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Novel Approach For 1D Resistivity Inversion Using a Systematically-determined Optimum Number Of Layers

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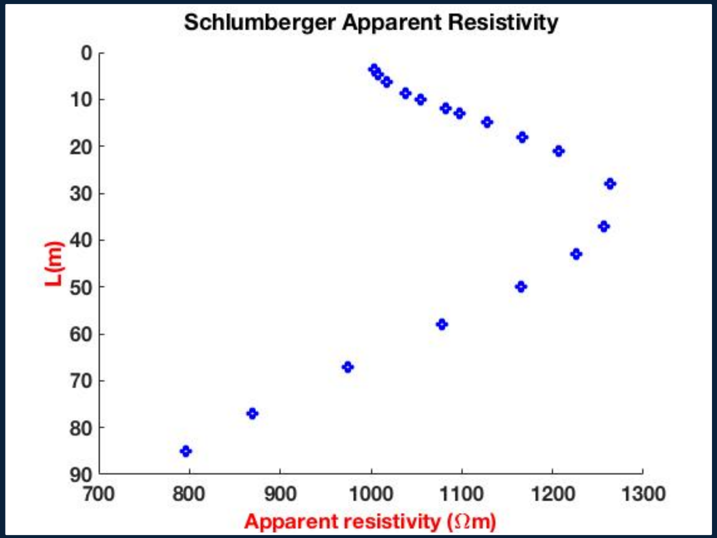
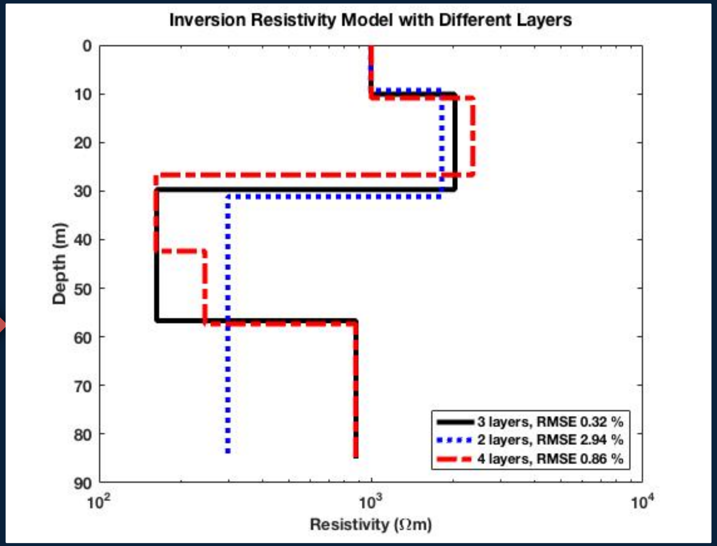
Motivation

- Inverting the apparent resistivity to obtain a "true" resistivity model entails *a priori* selection of the number of layers. Consequently, one can obtain different models for the *same* input apparent resistivity.
- ***Therefor, we need a solution to select the optimum number of layers a priori***



(One) Apparent Resistivity

Inversion

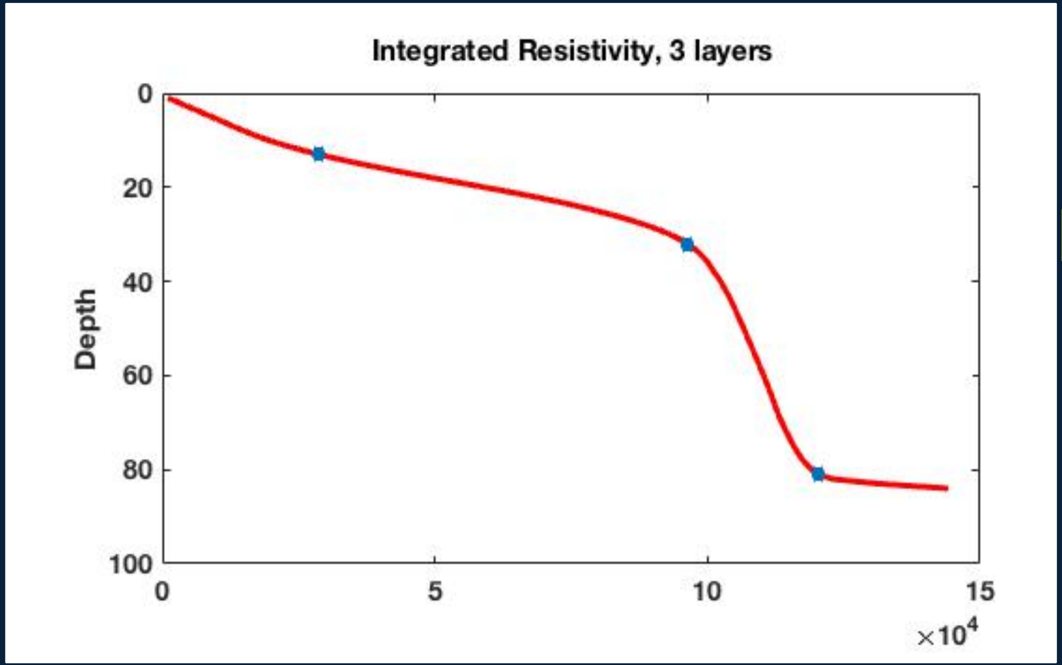


Inverted Model(s)

Proposed Solution

- A method to systematically determine the number of layers used in VES inversion.
- Use self-consistent, and stable in terms of convergence inversion algorithm.

Integrated Resistivity Model



3 Layers and a half space

Outline

- Background
- Methodology
- Synthetic Results
- Field Results
- Conclusion

Background

- Vertical electrical sounding (VES) is a geophysical method to determine the structure of the subsurface.

- *Schlumberger*, *Wenner*, Dipole-dipole Configurations.

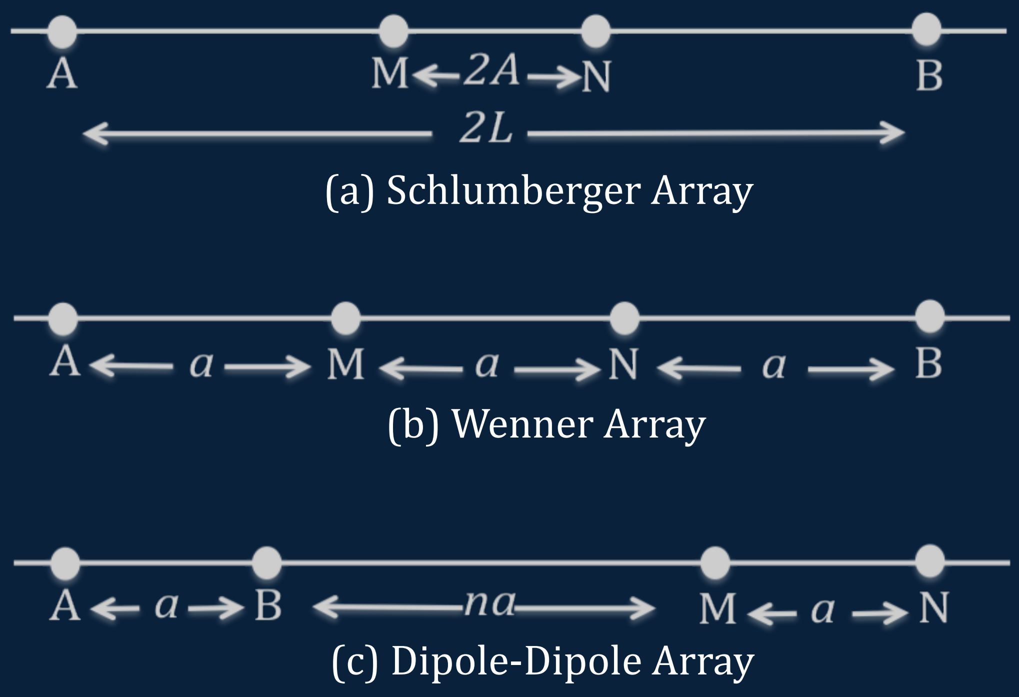


Illustration of VES configurations.

Vertical Electrical Sounding

The data collected by the electrodes is voltage. The apparent resistivity, (ρa) , is an Ohm's-law ratio of measured voltage V to applied current l, multiplied by a geometric constant k which depends on the electrode array:

$$\rho_a = k\,\frac{V}{I} \quad (\Omega{\cdot}m).$$

Methodology

Forward Model

Two Steps Approach

Inversion

Forward Model

Electric potential at a distance r from a point source of current I on the surface of a horizontally stratified earth is given as :

$$V(r) = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + 2\int_0^{\infty} B(\lambda)J_0\,(\lambda r)\;d\lambda\right].$$

Where:

r is the distance between the current source and the potential measuring station.

V(r) is the potential measured at a point on the surface at a distance ***r***.

B(λ) is the resistivity kernel function for an n- layered earth.

P_1 is the resistivity of the upper layer.

J_0 is the Bessel function of zero order.

Two-Steps Approach

Inversion Problem

(1) *High* number of **fixed** thickness layers and variable resistivity inversion

↓

Select optimum number of layers

↓

(2) *Optimum* number of **variable** thickness layers and variable resistivity inversion

↓

Final Product:
Each layer thickness and true resistivity

Inversion

To solve for:

$$\mathcal{A}(X) = b\,.$$

The inversion scheme used is a damped non-linear least square (NLLS).

$$\delta X = (\mathcal{A}^T\mathcal{A} + \propto I)^{-1}\mathcal{A}^T\delta b\,.$$

L2-norm of misfit to quantify the difference and minimize it.

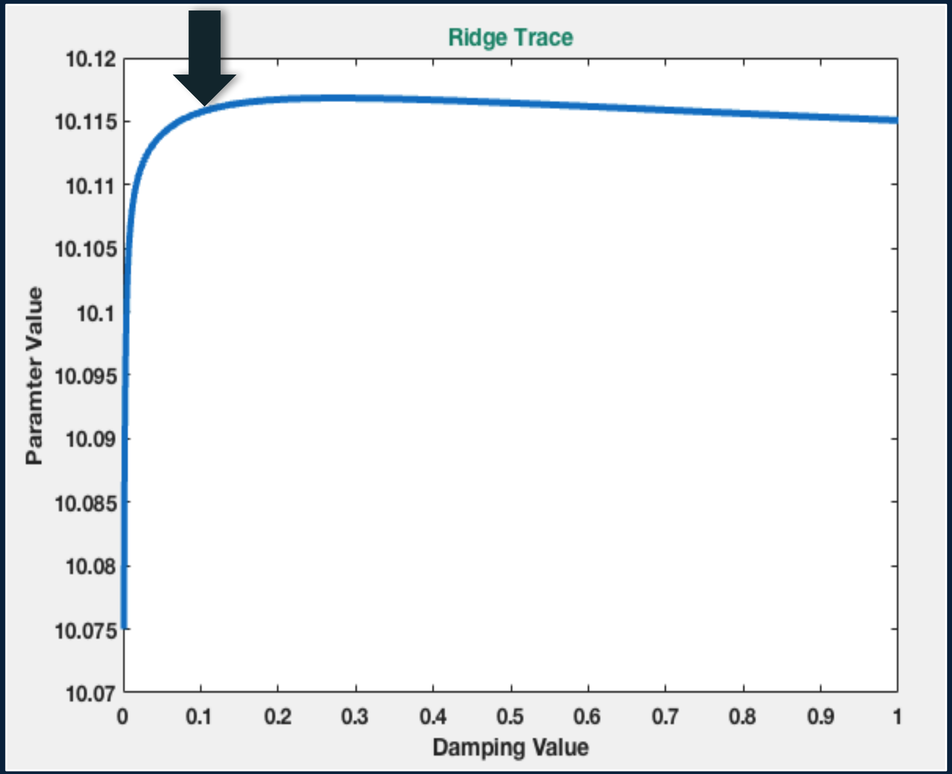
Ridge Trace Regression

Ridge trace determines which damping value to use for each parameter individually.

