Defmod, an earthquake simulator that adaptively switches between quasi-static and dynamic states

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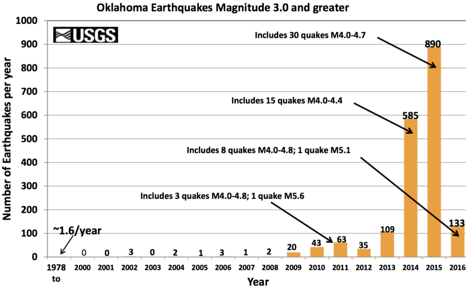
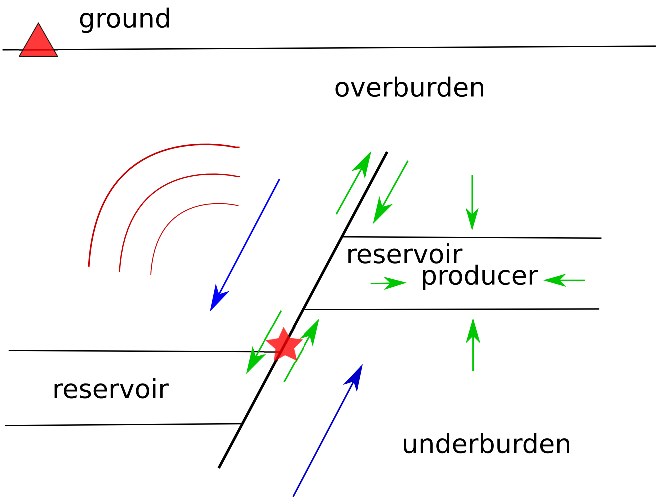
May 16, 2016

Reservoir production/injection induced seismicity

Groningen earthquakes, gas production induced.

Oklahoma earthquakes, waste water disposal induced.



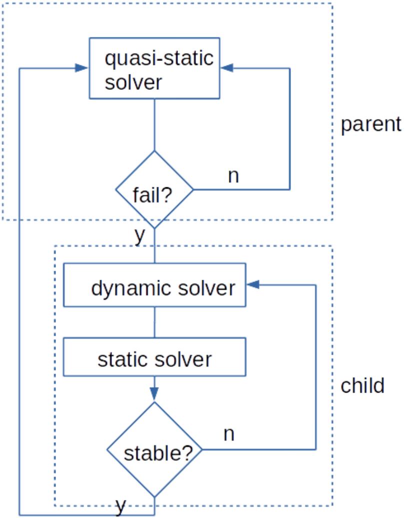

ground
overburden
reservoir
producer
reservoir
underburden

Defmod code is supposed to

- Capture poro-visco-elasticity processes due to fluid injection/production, viscoelastic flow and external loadings;
- When failure criterion is met, allow the fault to have frictional slip, accompanied by seismic radiation;
- Exchange the fault slip and stress perturbation between the quasi-static and dynamic solvers, forming a hybrid solver.
- History match the earthquake event occurrence and waveforms.

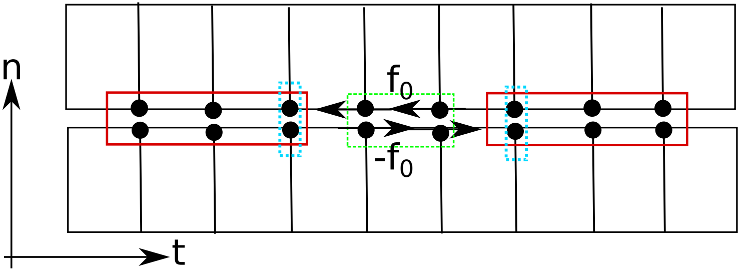
Hybrid solver flowchart

The hybrid model will return to the quasi-static (parent) loop once the dynamic run is over.



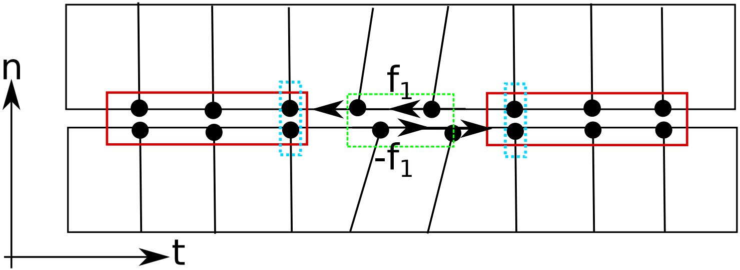
Dynamic fault slip schematic

Nucleation starts at a fault patch.



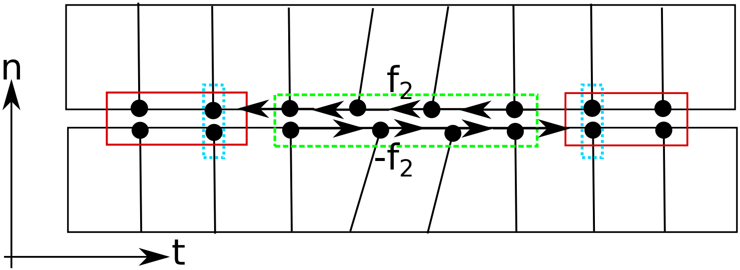
Dynamic fault slip schematic

Slip and stress concentration.



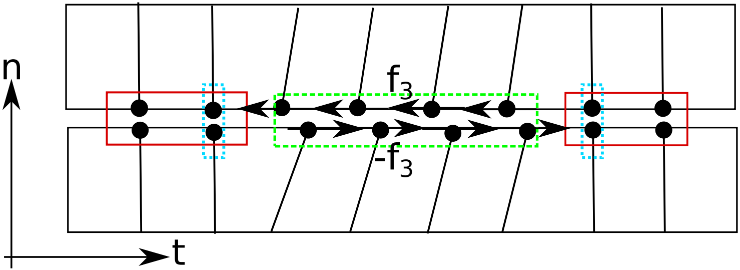
Dynamic fault slip schematic

Rupture propagation.



