MIT EARTH RESOURCES LABORATORY
ANNUAL FOUNDING MEMBERS MEETING 2018

Focused blind deconvolution of seismic signals

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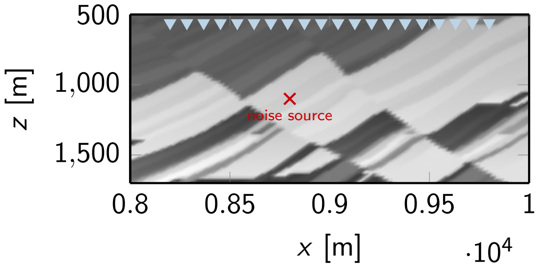
POSTDOC [DEPARTMENT OF MATHEMATICS / EARTH, ATMOSPHERIC AND PLANETARY SCIENCES]

In collaboration with Laurent Demanet and Aimé Fournier

Can We Use Noise-Source Records for Imaging?

- Noise source:
  - is uncontrollable and continuously inputs energy.
  - can be heavily *correlated* in time, i.e., its signal is non white.
- Imaging:
  - raw records is not possible because of unknown noise signature.
  - requires the subsurface Green's function that are not contaminated by noise.

Marmousi II Impedance Model



Drill Noise from Gradl et. al., 2012

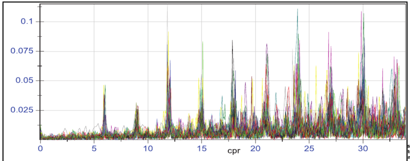


Fig. 2 Amplitude versus plot for PDC bit data

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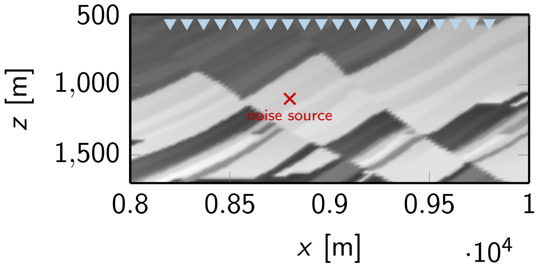

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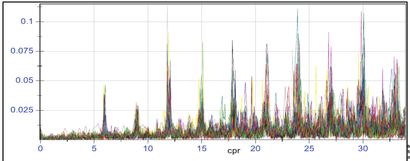
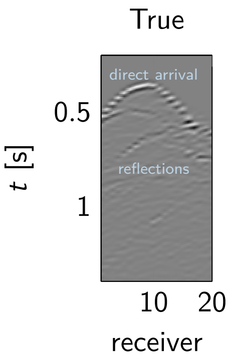
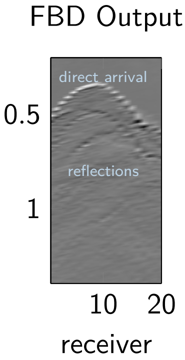
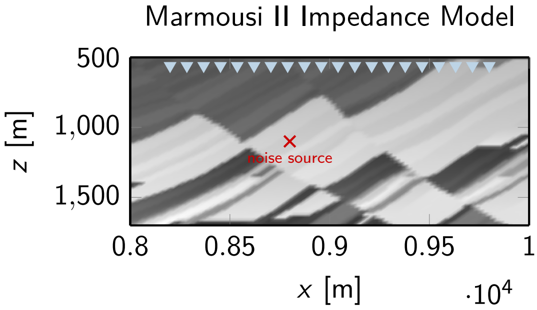


Fig. 2 Amplitude versus plot for PDC bit data

Yes, Ask FBD for Help!







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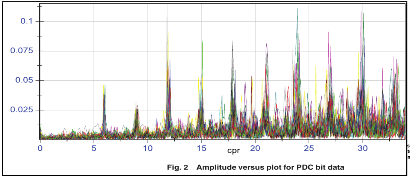
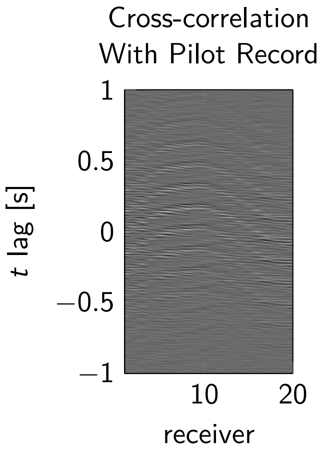
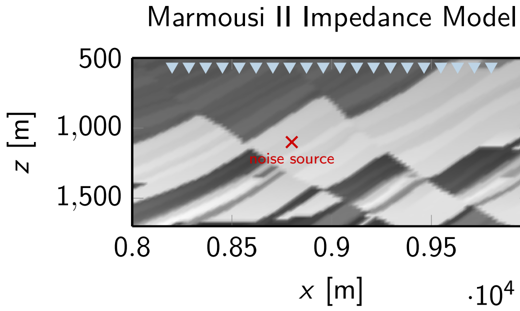