$$\tau = \left| {}^{\delta X_i}/_{\delta \propto_i} \right| .$$

$\propto$ is damping parameter

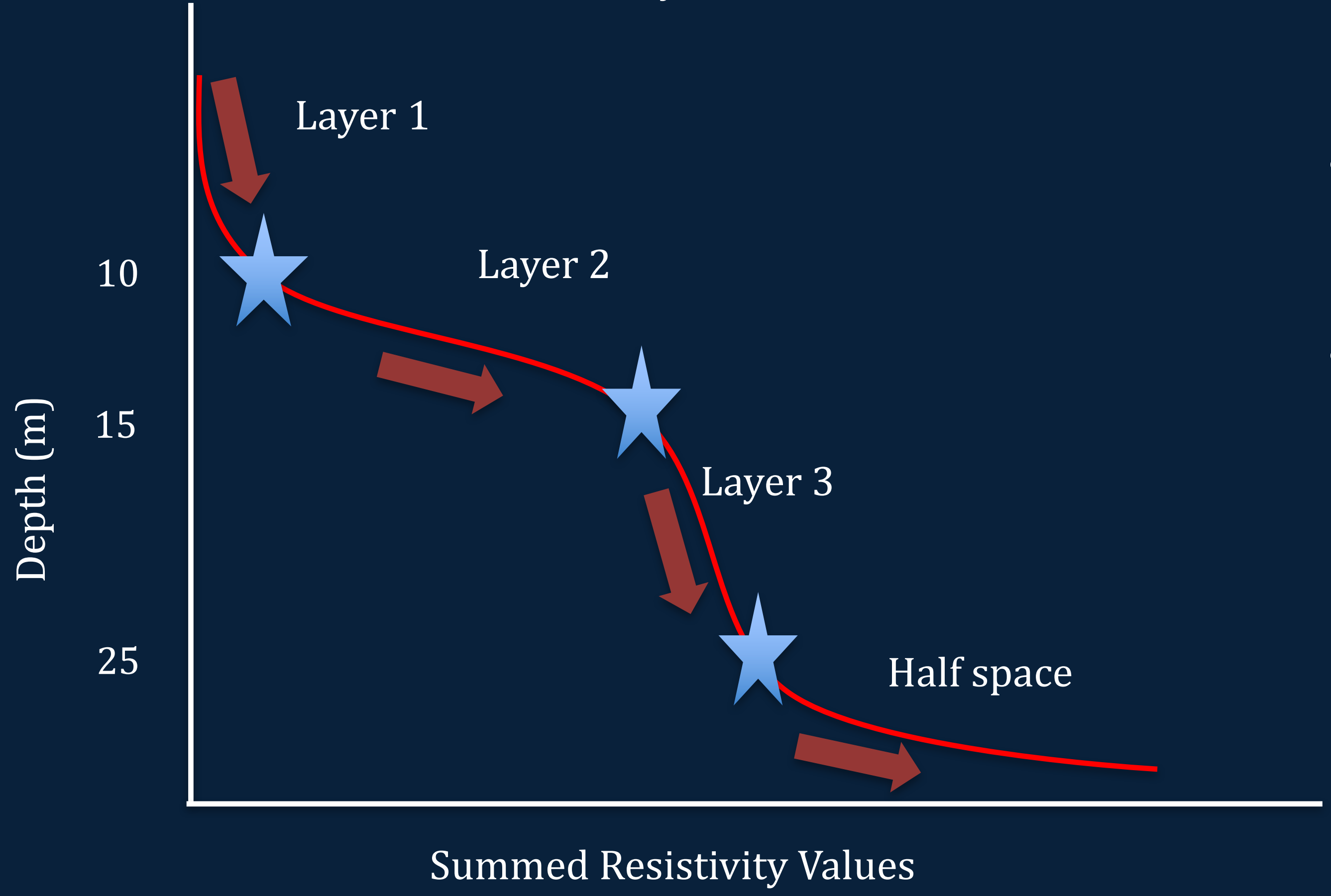
Minimum $\propto$ for which $\tau < \tau_{threshold}$.



Ridge Trace

Number of Layers

Graphic illustration of integrated resistivity curve of the fixed-layer resistivity model.



- The slope at each point in the curve is calculated *(f'(x))*.

- The point of changing slope will be interpreted for a layer boundary that represents a new layer in the model with a different true resistivity value .

Synthetic Examples

Case 1 Schlumberger Array

Case 2 Schlumberger Array

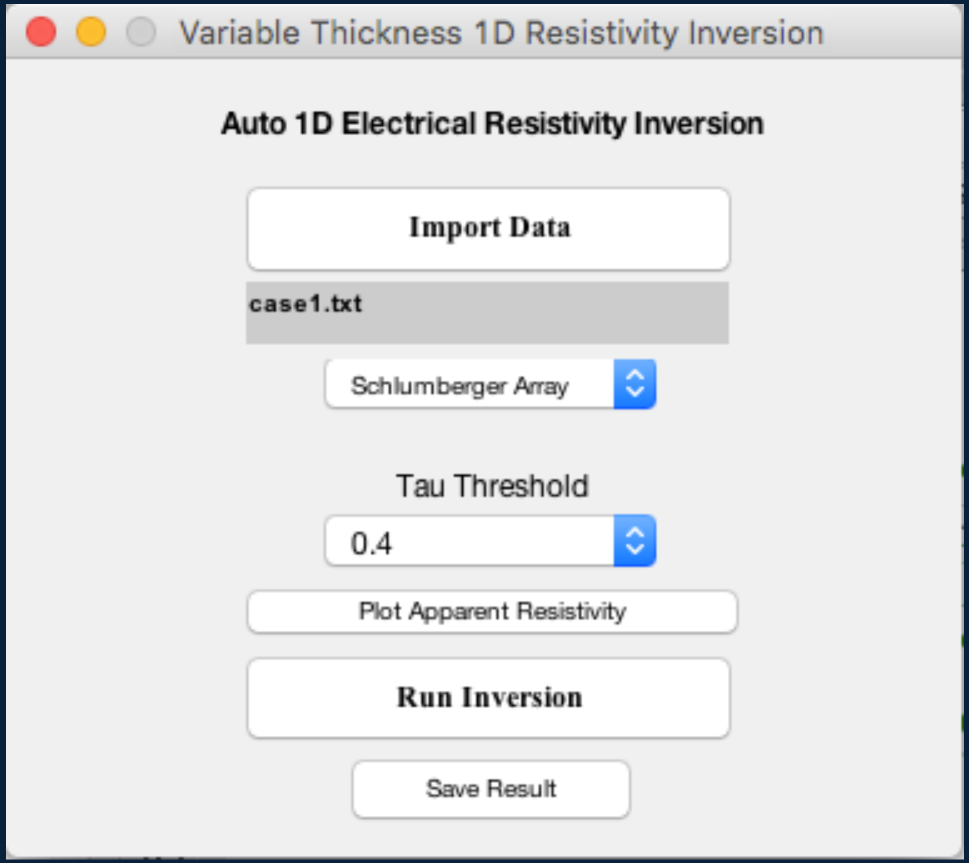
Case 1 Schlumberger Array

Forward Model

Type	Input
ρ [Ω·m]	100, 150, 200
Thickness [m]	5,7, ∞
Number of measurements (expand electrode separations logarithmically)	9

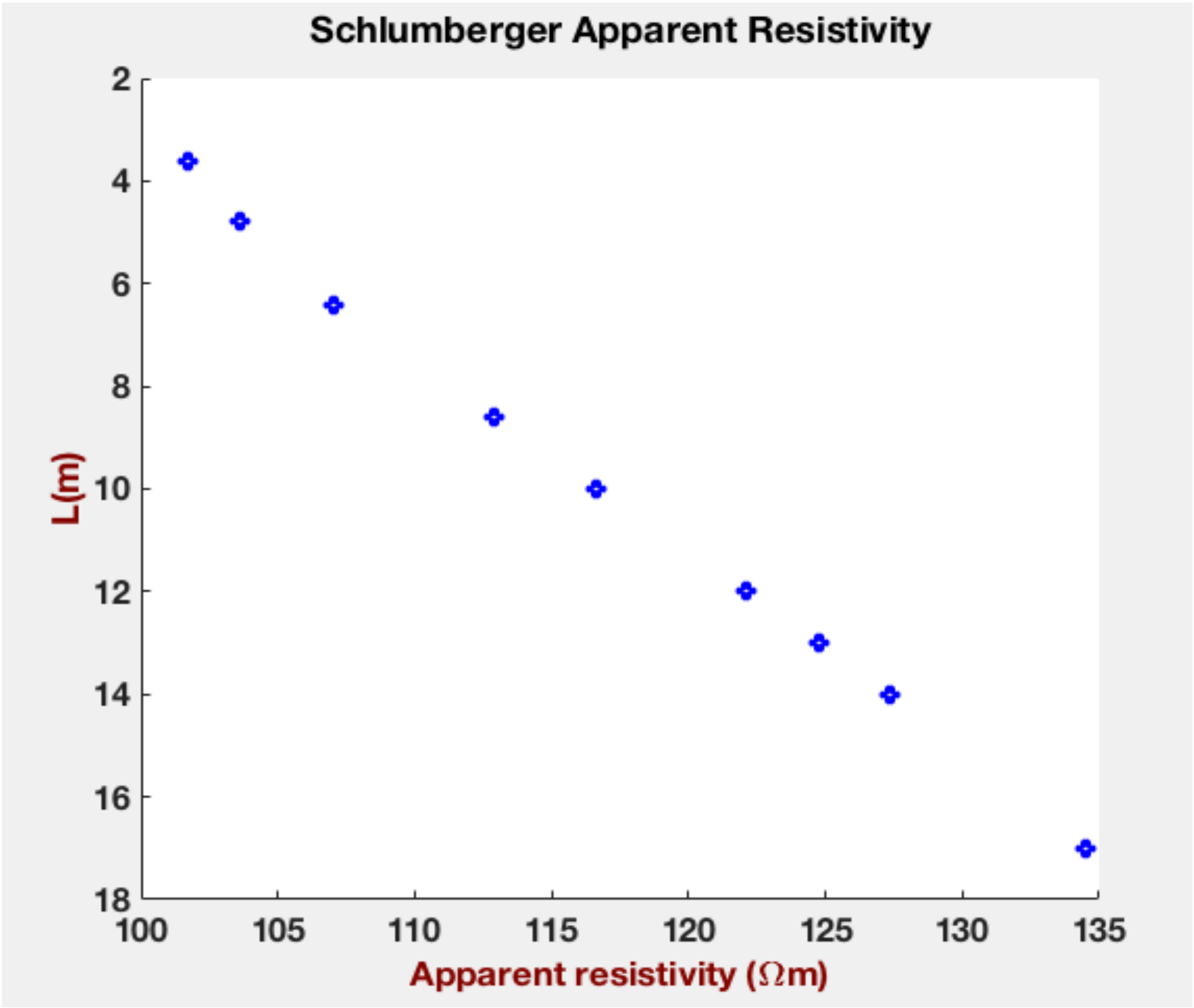
Software Interface

The user has the option to select between two different acquisition configurations.