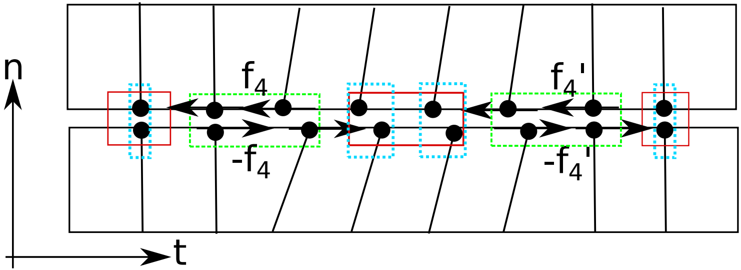
Dynamic fault slip schematic

More slip.



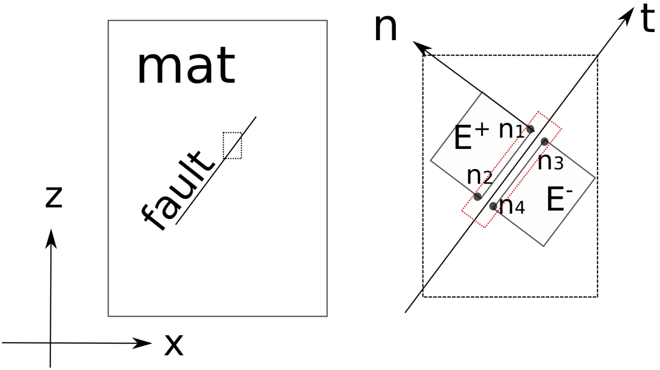
Dynamic fault slip schematic

Shear stress drop and rupture arrest.



Fault constraint

Coinciding nodes on the fault belong to different elements.



For locked and permeable fault,

$$\begin{bmatrix} n_x & n_z & 0 & -n_x & -n_z & 0 \\ t_x & t_z & 0 & -t_x & -t_z & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u_x^{(1)} \\ u_z^{(1)} \\ p^{(1)} \\ u_x^{(3)} \\ u_z^{(3)} \\ p^{(3)} \end{bmatrix} = \mathbf{0},$$

For slipping and permeable fault,

$$\begin{bmatrix} n_x & n_z & 0 & -n_x & -n_z & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u_x^{(1)} \\ u_z^{(1)} \\ p^{(1)} \\ u_x^{(3)} \\ u_z^{(3)} \\ p^{(3)} \end{bmatrix} = \mathbf{0}.$$

For locked and impermeable fault,

$$\begin{bmatrix} n_x & n_z & -n_x & -n_z \\ t_x & t_z & -t_x & -t_z \end{bmatrix} \begin{bmatrix} u_x^{(1)} \\ u_z^{(1)} \\ u_x^{(3)} \\ u_z^{(3)} \end{bmatrix} = \mathbf{0},$$

For slipping and impermeable fault,

$$\begin{bmatrix} n_x & n_z & -n_x & -n_z \end{bmatrix} \begin{bmatrix} u_x^{(1)} \\ u_z^{(1)} \\ u_x^{(3)} \\ u_z^{(3)} \end{bmatrix} = \mathbf{0},$$

A compact linear constraint equation can be written as

$$\mathbf{GU} = \mathbf{l},$$

where, **G** is constraint matrix, **l** is constraint function, nonzero for prescribed slip (pressure jump).

$$\text{governing equation} \begin{cases} \mathbf{KU} = \mathbf{F} & \text{(quasi-)static,} \\ \mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = \mathbf{f} & \text{dynamic,} \end{cases}$$

where,

- **K**, **M** and **C** are stiffness, mass and damping matrices;
- **U** is the solutions vector, e.g. $\mathbf{U} = \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix}$ displacement and pressure for poroelastic problem;
- **F** is exterior load, e.g. $\mathbf{F} = \begin{bmatrix} \mathbf{f} \\ \mathbf{q} \end{bmatrix}$, force and flow rate.

Write the time dependent quasi-static equation in the compact form:

$$\mathbf{K}\mathbf{U}_n = \mathbf{F}_n.$$

The constrained equation can be written as

$$\begin{bmatrix} \mathbf{K} & \mathbf{G}^T \\ \mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_n \\ \boldsymbol{\lambda}_n \end{bmatrix} = \begin{bmatrix} \mathbf{F}_n \\ \mathbf{I}_n \end{bmatrix},$$

- n is the time step index, and $\boldsymbol{\lambda}_n$ is Lagrange Multiplier, i.e. the nodal force/flux needed to let the solution honor the constraint function $\mathbf{I}_n$.
- Solution is solved by inverting the global matrix.

With the unconstrained solution obtained from the Newmark method,

$$\mathbf{u}_n = \mathbf{M}^{-1}\left(\left(\Delta t^2 \mathbf{f}_n - \mathbf{K}\mathbf{u}_{n-1}\right) - \Delta t \mathbf{C}\left(\mathbf{u}_{n-1} - \mathbf{u}_{n-2}\right)\right) + 2\mathbf{u}_{n-1} - \mathbf{u}_{n-2}.$$

The Forward Increment Lagrange Multiplier method [Carpenter et al. 1991] is applied to impose the constraint,