...just using a cross-correlation is not sufficient





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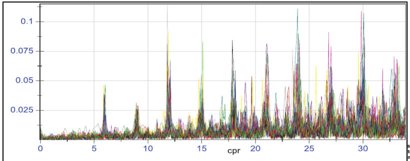
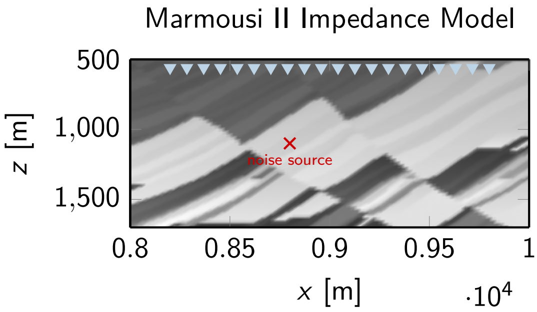


Fig. 2 Amplitude versus plot for PDC bit data

Blind Deconvolution

- This talk:
  - extraction of the Green's functions without any knowledge of the noise signature by deconvolution.



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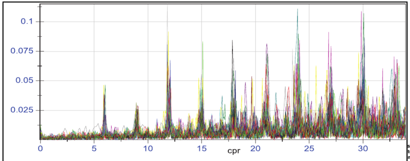

Fig. 2 Amplitude versus plot for PDC bit data

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From here on...

- $s(t)$: noise source signature in time
- $S(z)$: noise source signature after z-transform
- $d_i(t)$, $D_i(z)$: recorded noise at the i^{th} receiver, with length T
- $g_i(t)$, $G_i(z)$: subsurface Green's function at the i^{th} receiver, with length τ
- convolution in time $*$
- cross-correlation in time $\otimes$
- assume $T \geq 10\tau$

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Overview

1. Multichannel Blind Deconvolution
   - Non-uniqueness
2. Two Focusing Constraints
   - Maximally White
   - Maximally Front-loaded
3. Focused Blind Deconvolution
4. Numerical Experiments
5. Conclusions

Next Section

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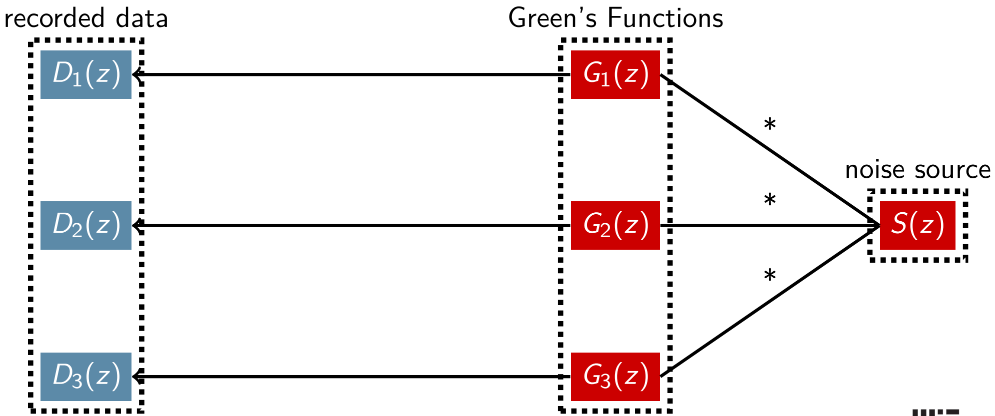
Convolutional Model



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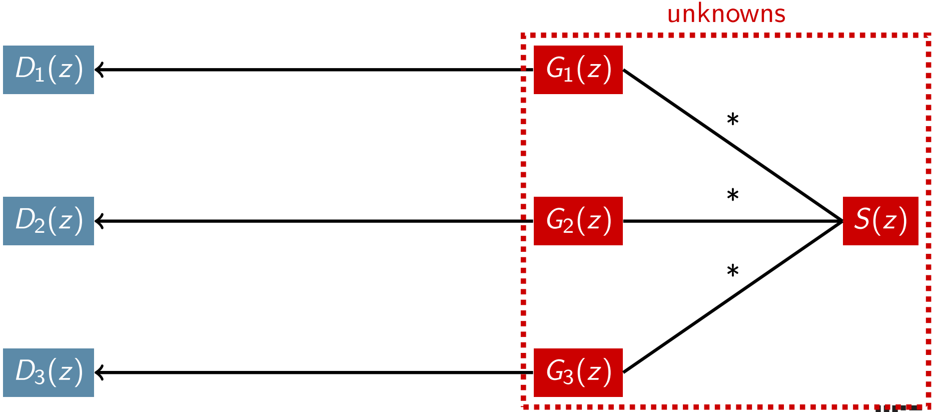

