MIT EARTH RESOURCES LABORATORY
ANNUAL FOUNDING MEMBERS MEETING 2020

Wide-band Butterfly Network:
An architecture for multifrequency sub-wavelength imaging

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In collaboration with Leonardo Zepeda-Núñez (UW Madison) and Laurent Demanet (MIT)

Acknowledgements

MIT | Earth Resources Laboratory

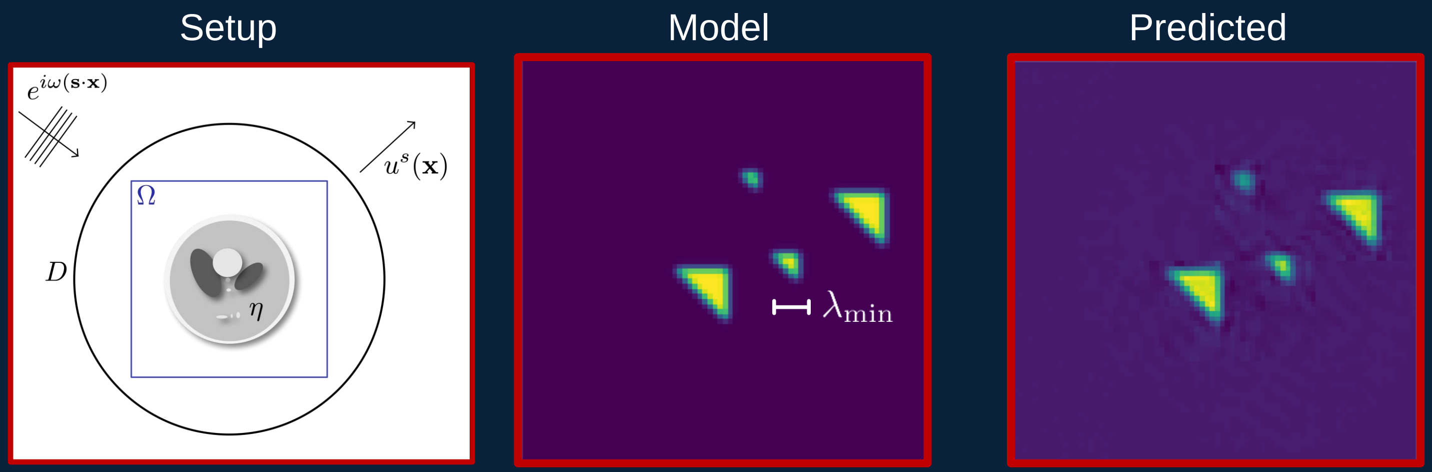
WARF
Wisconsin Alumni Research Foundation

AFOSR
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

TOTAL

Outline / Contributions

1) we introduce a neural network for **seismic inversion below the diffraction limit**



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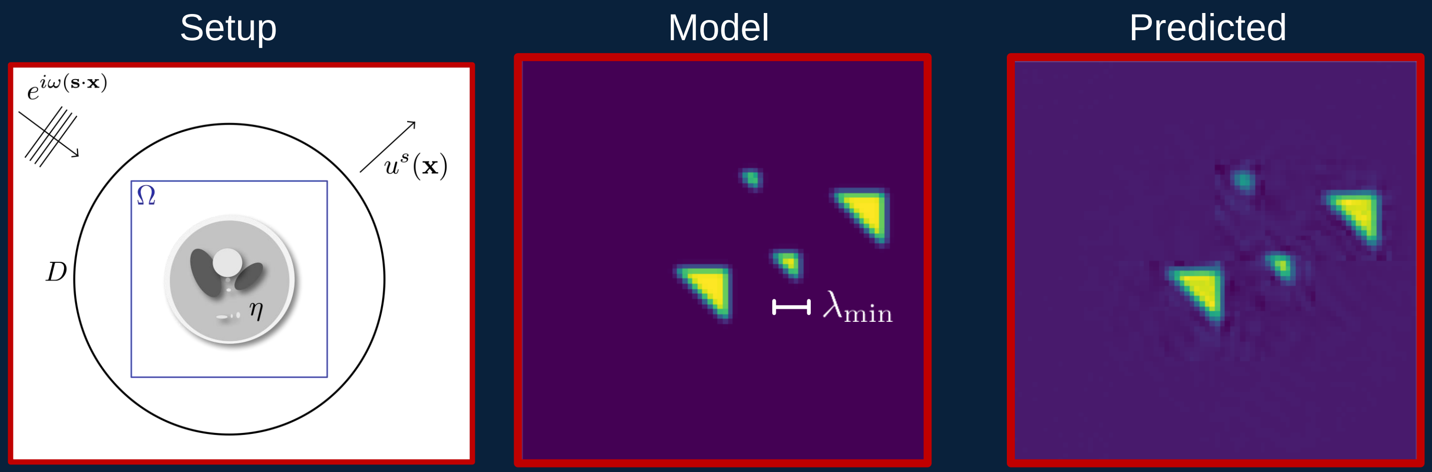
2) our architecture **incorporates the physics of wave propagation**