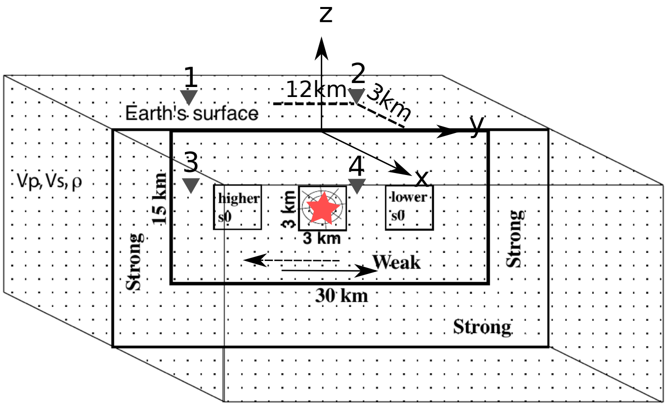
$$\begin{aligned}\boldsymbol{\lambda}_n &= \left(\Delta t^2 \mathbf{G}\mathbf{M}^{-1}\mathbf{G}^T\right)^{-1}\left(\mathbf{G}\mathbf{u}_n - \mathbf{l}_n\right),\\ \mathbf{u}_n &= \mathbf{u}_n - \Delta t^2 \mathbf{M}^{-1}\mathbf{G}^T \boldsymbol{\lambda}_n.\end{aligned}$$

Function and benchmark

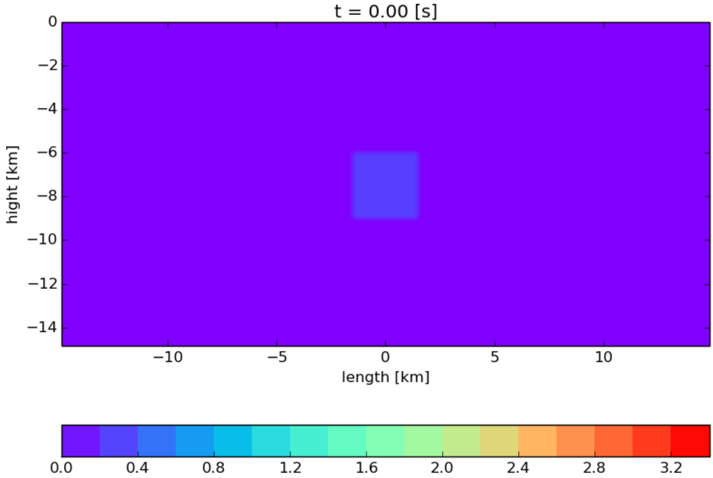
functionality	method	benchmark
poroelasticity	implicit, pore pressure stabilization	Mandel
viscoelastic power law	implicit	Abaqus
(quasi)static constraint	Lagrange Multiplier	Mohr-Coulomb
elastodynamic constraint absorbing fault/faulting	forward increment Lagrange Multiplier viscous damping implicit/explicit	SCEC

SCEC benchmark

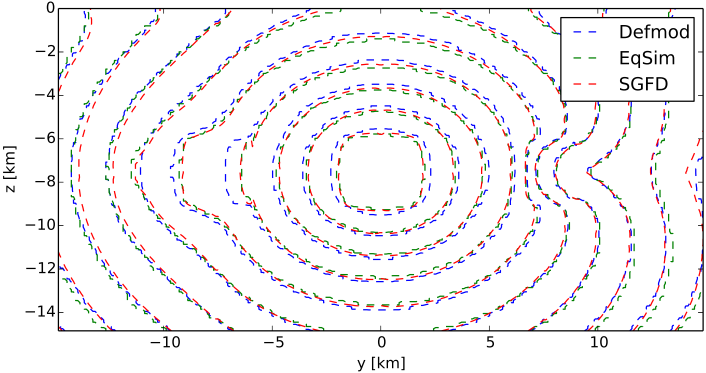
SCEC benchmark problem TPV5: Strike-slip rupture on a heterogeneously stressed fault patch.



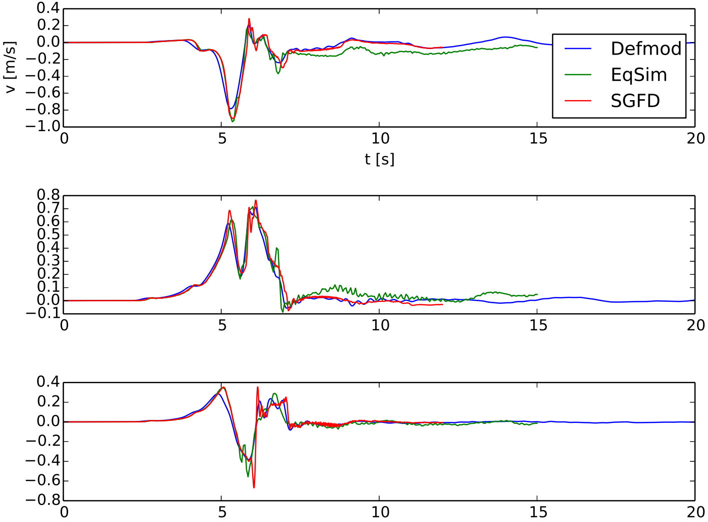
Fault slip rate magnitude [m/s]



Rupture front comparison against EqSim (Pylith) and SGFD.

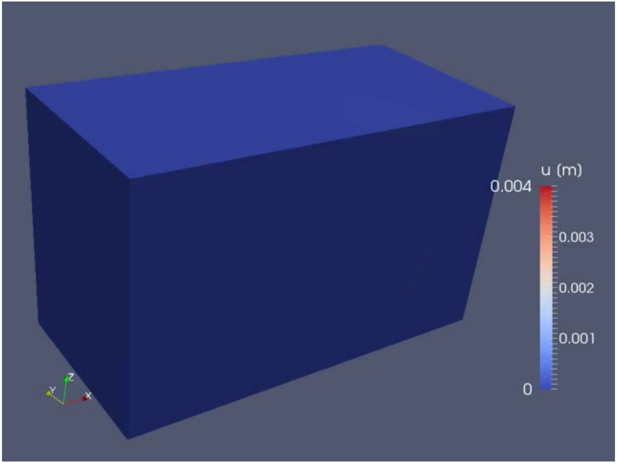


Waveform comparison for station 1.