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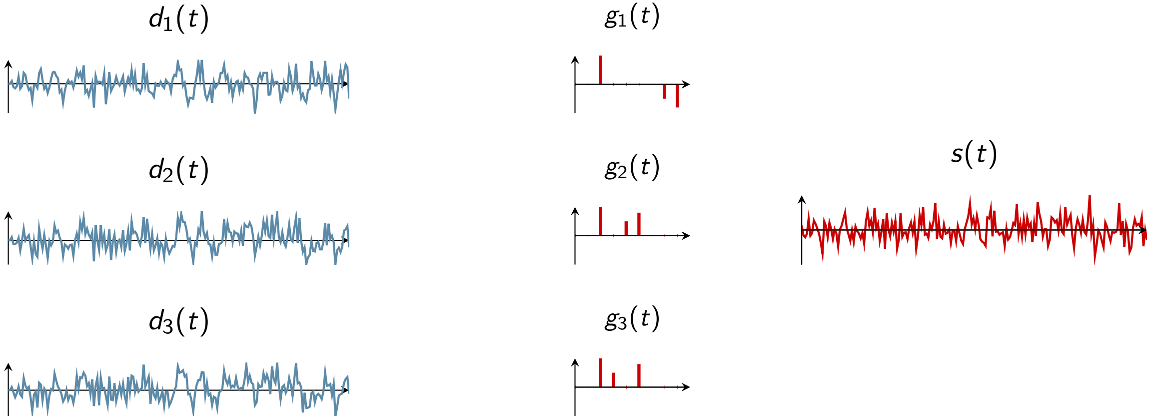
True Solution



Least-squares Blind Deconvolution

Definition

$$Q(s, g_i) = \sum_i \sum_t \{d_i(t) - [s * g_i](t)\}^2 + \alpha \underbrace{\sum_i \sum_t \frac{|t|}{T} g_{ii}^2(t)}_{\text{Maximally White}} + \beta \underbrace{\sum_i \sum_t \frac{|t|}{T} g_i^2(t)}_{\text{Maximally Front-loaded}}$$

$$(\widehat{s}, \widehat{g}_i) = \arg\min_{s, g_i} \quad Q$$

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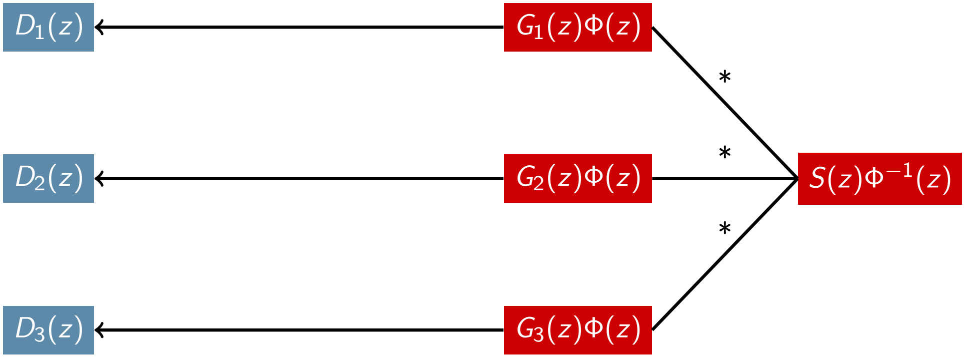
Next Subsection