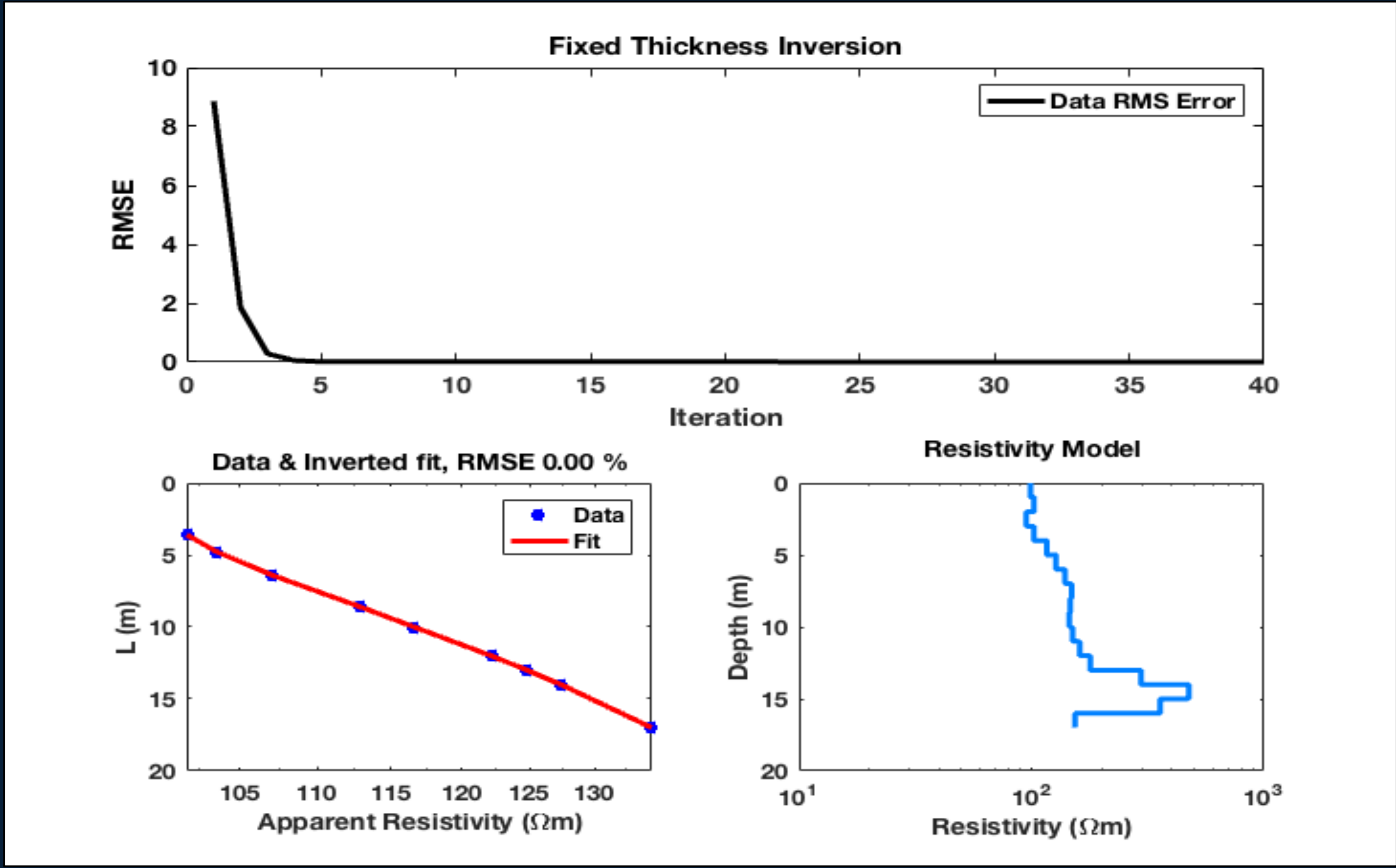
Schlumberger Apparent Resistivity

Plot of the apparent resistivity for *case 1*.



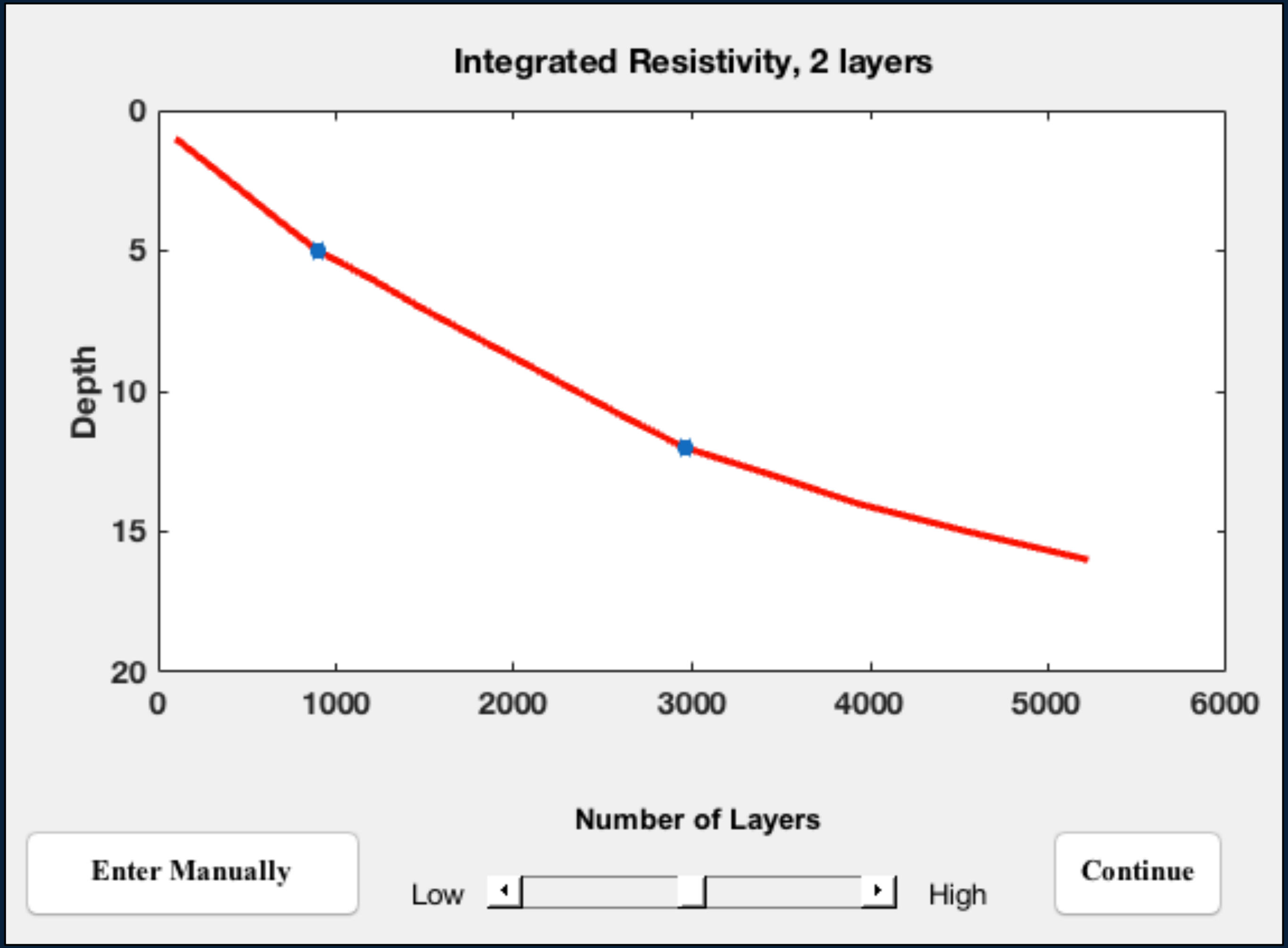
Fixed-Thickness Inversion

Fixed-thickness inversion for case 1. Top shows RMSE. Bottom left is data and inverted fit with RMSE. Bottom right is the resistivity model.



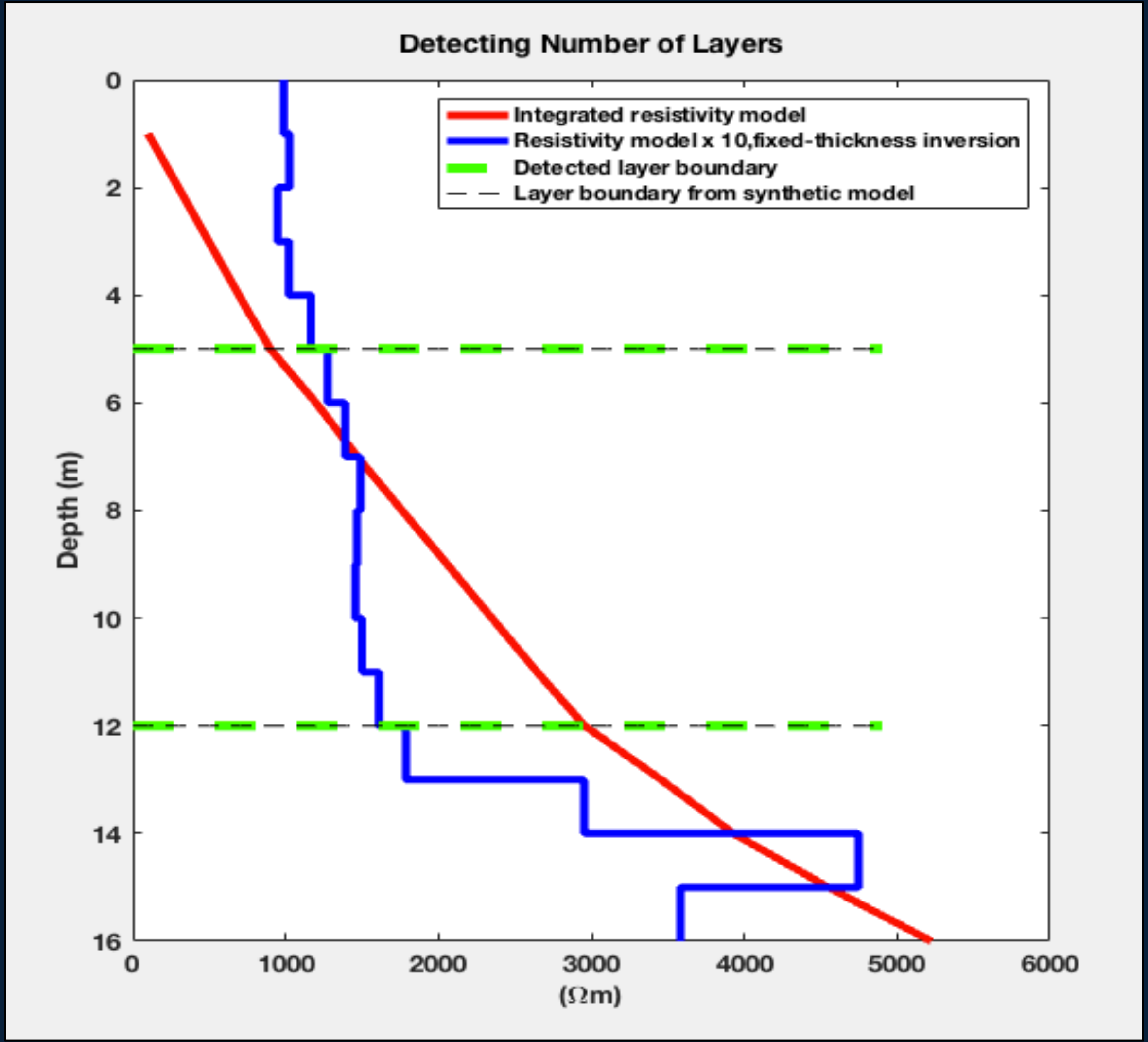
Detecting Number of Layers

Integrated resistivity model for *case 1*. The blue stars indicate the start of each new layer.