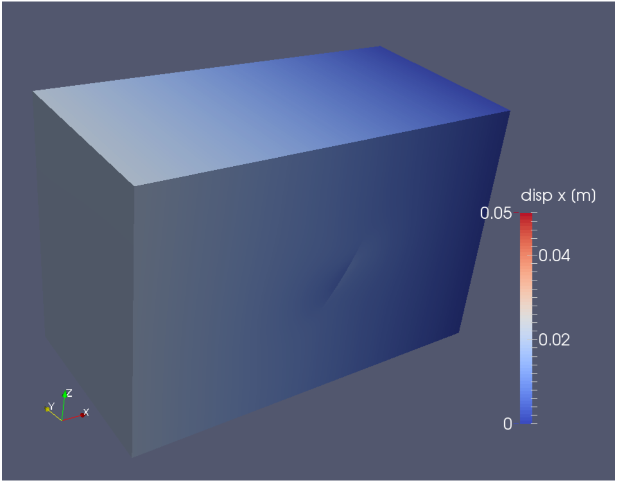


3D quasi-static loading and fault rupture

Hybrid output: seismic radiation, velocity magnitude [m/s].



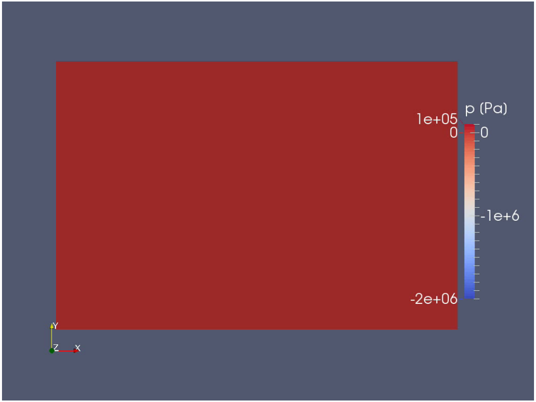
Hybrid output: quasi-static state after failure, x displacement [m] discontinuous across fault.



2D production induced earthquake

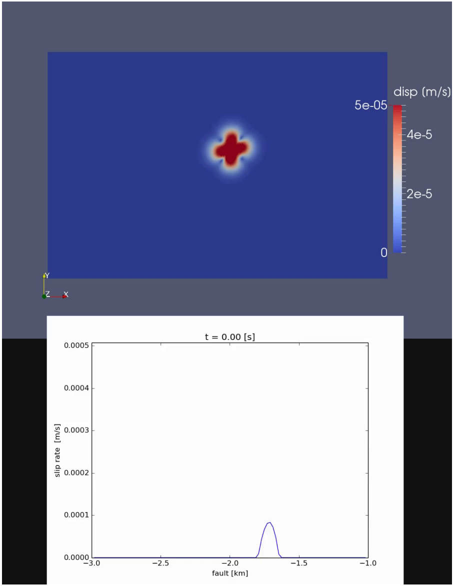
Quasi-static output: fault traction and pressure.

- Production/shut-in is indicated by pressure perturbation.
- Contact force/traction always appears in symmetric pairs.



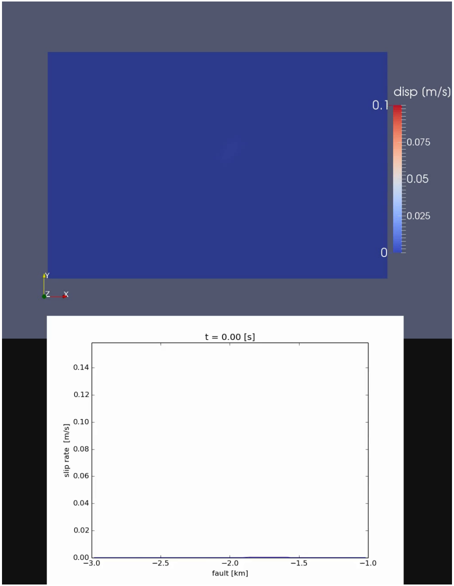
Induced earthquake: dynamic output for a small event.

- 2D plot of the velocity magnitude.
- 1D (fault profile) plot of the slip rate.



Induced earthquake: dynamic output for a large event.

- 2D plot of the velocity magnitude.
- 1D (fault profile) plot of the slip rate.



Summary

- The model adaptively switches between (quasi-)static and dynamic states describing the induced earthquake cycles.
- The model functionalities are well benchmarked against established results.

Future work

- The model will be used for history matching the induced earthquake occurrence and waveforms in Groningen.
- Once a reasonable history match is achieved, the model is capable of predicting induced seismic risks for given scenarios of production/injection.

Code availability

GNU General Public License, for bug report and contribution:

- Stable code: `https://bitbucket.org/stali/defmod`
- Developer version: `https://bitbucket.org/chunfangmeng/defmod-dev`

Poroelastic equation

The production/injection is typically of the time scale much longer than the seismic events. Such process can be considered quasi-static. The incremental loading scheme for poroelasticity [Smith and Griffiths, 2004] with the source and storage terms for the fluid:

$$\begin{bmatrix} \mathbf{K}_e & \mathbf{H} \\ -\mathbf{H}^T & \Delta t\mathbf{K}_c + \mathbf{S} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{u}_n \\ \Delta\mathbf{p}_n \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{f}_n \\ \mathbf{q}_n - \Delta t\mathbf{K}_c\mathbf{p}_{n-1} \end{bmatrix}$$