$\Phi(z) \neq 1$ Can Be Exchanged



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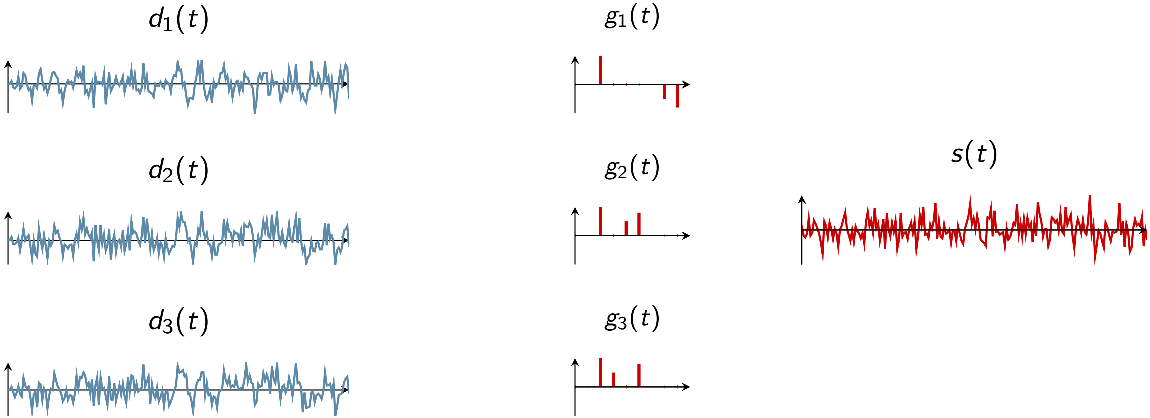
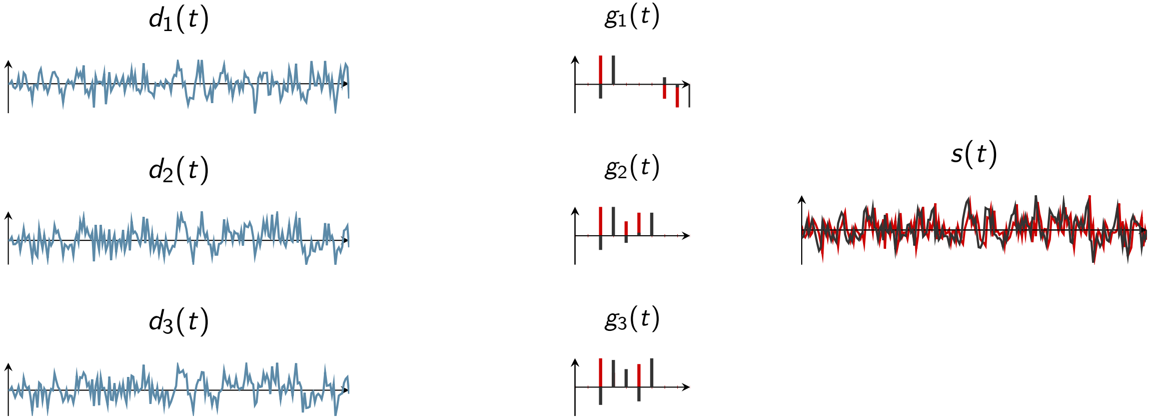
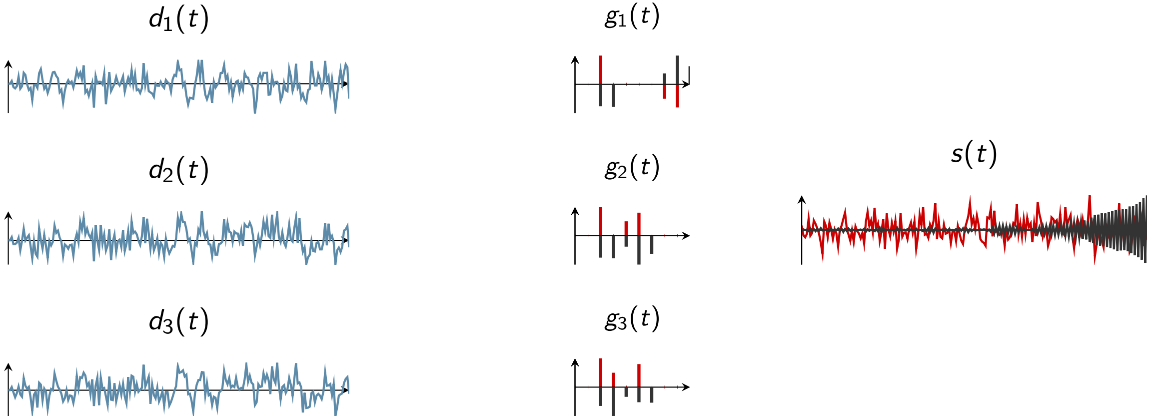


Undesired Solution 1



Undesired Solution 2



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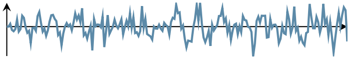
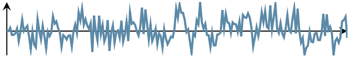
Undesired Solution 3

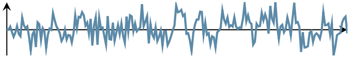
$d_1(t)$



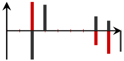
$d_2(t)$



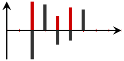
$d_3(t)$



$g_1(t)$



$g_2(t)$



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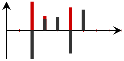
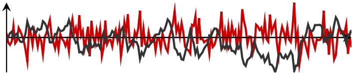


$s(t)$



Next Section

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Summary: Indeterminacy

Blind Deconvolution

$$D_i(z) = [G_i S](z) = \{[G_i\Phi][S\Phi^{-1}]\}(z)$$

Indeterminacy: $\Phi(z)$

Key Ideas

- For most of the physical systems, the Green's functions don't share the common roots, i.e., they are *coprime*.
- This constraint is sufficient to uniquely solve the multichannel blind deconvolution problem (Xu et al., 1995).
- FBD is a novel implementation of the BD problem with this constraint.