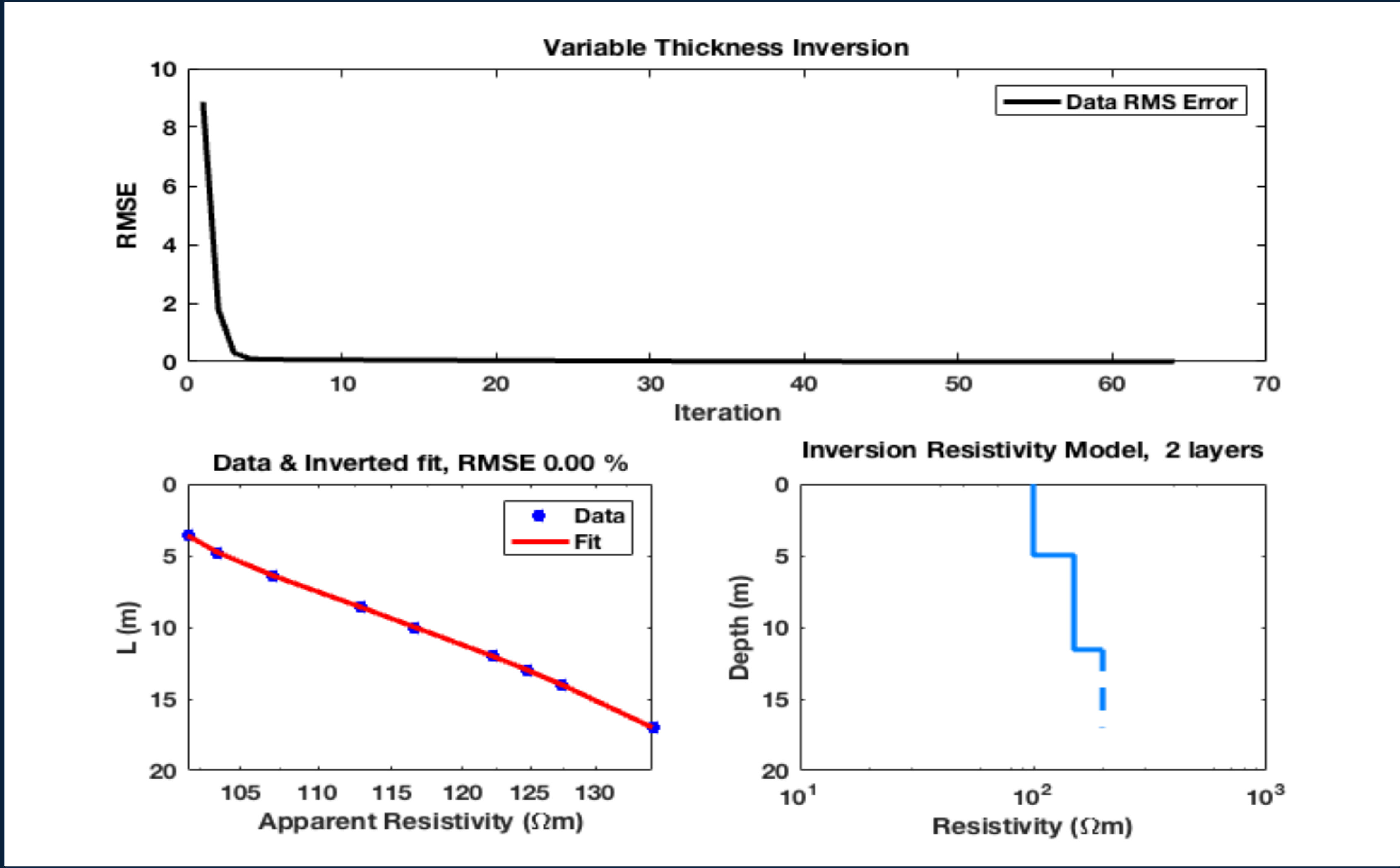
Detecting Number of Layers

Comparison between the detected layer boundaries, derived from the "integrated resistivity model" and in contrast to the "fixed-thickness resistivity model," to the actual boundaries from the synthetic model for the first case.



Variable-Thickness Inversion

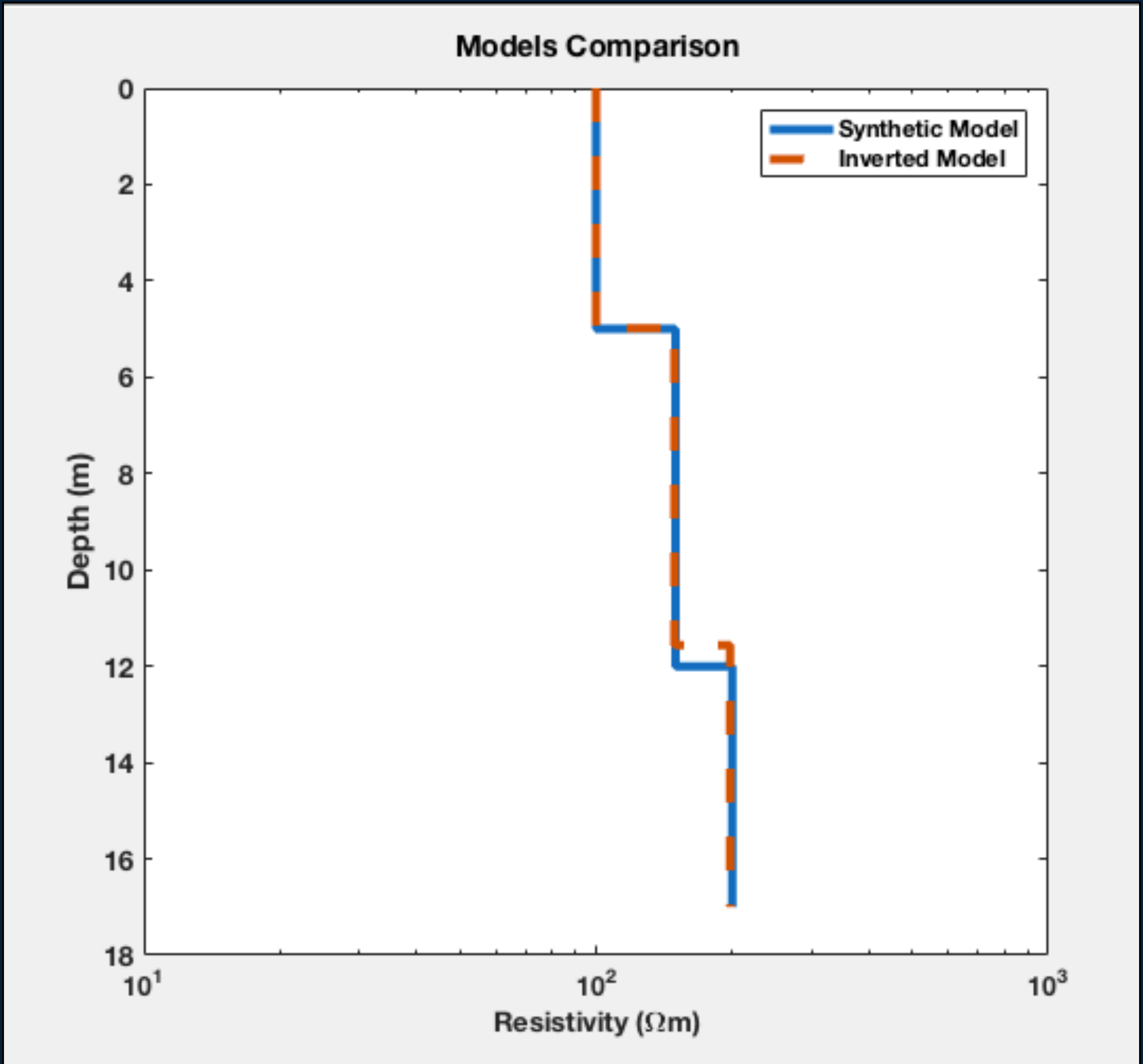
$\tau_{threshold} = 0.4$
Random Noise 0%
RMSE = 0.00%



Final Inverted Model

Comparison between synthetic (data) and inverted model.

Type	Input	Output
ρ [Ω·m]	100, 150, 200	100, 150, 197
Thickness [m]	5, 7, ∞	5, 6.5, ∞
Data RMSE		0.00%