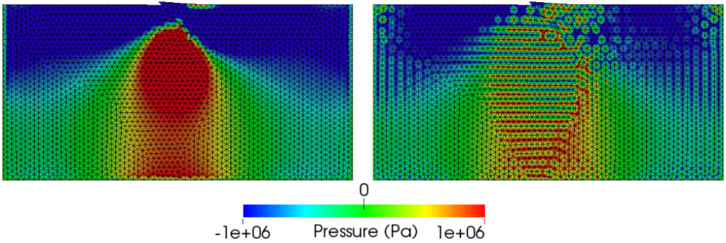
Where, $\mathbf{K}_e$ and $\mathbf{K}_c$ are the solid and fluid stiffness matrices; $\mathbf{H}$ is the coupling matrix; $\mathbf{S}$ is the storage matrix, $\Delta\mathbf{f}$ and $\mathbf{q}$ are the nodal force increment and the fluid flux during one time step Δt. Absolute solution at step n:

$$\mathbf{U}_n = \begin{bmatrix} \mathbf{u}_{n-1} \\ \mathbf{p}_{n-1} \end{bmatrix} + \begin{bmatrix} \Delta\mathbf{u}_n \\ \Delta\mathbf{p}_n \end{bmatrix}$$

where, $\mathbf{u}_{n-1}$ and $\mathbf{p}_{n-1}$ are the pressure of previous step.

Stabilizing fluid pressure

Unstable pressure is caused by using linear element known as the Ladyzenskaja-Babuska-Brezzi restrictions.



Pore pressure changes, 2D triangular element domain, following co-seismic slip on a thrust fault, with (left) and without (right) stabilization.

Local pressure projection scheme [Bochev and Dohrmann, 2006] is implemented to stabilize the pore pressure, [White and Borja, 2008] for quad/hex element.

$$\mathbf{F}_{n+1} = \mathbf{F}_{n+1} - \begin{bmatrix} \mathbf{0} \\ \mathbf{H}_s \mathbf{p}_n \end{bmatrix},$$
$$\mathbf{K}_{n+1} = \mathbf{K}_{n+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_s \end{bmatrix},$$

,

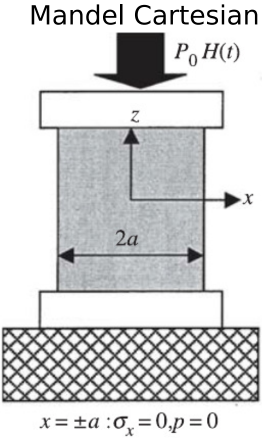
where,

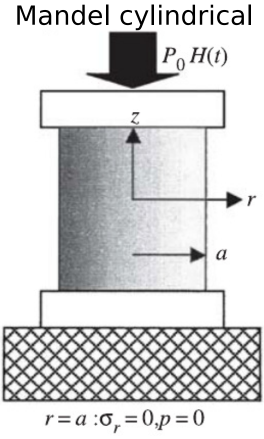
$$\mathbf{H}_s = \int_\Omega (\mathbf{N} - (1/n_e) I_{\mathbf{N}})^{\mathrm{T}} (\mathbf{N} - (1/n_e) I_{\mathbf{N}}) / (2G) d\Omega,$$

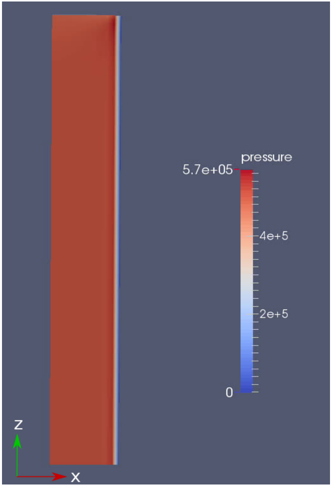
where, **N** is the shape function; $\overline{(\cdot)}$ is a function averaged over all nodal points of each element; G is the shear modulus; τ_{scale} is a scale constant.

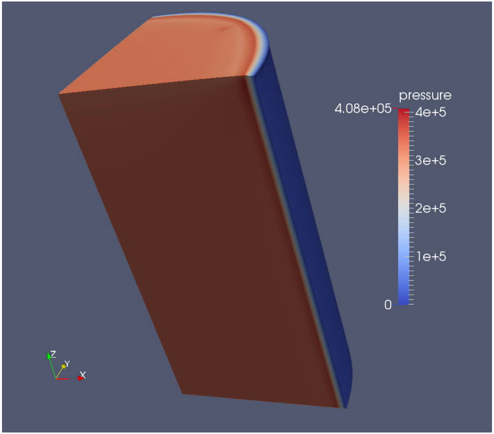
Poroelastic benchmark: Mandel solution

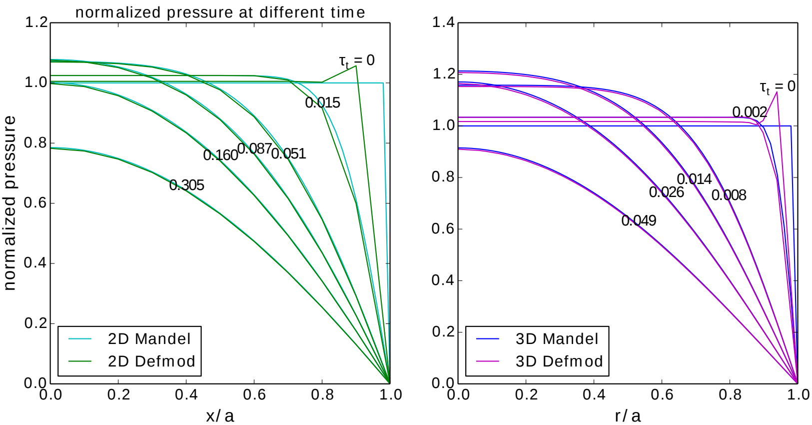
[M. Kurashige et al. 2004]