Constraint: The Estimated Green's Functions Shouldn't Have a Common Root

Summary: Indeterminacy

Blind Deconvolution

$$D_i(z) = [G_i S](z) = \{[G_i \Phi][S\Phi^{-1}]\}(z)$$

Indeterminacy: $\Phi(z)$

Indeterminacy: $|\Phi(z)|$

Indeterminacy: $\arg[\Phi(z)]$

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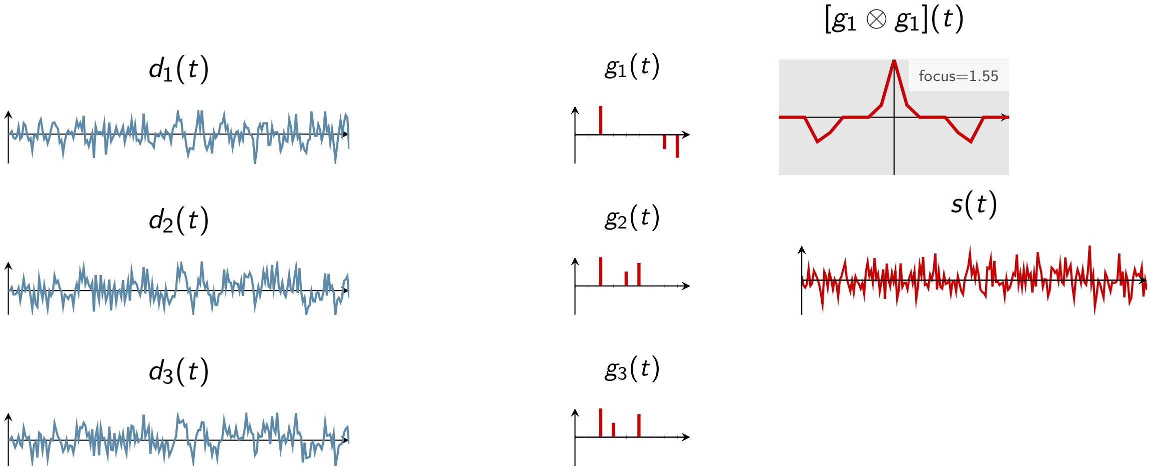
Next Subsection

True Solution: Amplitude Spectrum of the Green's Functions



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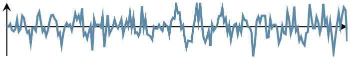
Undesired Solution 1: Amplitude Spectrum of the Green's Functions

$[g_1 \otimes g_1](t)$

$d_1(t)$



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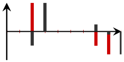
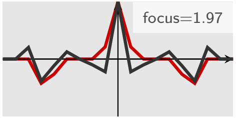
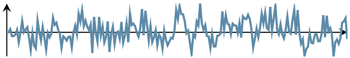




$d_2(t)$



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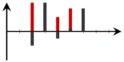
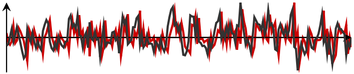


$s(t)$



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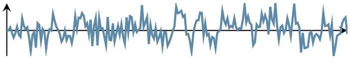
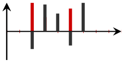


$g_3(t)$



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Undesired Solution 2: Amplitude Spectrum of the Green's Functions

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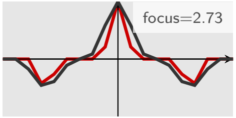

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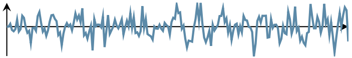

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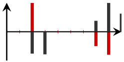

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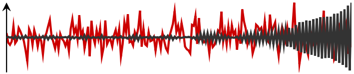
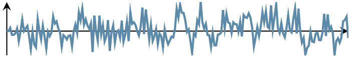


$d_2(t)$



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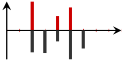

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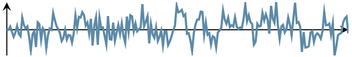

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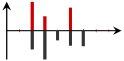
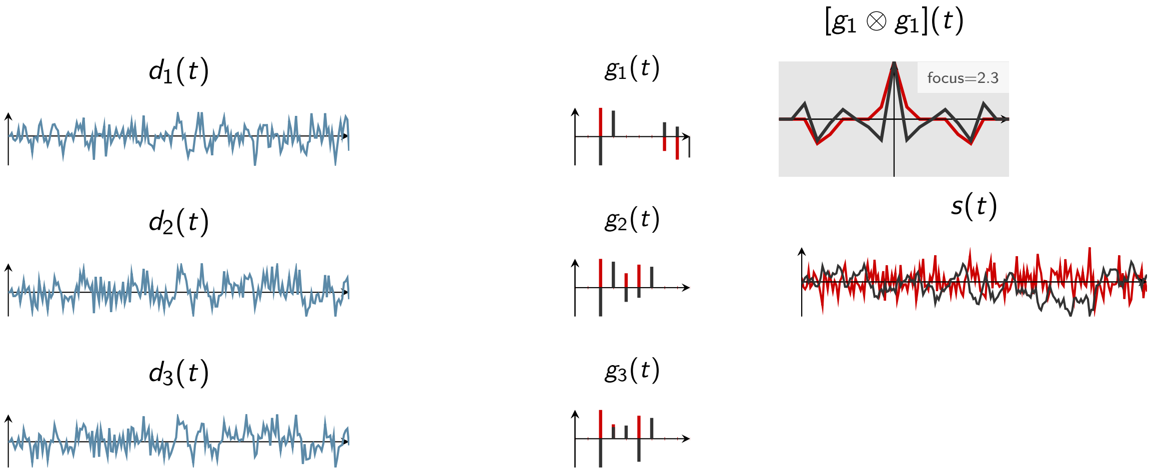


Undesired Solution 3: Amplitude Spectrum of the Green's Functions



The True Green's Functions Are...

Maximally White, so

- their auto-correlations are maximally focused at $t = 0$.

Maximally Front-loaded, so

- their phase spectra show minimal variation.
- they are maximally focused at $t = 0$.