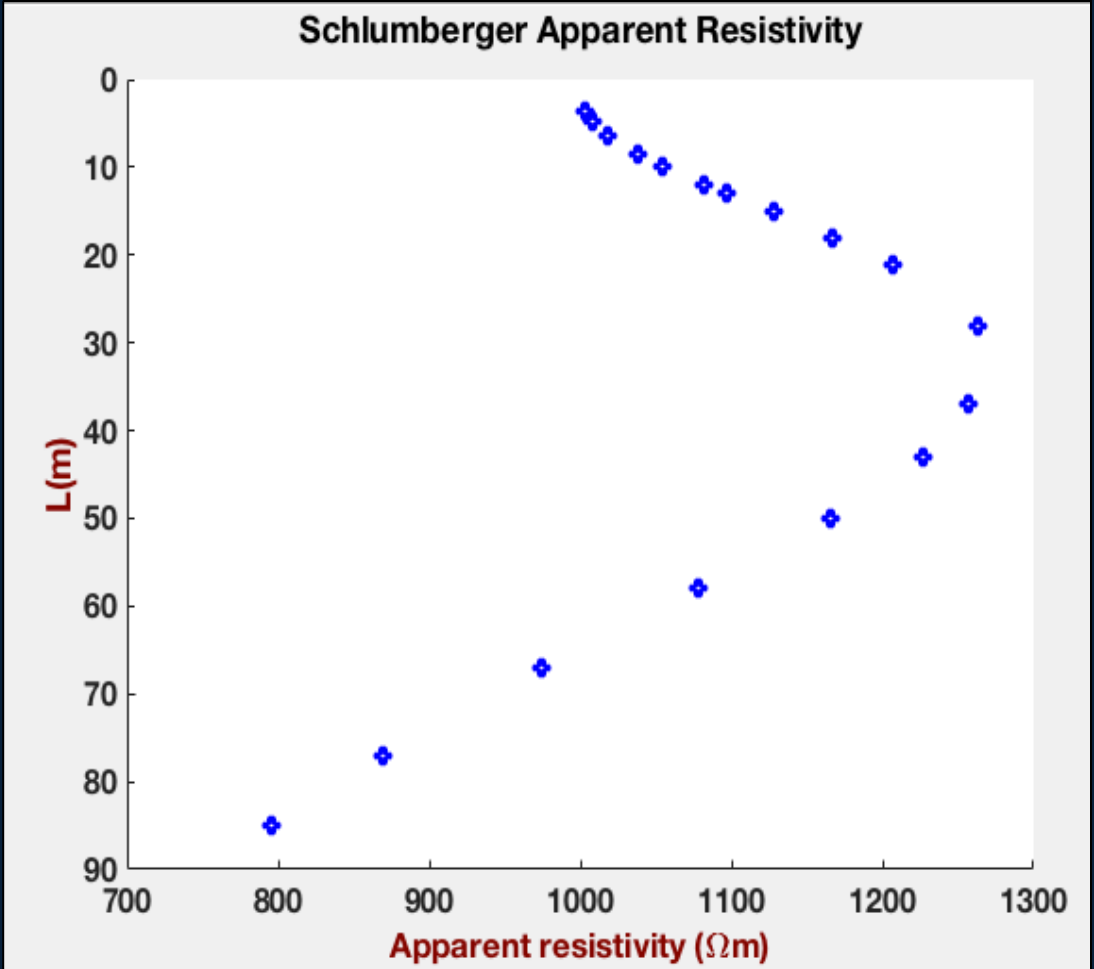


Case 2 Schlumberger Array

Forward Model

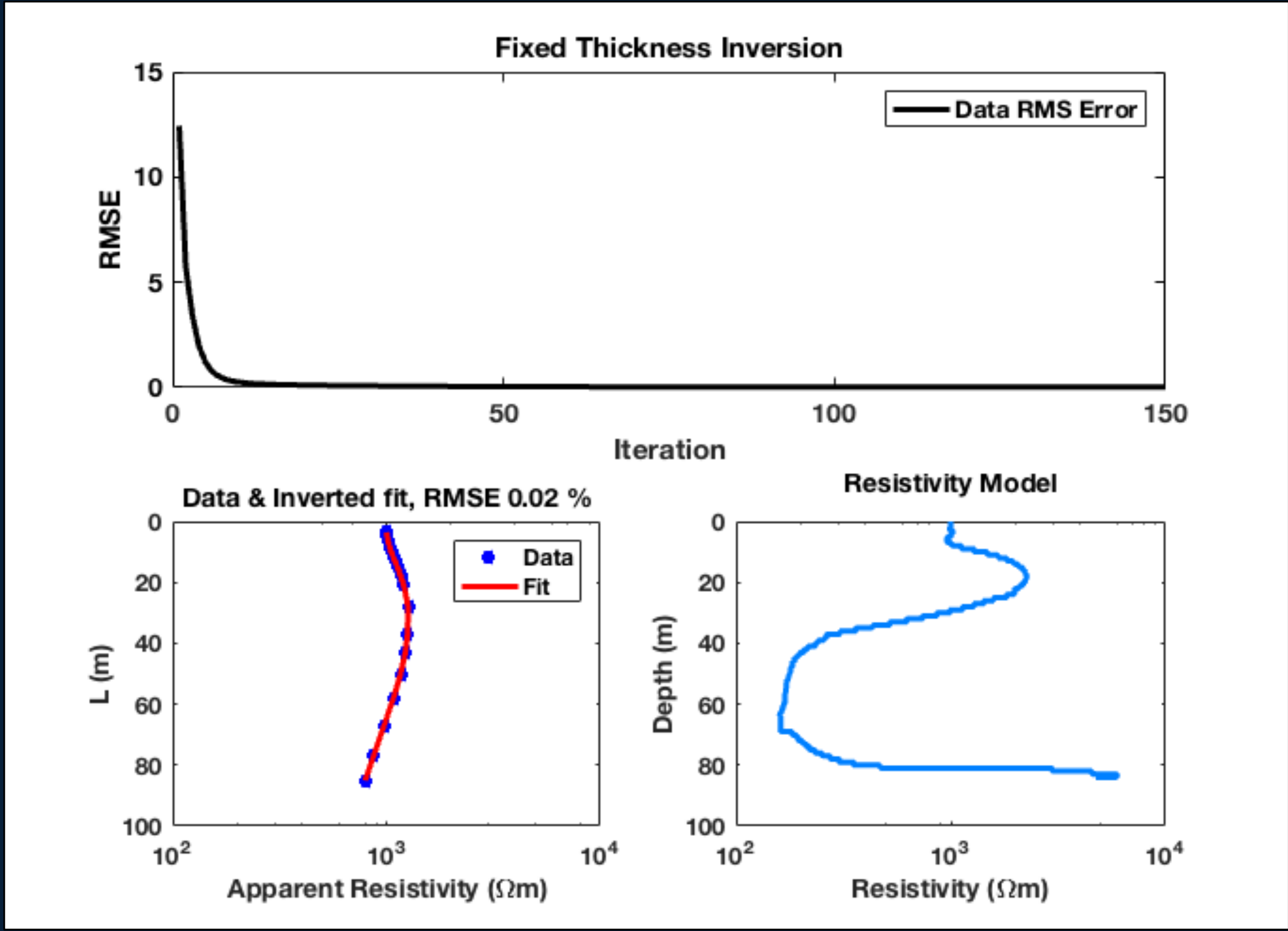
Type	Input
ρ [Ω·m]	1000, 2000, 200, 500
Thickness [m]	10, 20, 30, ∞
Number of measurements	18

Plot of the apparent resistivity for *case 2*.



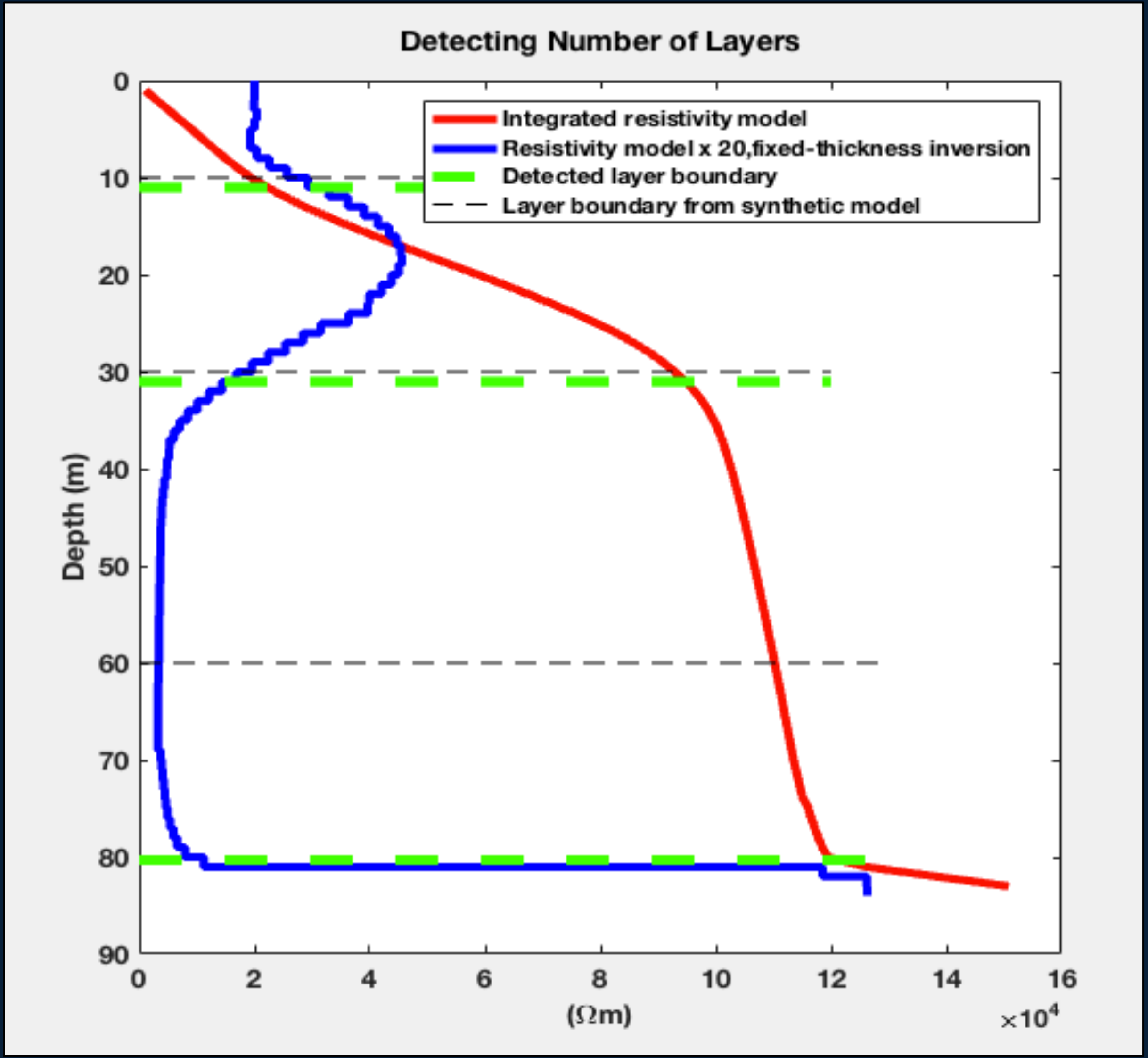
Fixed-Thickness Inversion

Fixed-thickness inversion for case 2.



Detecting Number of Layers

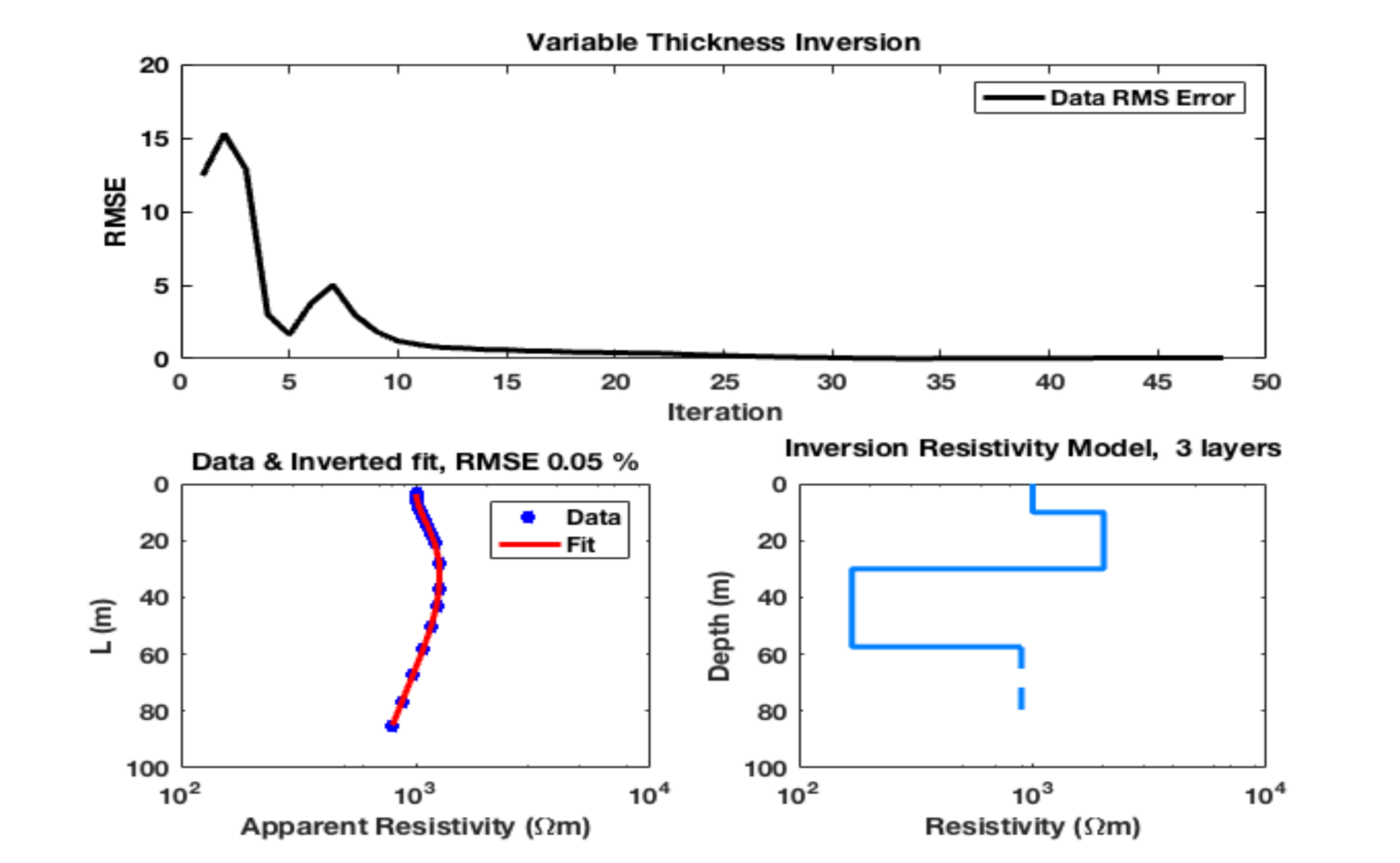
Comparison between the detected layer boundaries, derived from the "integrated resistivity model" and in contrast to the "fixed-thickness resistivity model," to the actual boundaries from the synthetic model for the second case.