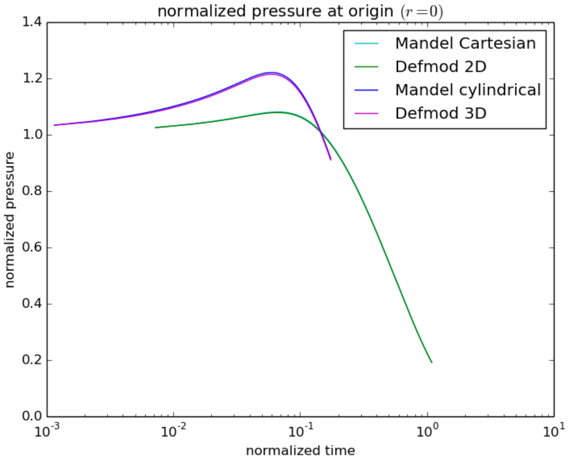



pressure
5.7e+05
4e+5
2e+5
0
Z
X


pressure
4.08e+05
4e+5
3e+5
2e+5
1e+5
0
Z
Y
X

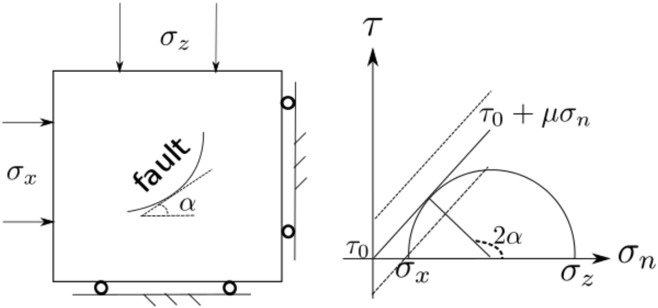

normalized pressure at different time
normalized pressure
x/a
$\tau_t = 0$
0.015
0.160
0.087
0.051
0.305
2D Mandel
2D Defmod
r/a
$\tau_t = 0$
0.002
0.014
0.026
0.008
0.049
3D Mandel
3D Defmod

Pressure at origin, 2D vs 3D

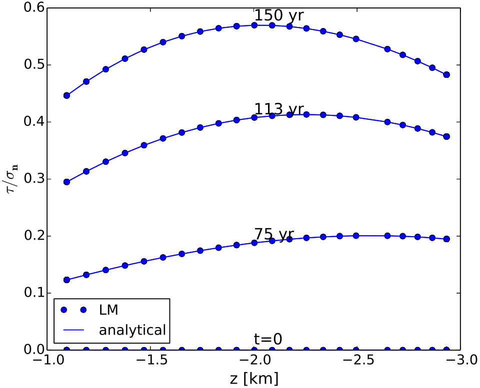


Static fault model benchmark

Anisotropic loading model: Incrementally load the sample at the rate of $\Delta\sigma_H = \Delta\sigma_V = 0.2$ MPa/yr. At year 50, stop the loading increment in the horizontal direction, and keep the increment in the vertical direction until year 150.



Stress ratio $\tau/\sigma_{\mathbf{n}}$ calculated by Defmod at different depth and time against the analytical values.



Viscoelastic problem

The stiffness matrix and RHS vector of a viscoelastic media have [Melosh and Raefsky 1980]

$$\mathbf{K}_{n+1} = \int_{\Omega} \mathbf{B}^T(\mathbf{D}^{-1} + \alpha\Delta t\beta'_n)^{-1}\mathbf{B}d\Omega$$
$$\mathbf{F}_{n+1} = \int_{\Omega} \mathbf{B}^T(\mathbf{D}^{-1} + \alpha\Delta t\beta'_n)^{-1}(\Delta t\boldsymbol{\beta}_n)d\Omega + \mathbf{F}_{n+1},$$

where, **B** is displacement to strain matrix depending on the element geometry, **D** is the element stiffness matrix depending on the elastic constants,

$$\beta(\boldsymbol{\sigma}) = \frac{\sigma^{e-1}}{4\eta}\mathbf{C}_c : \boldsymbol{\sigma},\ e \geq 1,$$
$$\sigma = \sqrt{\sum_{i\neq j}(\boldsymbol{\sigma}_{ii} - \boldsymbol{\sigma}_{jj})^2/(2d) + \boldsymbol{\sigma}_{ij}^2},\ \ d = 2 \text{ or } 3,$$

Viscoelastics: β and β'

$$\beta(\boldsymbol{\sigma}) = \frac{\sigma^{e-1}}{4\eta}\mathbf{C}_c : \boldsymbol{\sigma}$$

$$\mathbf{C}_c = \begin{cases} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}, & \text{2D,} \\ \begin{bmatrix} 4/3 & -2/3 & -2/3 & & & \\ -2/3 & 4/3 & -2/3 & & \mathbf{0} & \\ 2/3 & -2/3 & 4/3 & & & \\ & & & 4 & & \\ & \mathbf{0} & & & 4 & \\ & & & & & 4 \end{bmatrix}, & \text{3D.} \end{cases}$$

$$\beta' = \begin{cases} \frac{\sigma^{e-1}}{4\eta}\begin{bmatrix} c_1 & -c_1 & c_3 \\ -c_1 & c_1 & -c_3 \\ c_3 & -c_3 & 4c_2 \end{bmatrix}, & \text{2D,} \\ \frac{\sigma^{e-1}}{4\eta}\begin{bmatrix} 4/3 + S_x^2 & -2/3 + S_xS_y & -2/3 + S_xS_z & S_xT_1 & S_xT_2 & S_xT_3 \\ -2/3 + S_xS_y & 4/3 + S_y^2 & -2/3 + S_yS_z & S_yT_1 & S_yT_2 & S_yT_3 \\ -2/3 + S_xS_z & -2/3 + S_yS_z & 4/3 + S_z^2 & S_zT_1 & S_zT_2 & S_zT_3 \\ S_xT_1 & S_yT_1 & S_zT_1 & 4 + T_1^2 & T_1T_2 & T_1T_3 \\ S_xT_2 & S_yT_2 & S_zT_2 & T_2T_1 & 4 + T_2^2 & T_2T_3 \\ S_xT_3 & S_yT_3 & S_zT_3 & T_3T_1 & T_3T_2 & 4 + T_3^2 \end{bmatrix}, & \text{3D.} \end{cases}$$