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Least-squares Blind Deconvolution

Definition

$$Q(s, g_i) = \sum_i \sum_t \{d_i(t) - [s * g_i](t)\}^2 + \alpha \underbrace{\sum_i \sum_t \frac{|t|}{T} g_{ii}^2(t)}_{\text{Maximally White}} + \beta \underbrace{\sum_i \sum_t \frac{|t|}{T} g_i^2(t)}_{\text{Maximally Front-loaded}}$$

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Adding Focusing Constraint

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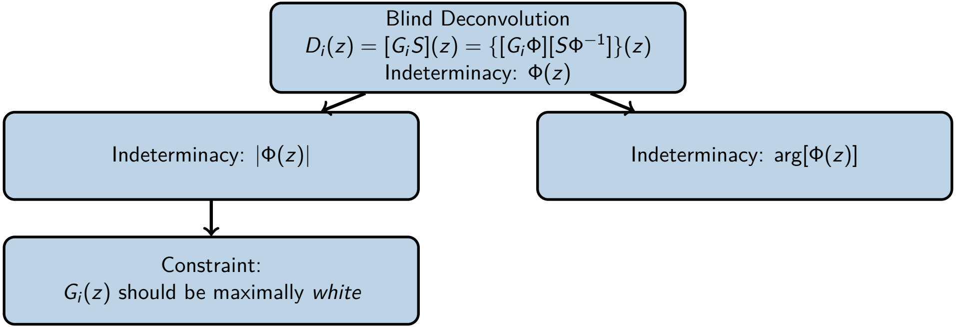
Summary: Maximally White



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Next Subsection

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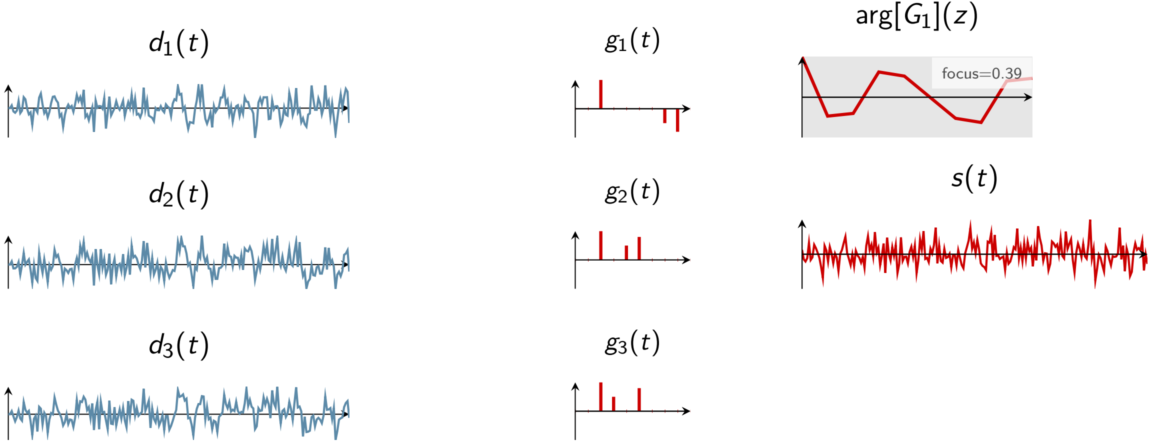
True Solution: Phase Spectrum of the Green's Functions