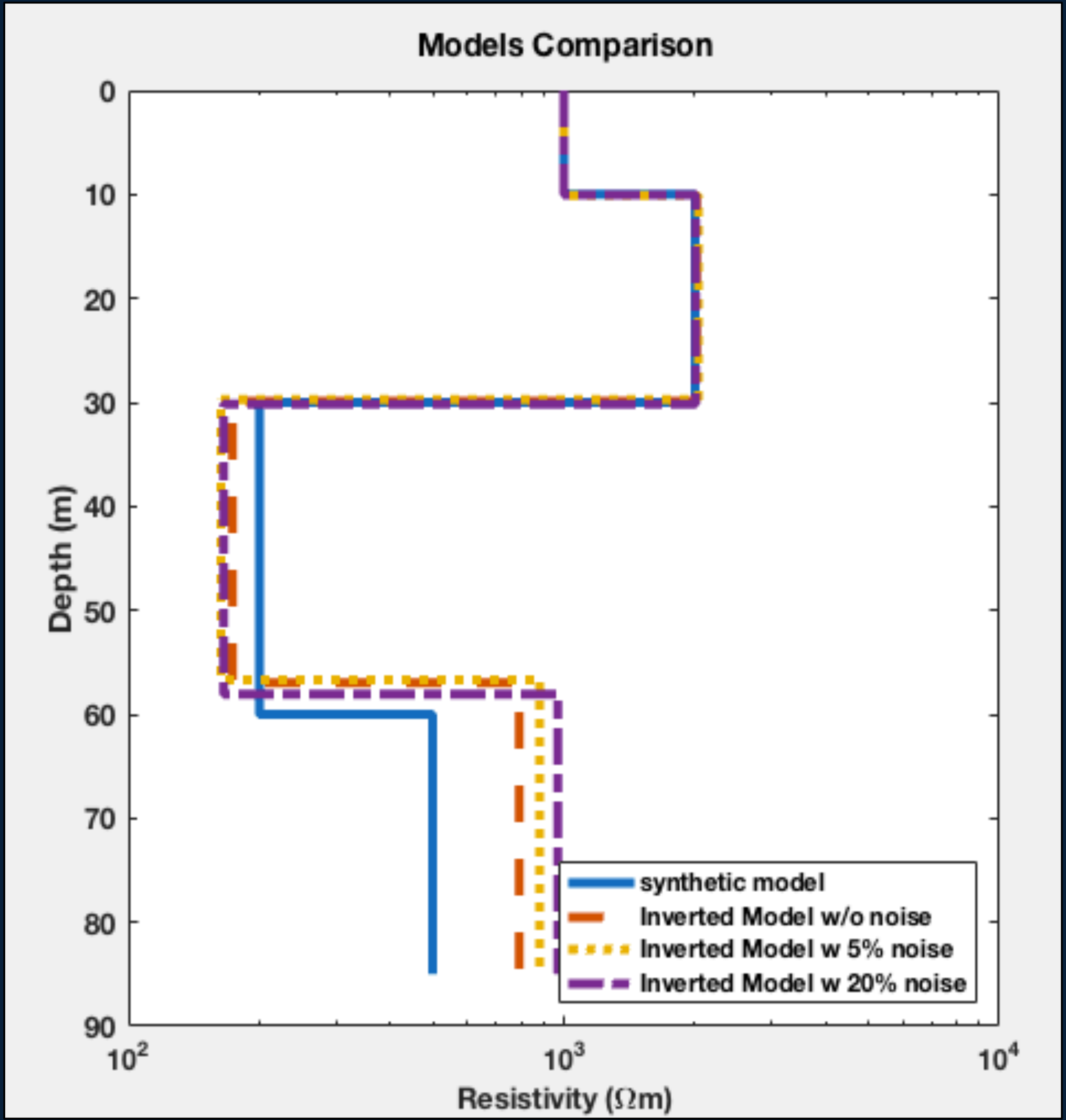
Variable-Thickness Inversion

The result of the variable-thickness inversion, *Case 2.*

$\tau_{threshold} = 0.4$
Random Noise 0%
RMSE = 0.05%



Final Inverted Model



Comparison between synthetic (data) and inverted model with:

- Zero noise.
- 5% Gaussian noise.
- 20% Gaussian noise.

Inversion Result

Inversion Results: Comparison between input parameters, inverted parameters without noise and with added noise.

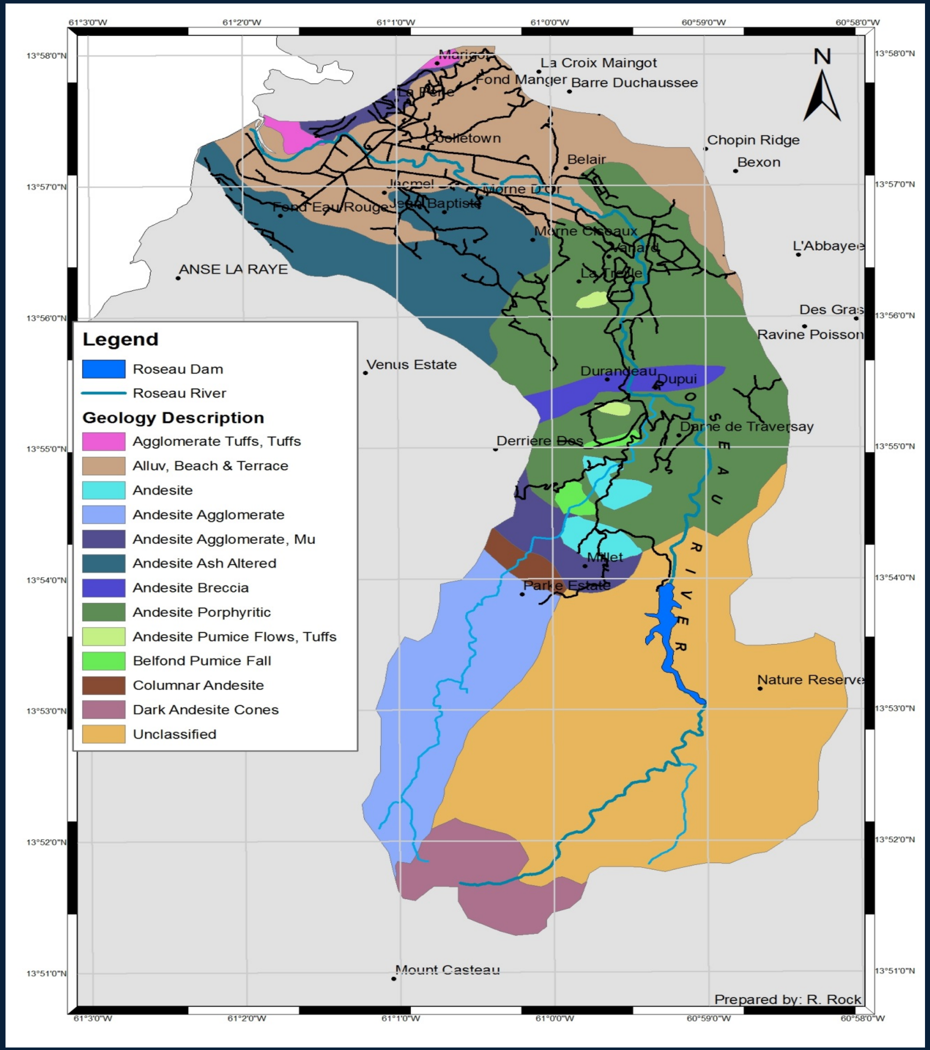
Type	Input	Output without noise	Output with 5% noise	Output with 20% noise
ρ [Ω·m]	1000, 2000, 200, 500	999, 2017, 173, 789	1003,2035,163,850	1000,2007,165,970
Thickness[m]	10, 20, 30, ∞	10, 19.8, 29.6, ∞	10, 19.7, 29.5, ∞	10, 20.2, 29.2, ∞
Data RMSE		0.05%	0.08%	0.20%

Field Data Example

Roseau Watershed of Saint Lucia

Geographical location of Roseau Watershed in Saint Lucia (King and Cole, 2008).

Geology



Site Selection:

1) High porosity and permeability.

1) A normalized chargeability value close to zero indicates a near clay-free zone.

1) High Resistivity (200 – 3000 Ω·m).

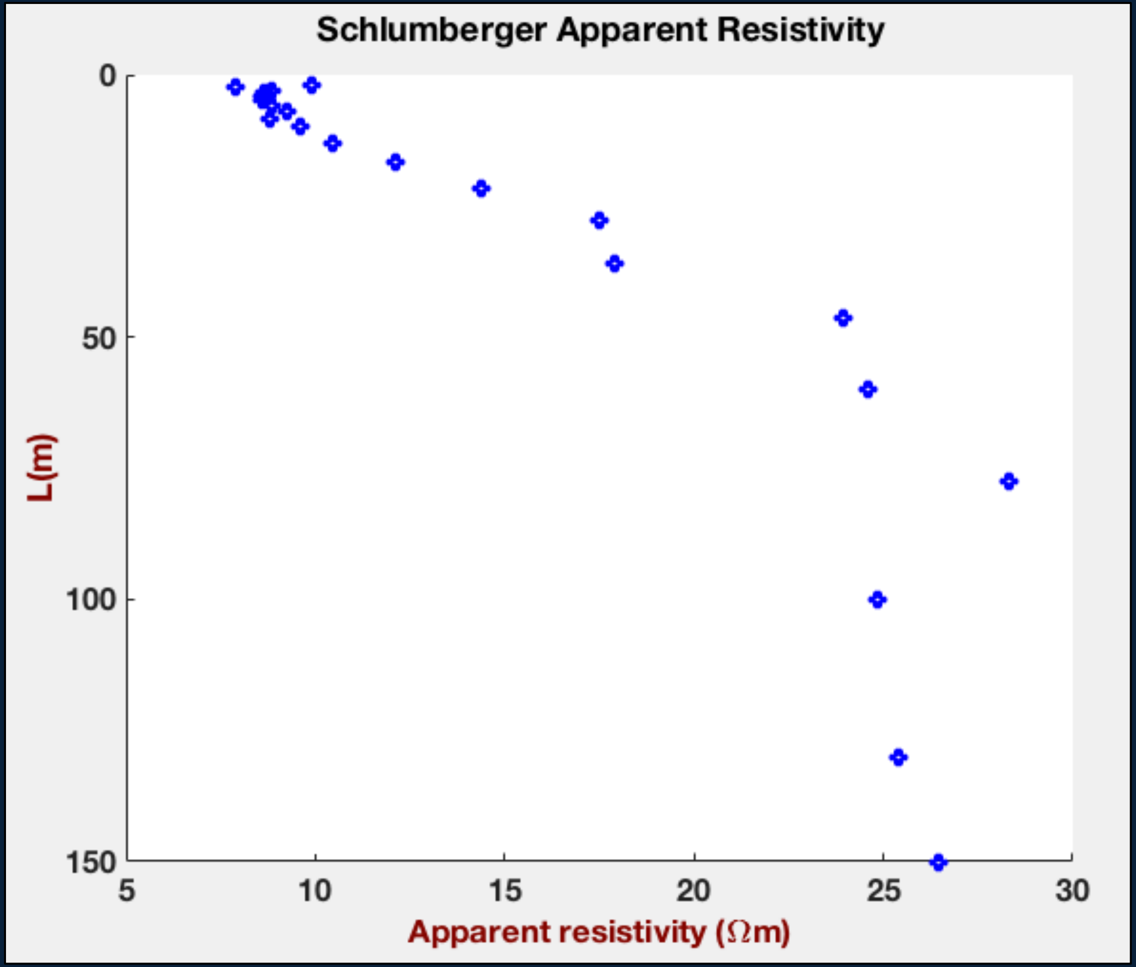
Geological map of Roseau (Vanard) region, prepared by Rebecca Rock (Morgan et al., 2013).