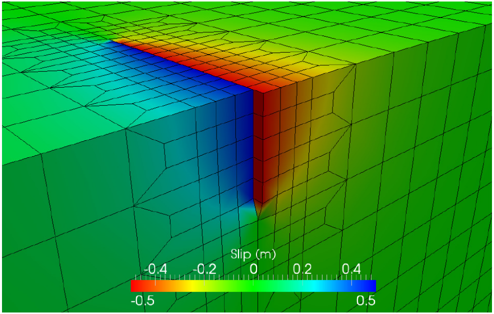
$$
\begin{aligned}
c_1 =& 1 + (e-1)((\boldsymbol{\sigma}_{xx} - \boldsymbol{\sigma}_{yy})/(2\sigma))^2 \\
c_2 =& 1 + (e-1)(\boldsymbol{\sigma}_{xy}/\sigma)^2 \\
c_3 =& (e-1)(\boldsymbol{\sigma}_{xx}\boldsymbol{\sigma}_{yy} - \boldsymbol{\sigma}_{yy}\boldsymbol{\sigma}_{xy})/\sigma^2 \\
S_x =& c(2\sigma_{xx} - \sigma_{yy} - \sigma_{zz})/(3\sigma) \\
S_y =& c(2\sigma_{yy} - \sigma_{zz} - \sigma_{xx})/(3\sigma) \\
S_z =& c(2\sigma_{zz} - \sigma_{xx} - \sigma_{yy})/(3\sigma) \\
T_1 =& 2c\sigma_{xy}/\sigma, T_2 = 2c\sigma_{yz}/\sigma, T_3 = 2c\sigma_{xz}/\sigma
\end{aligned}
$$

where $c = \sqrt{e-1}$

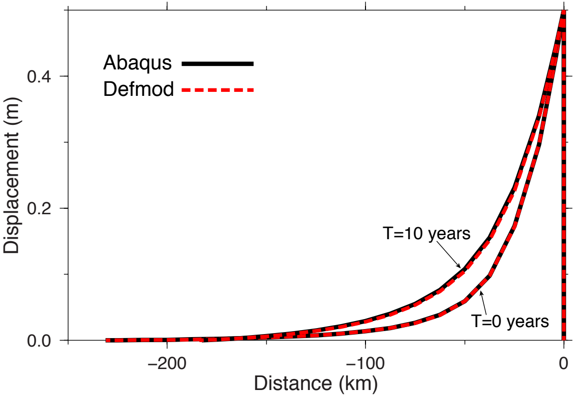
- When $e = 1$, $\beta' = \frac{1}{4\eta}\mathbf{C}_c$, the matrix $\mathbf{K}_n$ is then independent of $\boldsymbol{\sigma}_{ij}$ and n, as long as the step length Δt is a constant. In this case we only need to assemble the viscoelastic stiffness matrix **K** once for the first step, and keep updating the RHS $\mathbf{F}_n$. When $e > 1$ however, both the stiffness matrix and RHS need to be reassembled for every step.
- Since the scale factor α and the effective viscosity η always appear as $\frac{\alpha}{4\eta}$, they are treated as one parameter. Therefore, in addition to the elastic constants, two parameters, e and η, need to be specified.

Viscoelastic benchmark

Slip on a strike-slip fault: Elastic crust over viscoelastic mantle. Only a part of model domain is shown.



Comparison against Abaqus at t = 0 and t = 10 years. The displacement is plotted along a trajectory perpendicular to the fault plane through the viscoelastic layer.



Explicit solver for the seismic radiation

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f},$$

where, $\mathbf{M}$ is mass matrix; $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ is damping matrix; α and β are Rayleigh damping coefficients. Newmark explicit scheme has,

$$\mathbf{u}_n = \mathbf{M}^{-1}\Big((\Delta t^2\mathbf{f}_n - \mathbf{K}\mathbf{u}_{n-1}) - \Delta t\mathbf{C}\left(\mathbf{u}_{n-1} - \mathbf{u}_{n-2}\right)\Big) + 2\mathbf{u}_{n-1} - \mathbf{u}_{n-2}$$

The incremental form,

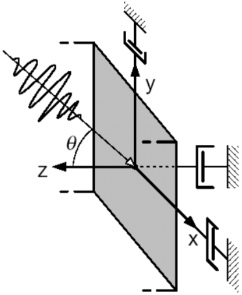
$$\begin{aligned}\Delta\mathbf{u}_n =& \mathbf{M}^{-1}\Big((\Delta t^2\Delta\mathbf{f}_n - \mathbf{K}\Delta\mathbf{u}_{n-1}) \\ &- \Delta t\mathbf{C}\left(\Delta\mathbf{u}_{n-1} - \Delta\mathbf{u}_{n-2}\right)\Big) + 2\Delta\mathbf{u}_{n-1} - \Delta\mathbf{u}_{n-2}\end{aligned}$$

Absorbing boundary

[Lysmer and Kuhlemeyer, 1969] propose that the absorbing boundary for the elastodynamic model can be achieved by adding additional terms to the damping matrix as,