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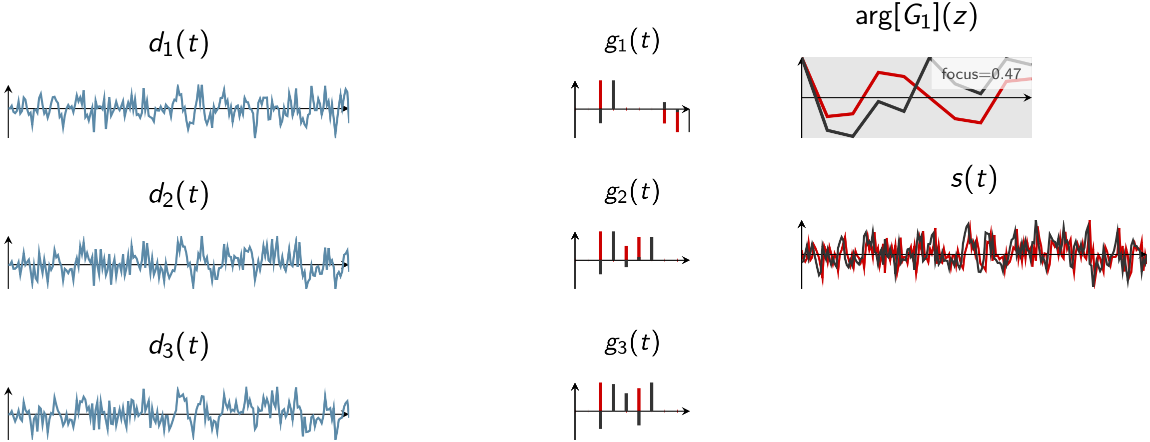
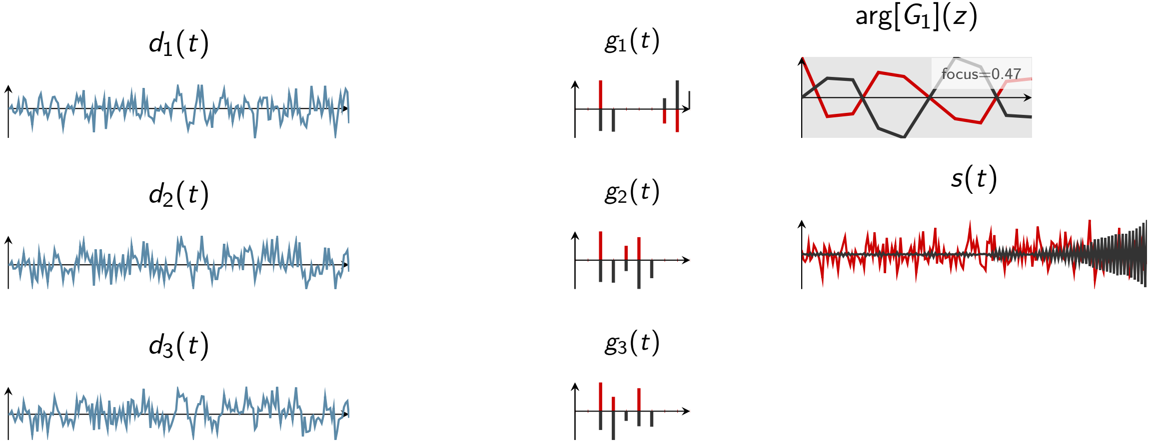
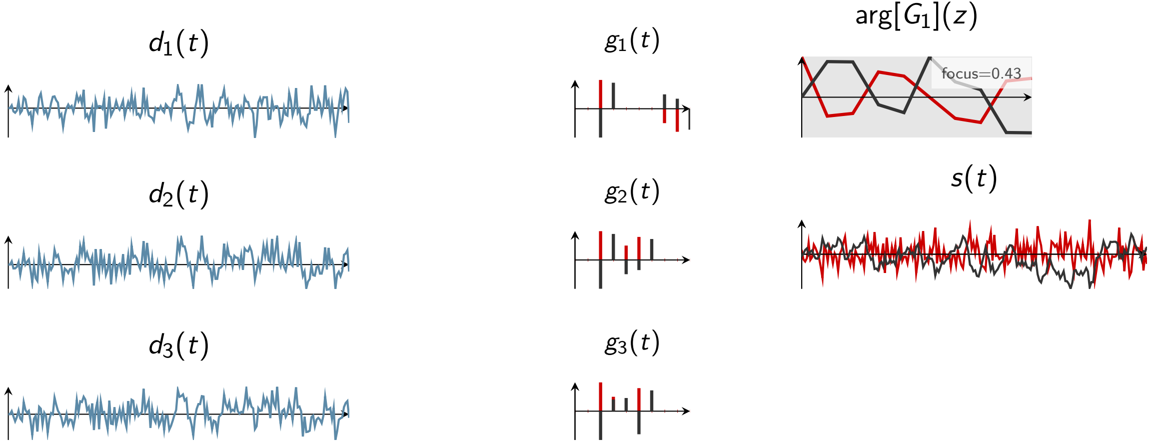
Undesired Solution 1: Phase Spectrum of the Green's Functions



Undesired Solution 2: Phase Spectrum of the Green's Functions



Undesired Solution 3: Phase Spectrum of the Green's Functions



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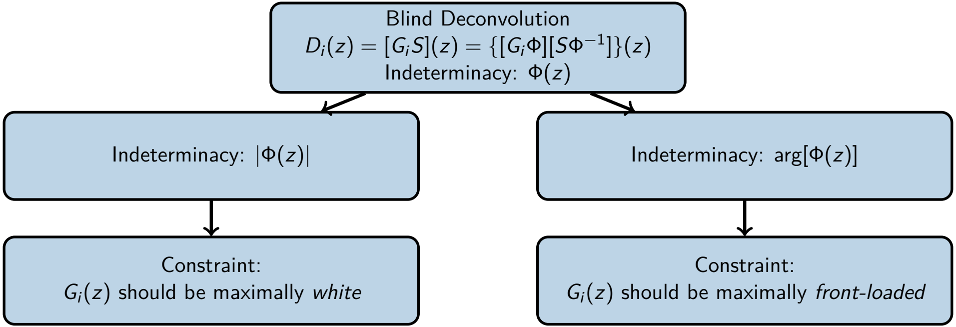
$$(\widehat{s}, \widehat{g}_i) = \arg\min_{s, g_i} \quad Q$$

