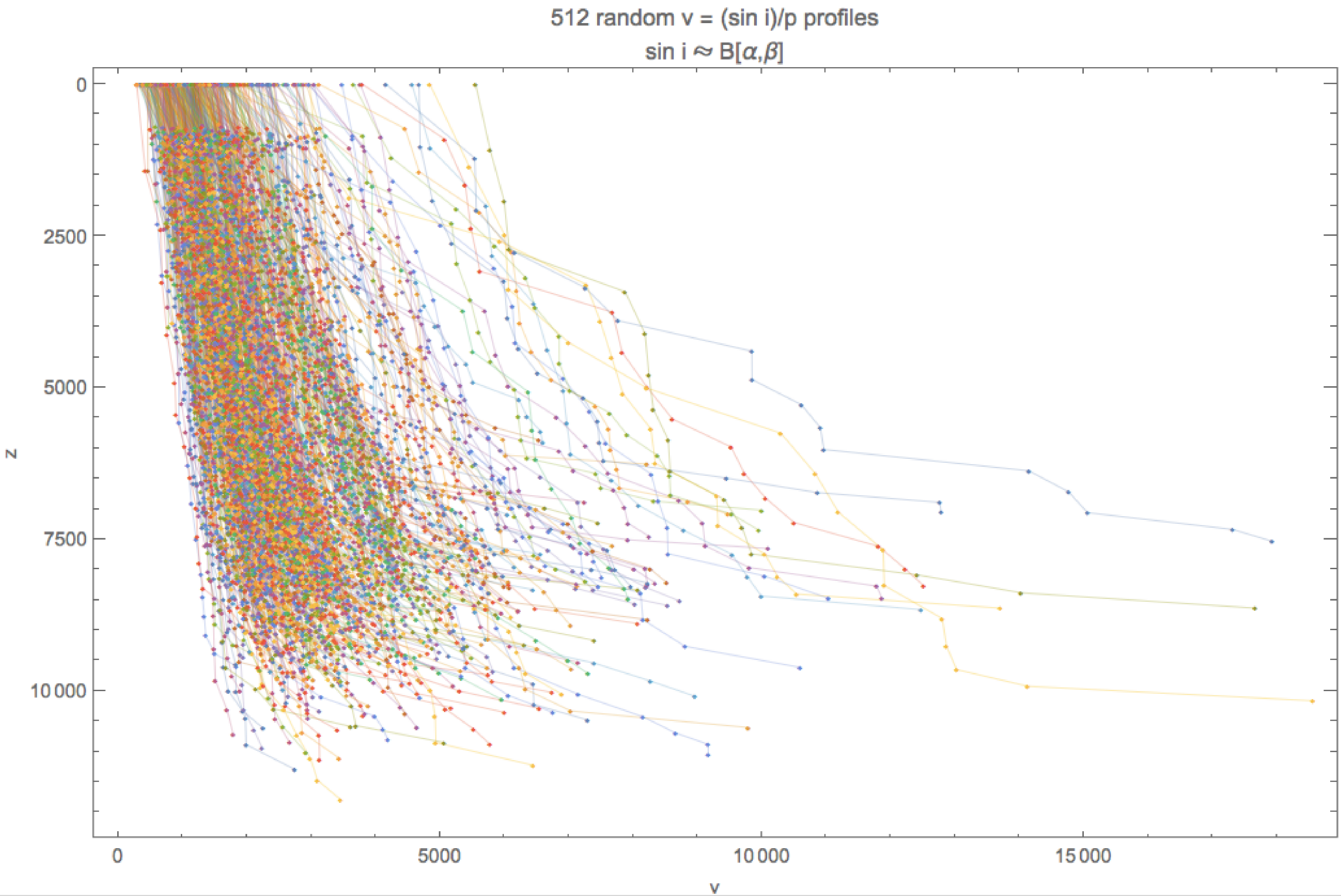
Simulate random *v* profiles



Travel time from stochastic simulation

Sum of $\triangle \tau$

from $z_0 = 0$

to full depth

may recover

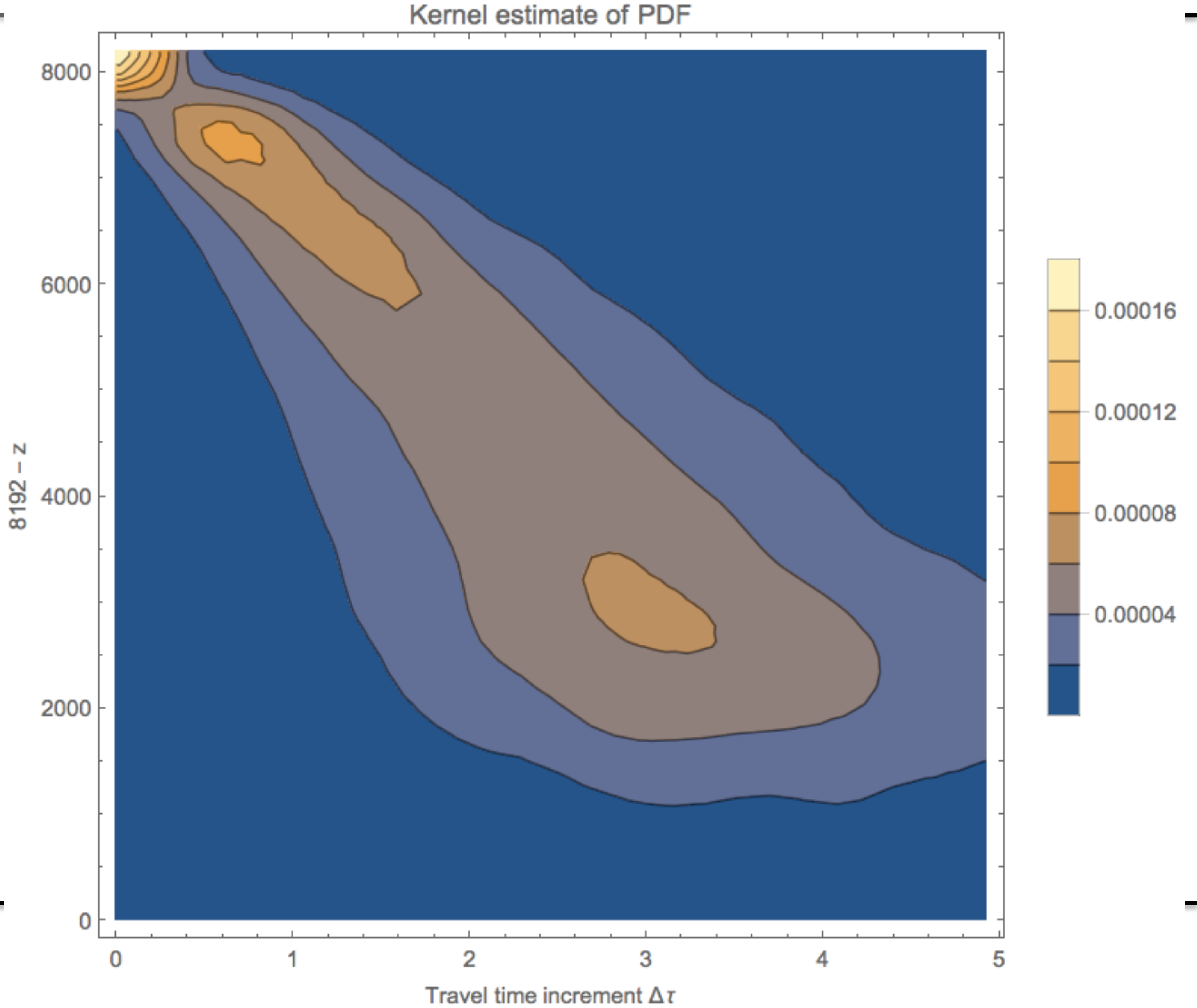
some

Gausianity

(Central

Limit

Theorem).



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Conclusions and non-conclusions

1. Mathematical evidence (and experience) $\Rightarrow$ uncertainty propagation from data to modeling to imaging involves significant nonlinearity.
2. Differential geometry analysis of idealized models can optimize update of measurements and/or model properties.
3. Simplistic stochastic simulation with non-Gaussian distributions can provide travel-time (and eventually image) distributions to compare with real-world results. Markov-chain Monte-Carlo will improve reliable inference.