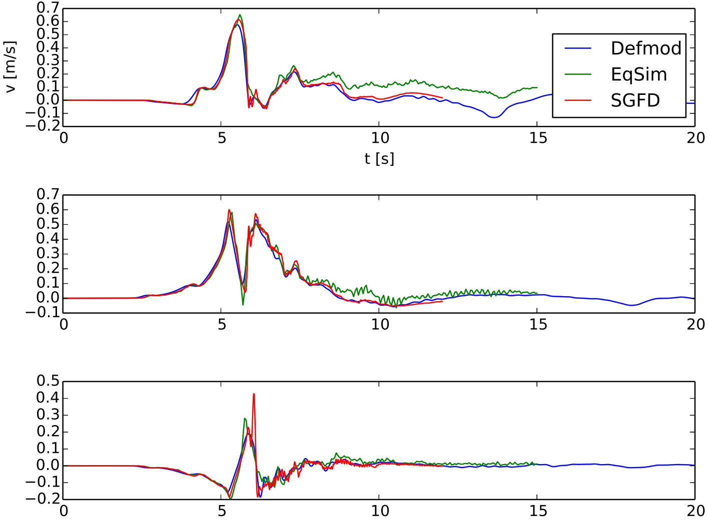
$$c_{ii} = \begin{cases} c_{ii} + \int_\Gamma \rho V_p d\Gamma, & \text{p wave} \parallel i^{th} \text{ axis}, \\ c_{ii} + \int_\Gamma \rho V_s d\Gamma, & \text{p wave} \perp i^{th} \text{ axis}, \end{cases} \quad \text{for all } i\text{-axes.}$$

Assuming small incident angles ($\theta < 30^\circ$), that the *p*-wave can be considered roughly perpendicular to the absorbing boundary. Therefore, this addition is irrespective to the coming wave directions.



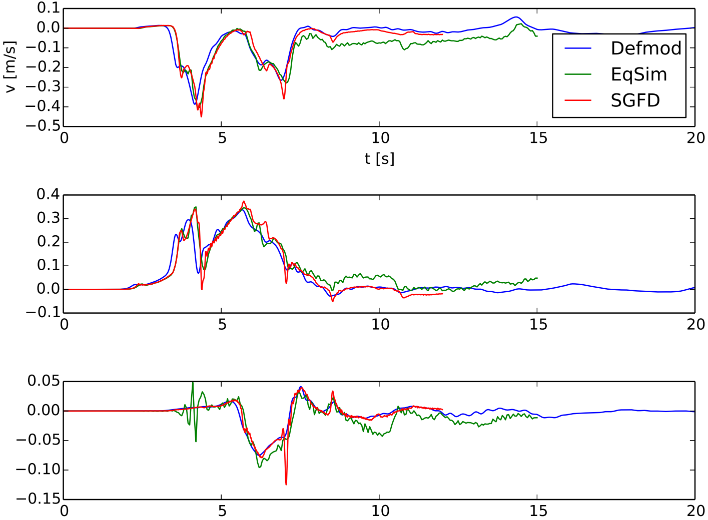
TVP 5 wave forms

Waveform comparison for station 2.



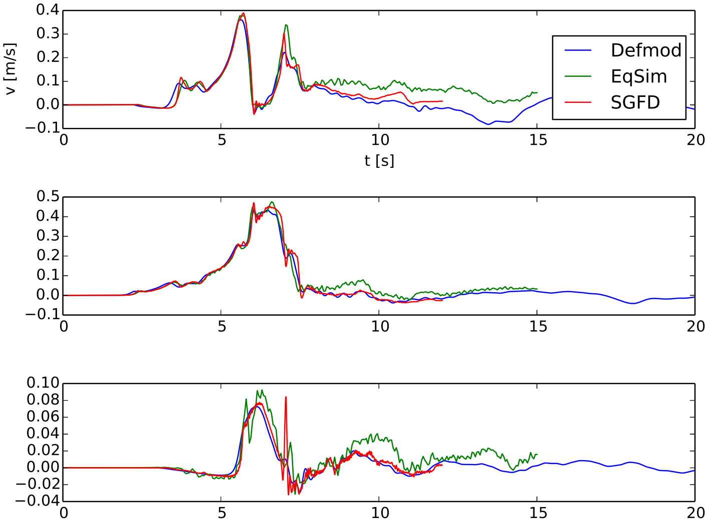
TVP 5 wave forms

Waveform comparison for station 3.



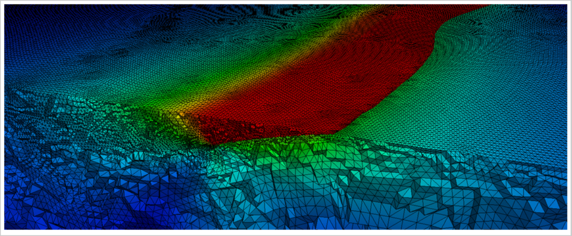
TVP 5 wave forms

Waveform comparison for station 4.

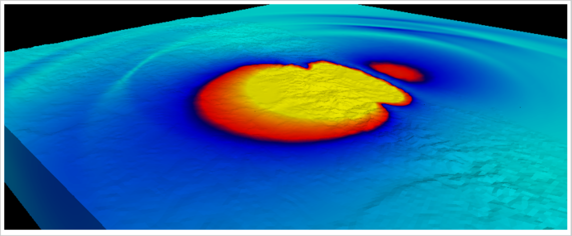


Performance

Subduction, $\sim$ 8 m elements, implicitly solved, under 120 secs.



Thrust fault, $\sim$ 6 m elements, 2000 explicit steps, under 60 secs.



Winkler Foundation

Winkler Foundation is to consider the gravity caused anisotropy by adding linear springs in the gravity direction. The element stiffness matrix is therefore modified by

$$k_{ii}^{(p)} = k_{ii}^{(p)} + \frac{1}{n_p}\int_{\Gamma_g} \rho g d\Gamma_g, \text{ for } p \in \Gamma_g, i^{\text{th}} \text{ axis} \parallel \vec{g}, \Gamma_g \perp \vec{g},$$

where, n_p is the number of Gauss points on the facet Γ_g.