Roseau Watershed of Saint Lucia

Elevation map with sounding locations along Roseau 10 line.

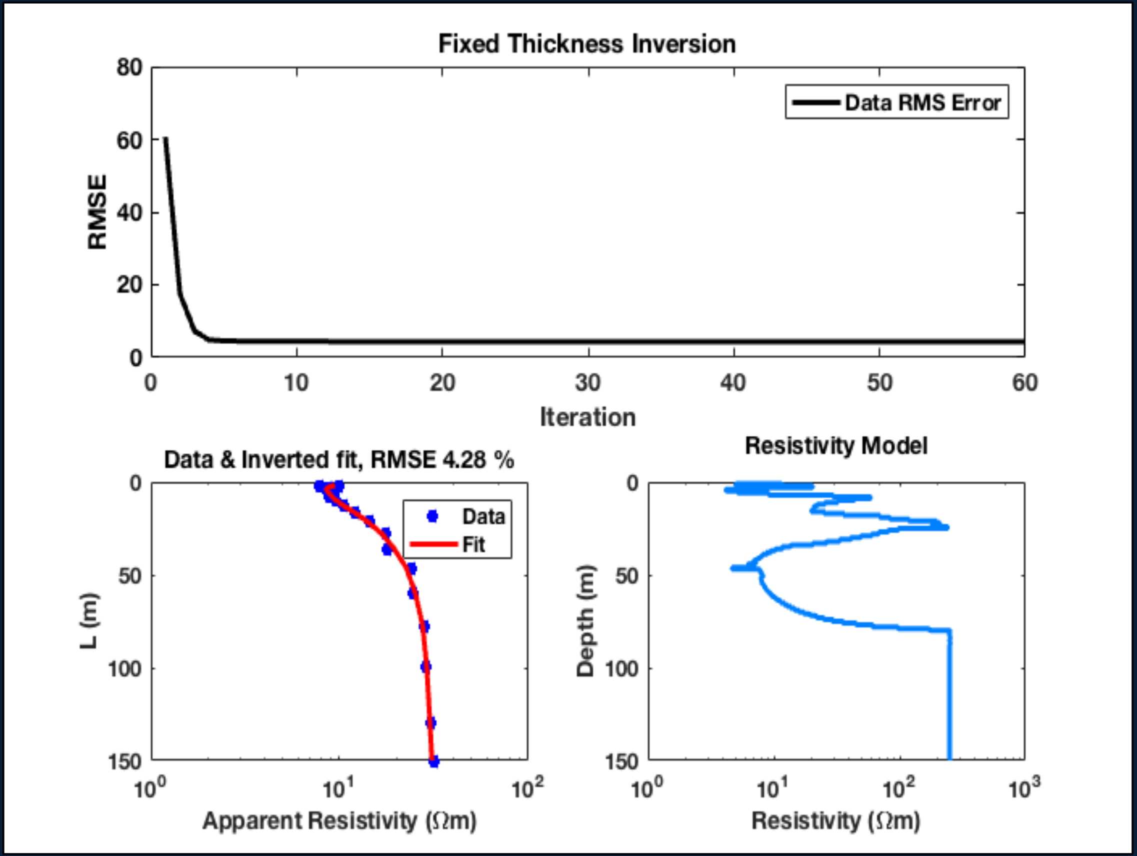
Apparent Resistivity

Plot of current spacing (AB/2) and apparent resistivity for Roseau 10-600 VES survey in Saint Lucia.

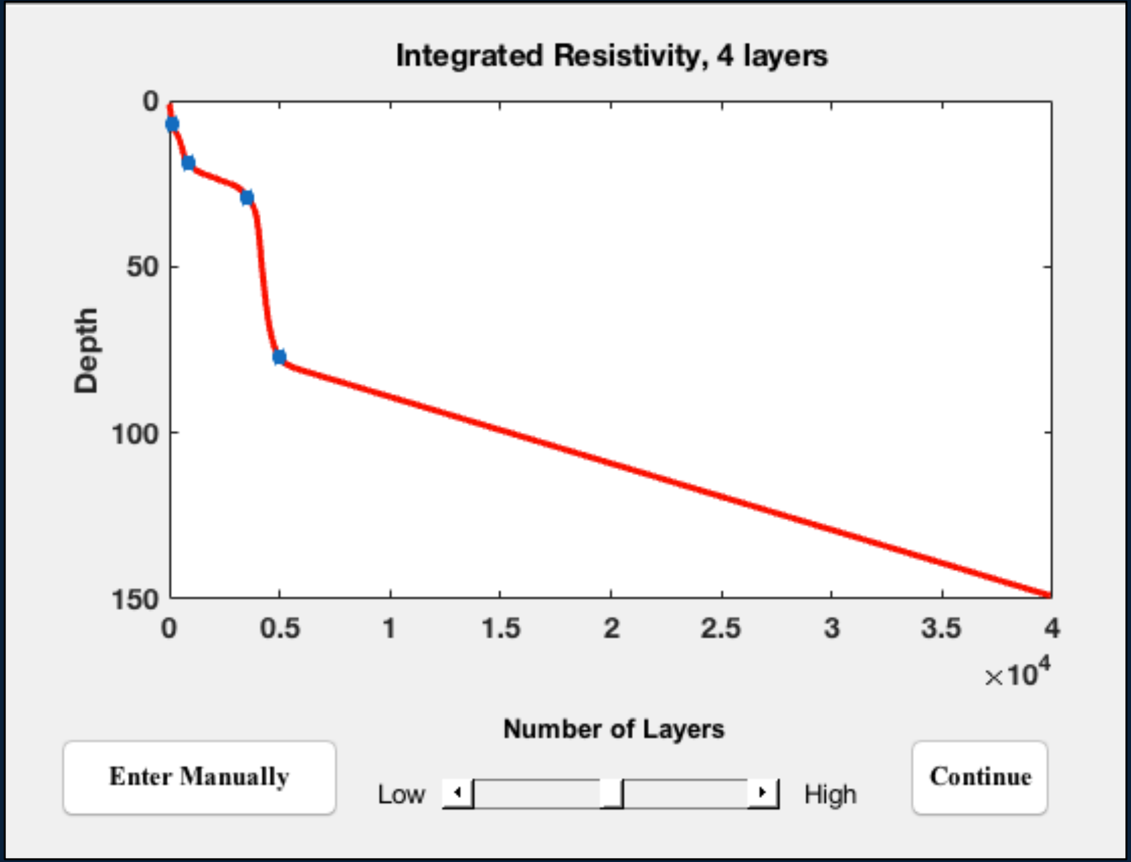


Fixed-Thickness Inversion

Fixed-thickness inversion.

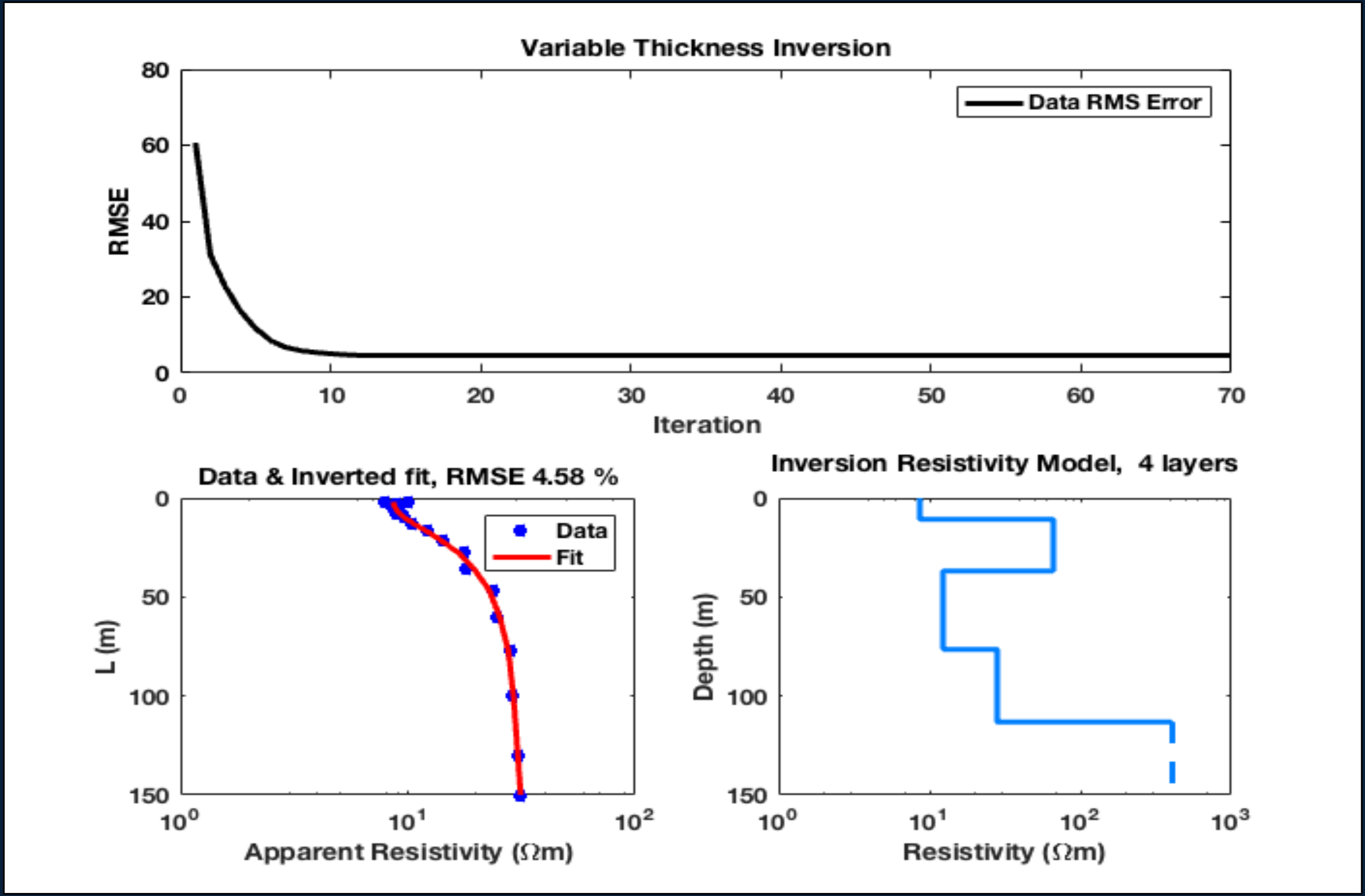


The integrated resistivity for Roseau10-600 VES.