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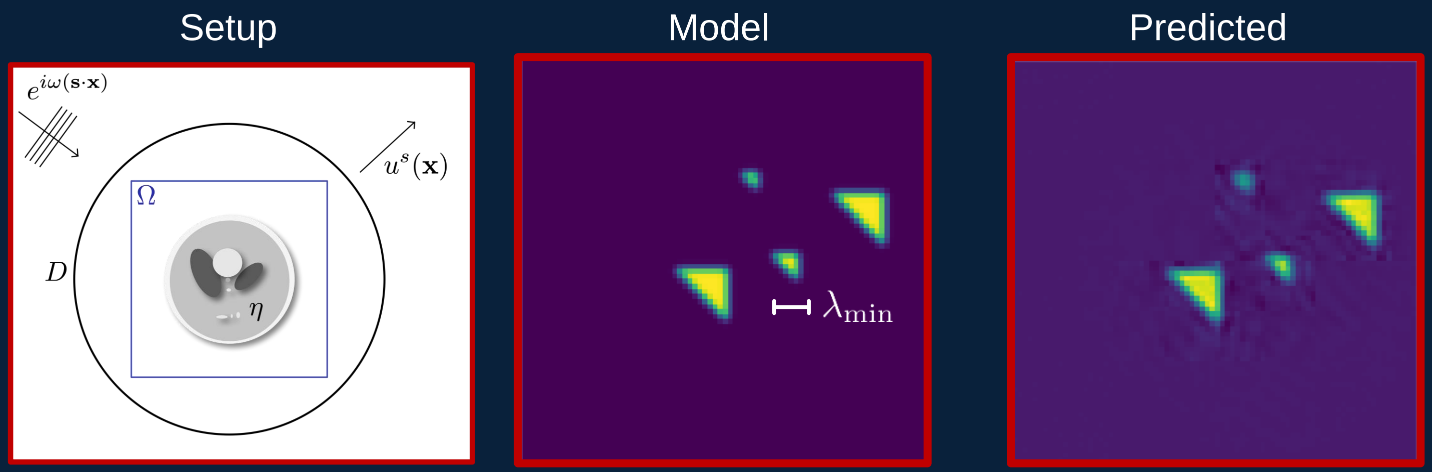


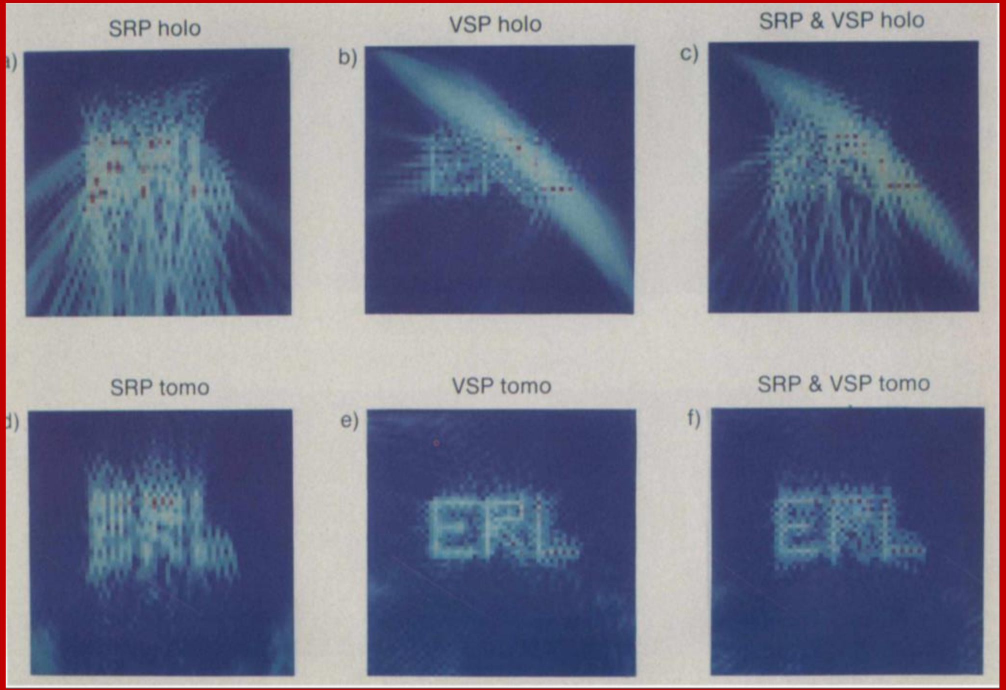
... but why machine learning?:

- *as a signal processing problem this is genuinely difficult*
- *it's unclear what are the resolution limits to FWI* [Fichtner & Trampert 2011]
- *machine learning **provides evidence of whether this task is even possible!***

Geophysical Relevance

Diffraction tomography and multisource holography applied to seismic imaging

Ru-Shan Wu* and M. Nafi Toksöz*

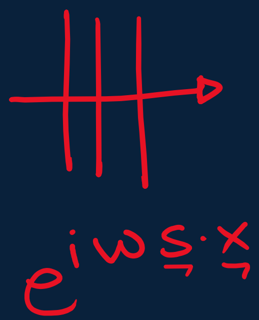


Thanks @**Nori Nakata** for this reference!

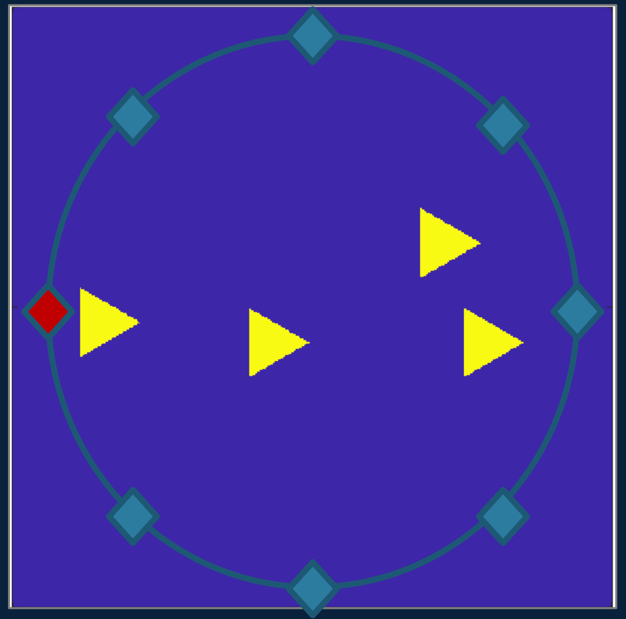
Imaging Setup

ASSUMPTIONS:

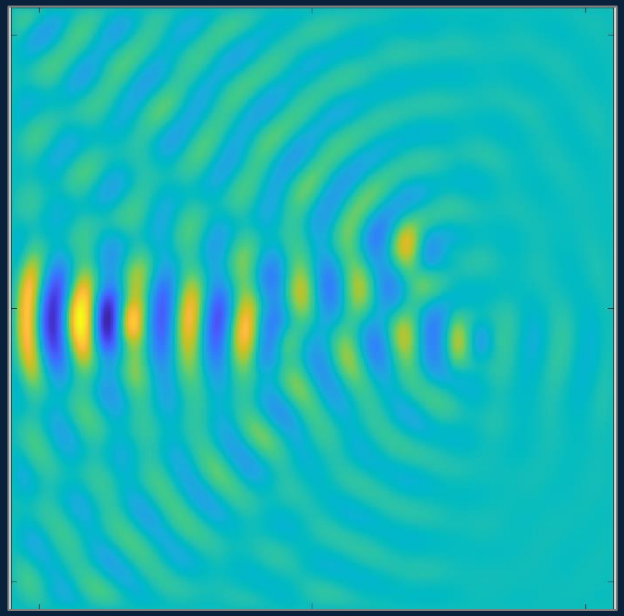
- full aperture
- far field
- noiseless
- known background velocity