Summary: Maximally Front-loaded



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Next Section

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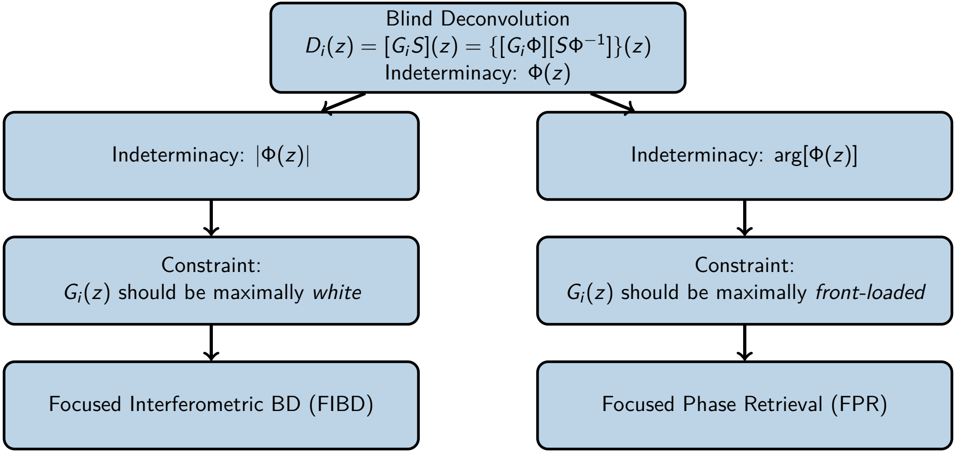
Summary: Focused Blind Deconvolution (FBD)



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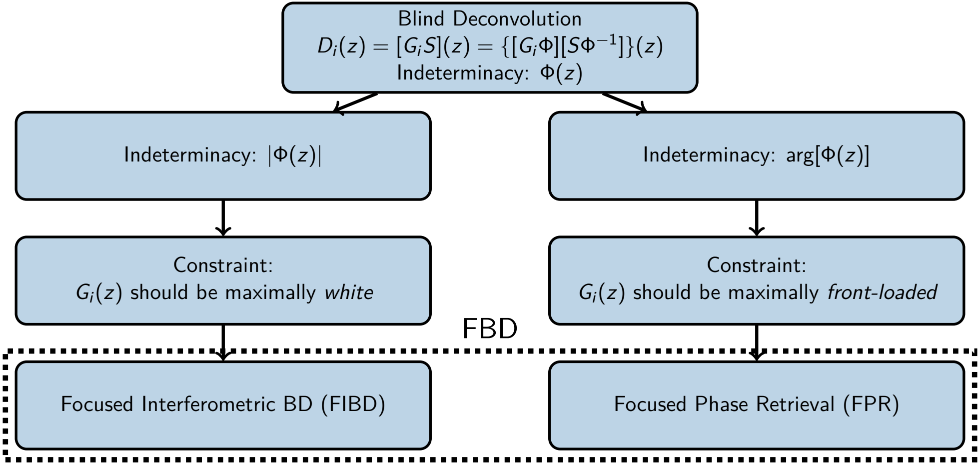
Summary: Focused Blind Deconvolution (FBD)



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Definition

$$Q(s, g_i) = \sum_i \sum_t \{d_i(t) - [s * g_i](t)\}^2 + \alpha \underbrace{\sum_i \sum_t \frac{|t|}{\tau} g_{ii}^2(t)}_{\text{Maximally White}} + \beta \underbrace{\sum_i \sum_t \frac{|t|}{\tau} g_i^2(t)}_{\text{Maximally Front-loaded}}$$

$$(\hat{s}, \hat{g}_i) = \underset{s, g_i}{\arg\min} \quad Q$$

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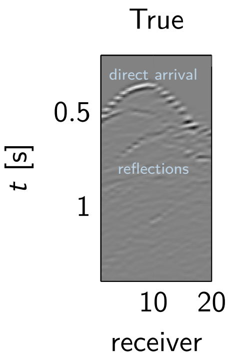
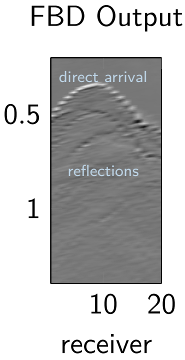
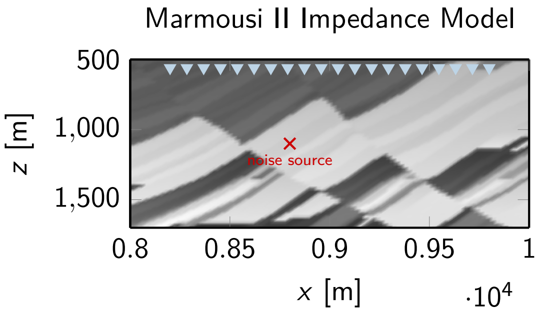
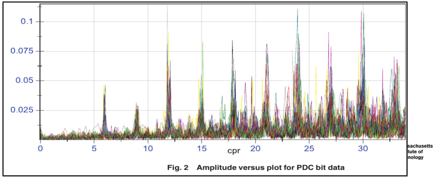
Yes, Ask FBD for Help!







Drill Noise from Gradl et. al., 2012



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Conclusions

Focused Interferometric Blind Deconvolution

- Resolves the indeterminacy due to $|\Phi(z)|$, by choosing the Green's functions that are maximally white.

Focused Phase Retrieval

- Resolves the indeterminacy due to $\arg[\Phi(z)]$, by choosing the Green's functions that are maximally front-loaded.

Focused Blind Deconvolution

- Resolves the indeterminacy due to $\Phi(z)$, such that the Green's functions are *coprime*, i.e., they don't share common roots.

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Thank You! Any Questions?

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