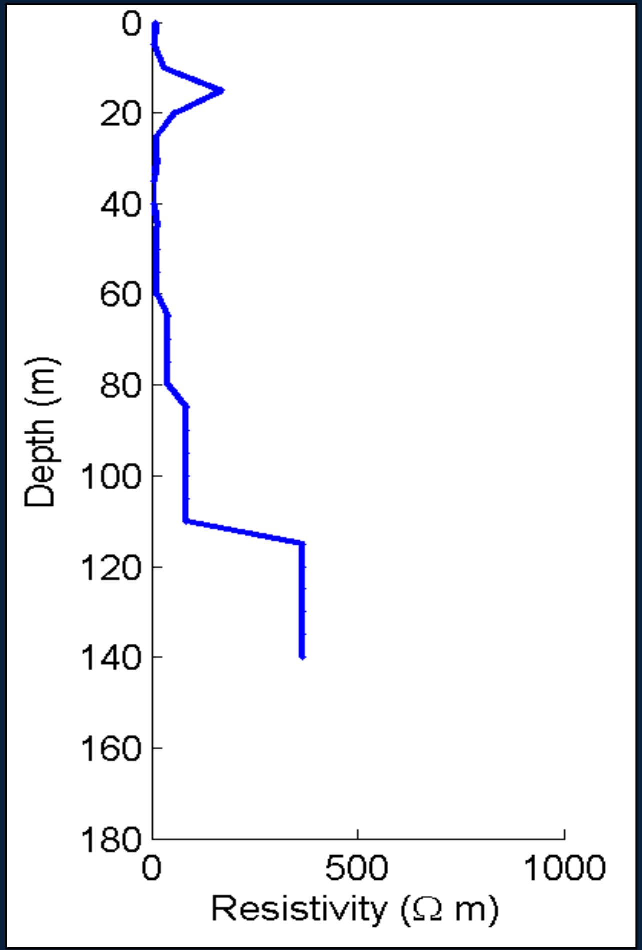
Variable-Thickness Inversion

$\tau_{threshold} = 0.4$
Field Data
RMSE = 5.26%

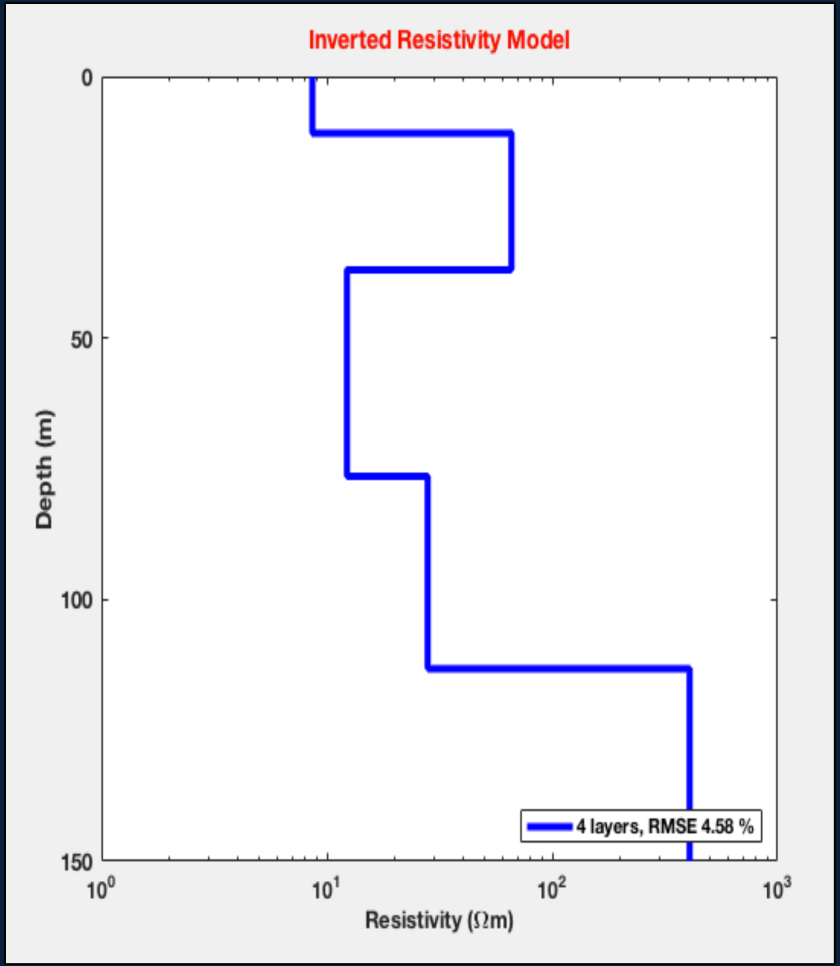


Final Inverted Model

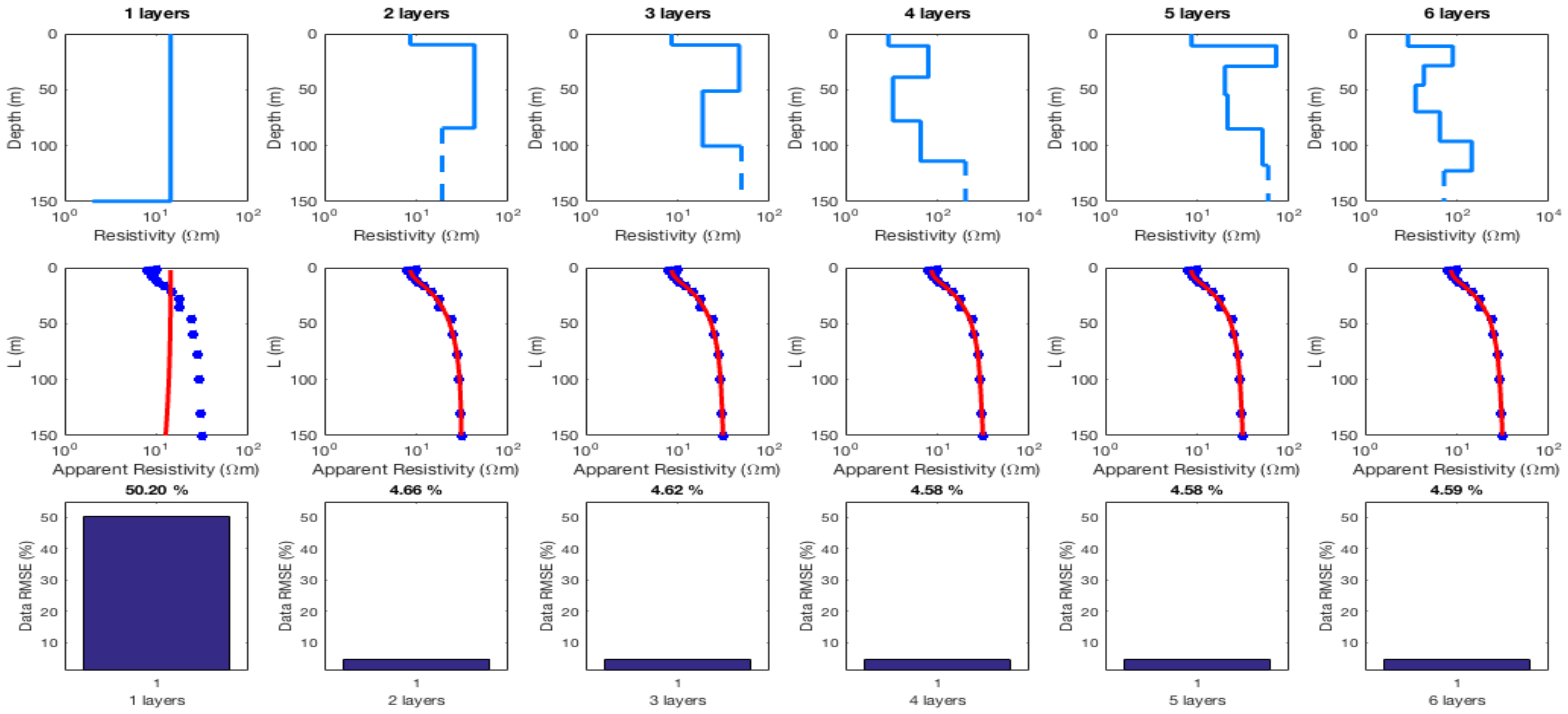
Final result after tens of trials that lasted for hours (2014).



Our Result.



Different Models



Conclusion

- VES Variable thickness inversion is the best solution to resolve the subsurface and it requires number of layers.

- We have demonstrated a systematic 2-steps approach to determine the number of layers.

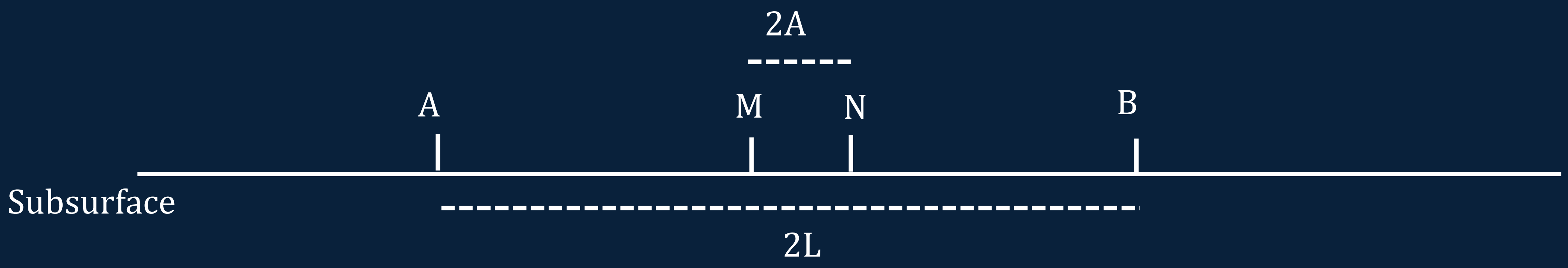
- The proposed approach is at least 100 times faster than the alternative methods.

- A similar method can be implemented on other configurations like Dipole-dipole array.

Thank you

Questions ?

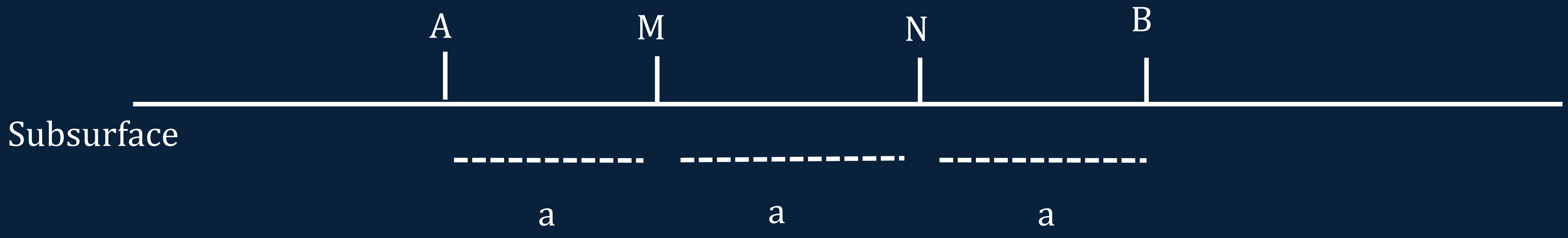
Schlumberger Apparent Resistivity



$$\rho_{as} = \pi \frac{L^2}{I}\left(-\frac{\mathrm{dV}}{\mathrm{dx}}\right).$$

M,N Potential electrodes
A,B Current electrodes
V Measured voltage difference (M & N)
I Electrical current
X Measuring distance from center of line.

Wenner Array



M,N Potential electrodes

A,B Current electrodes

V Measured voltage difference (M & N)

I Electrical current

$$\rho_a = 2\pi a \left(\frac{V}{I}\right).$$