$\eta(x)$



$u_{\mathrm{sc}}(x)$



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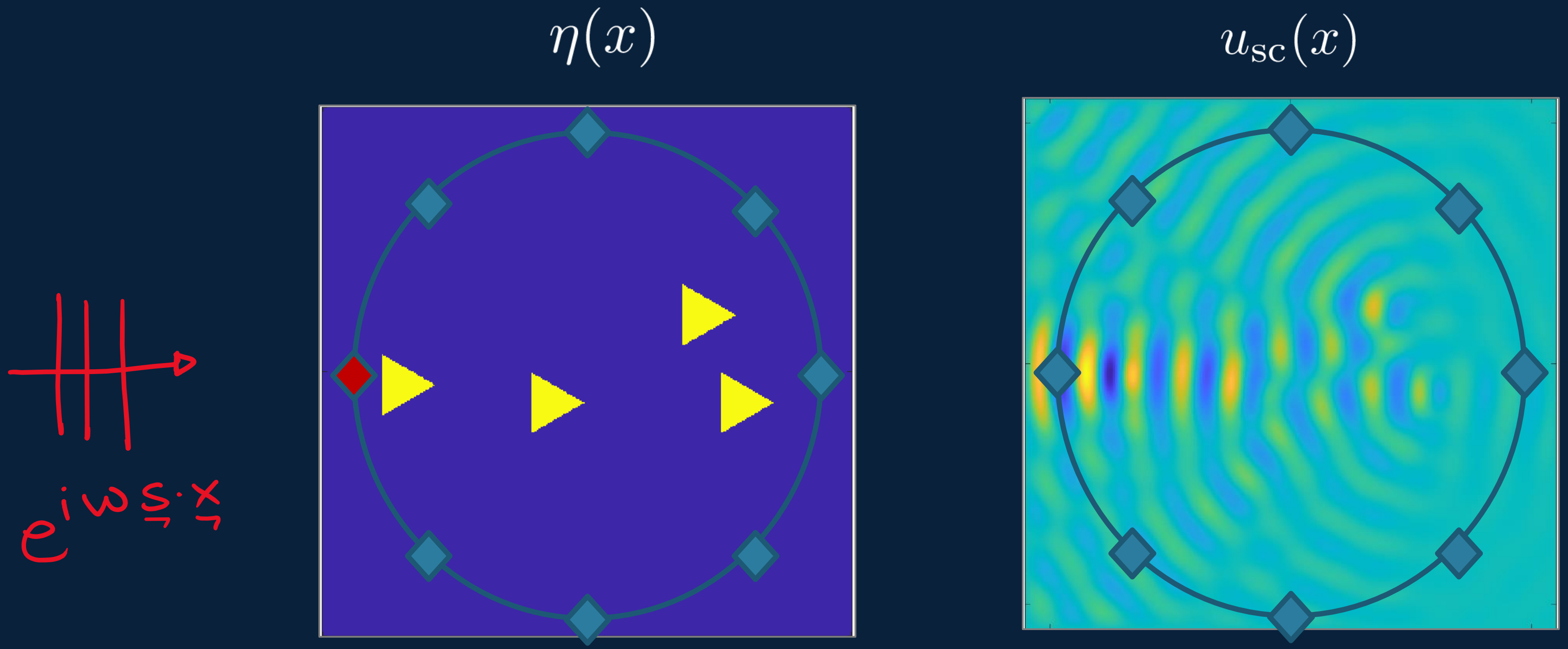
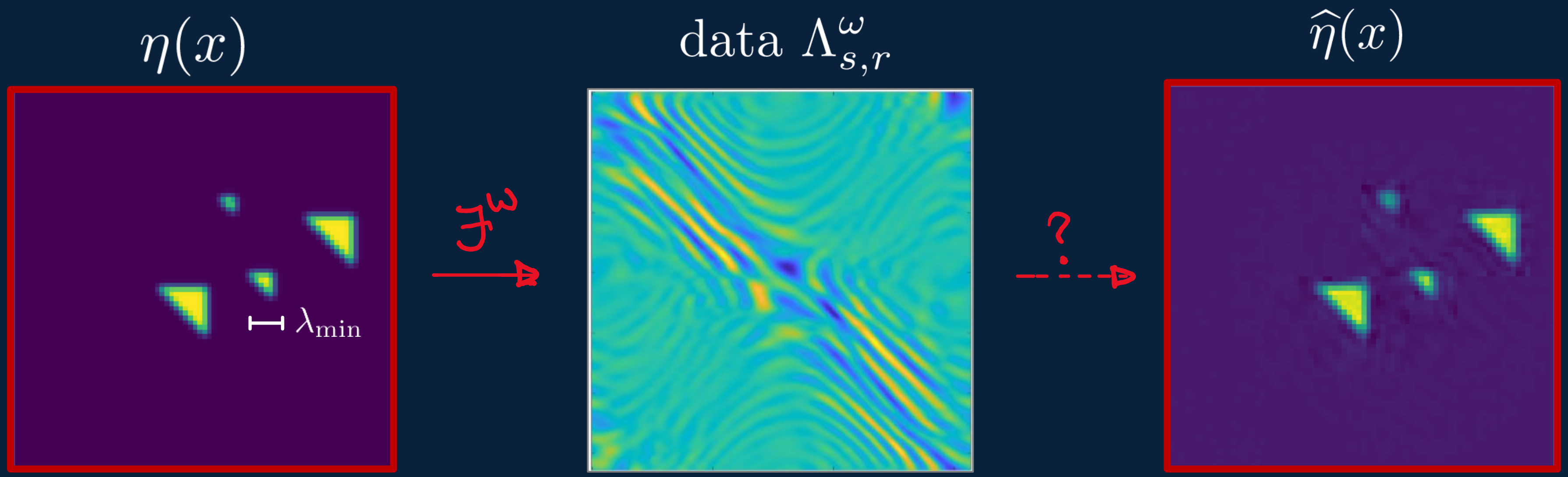


Inverse Problem

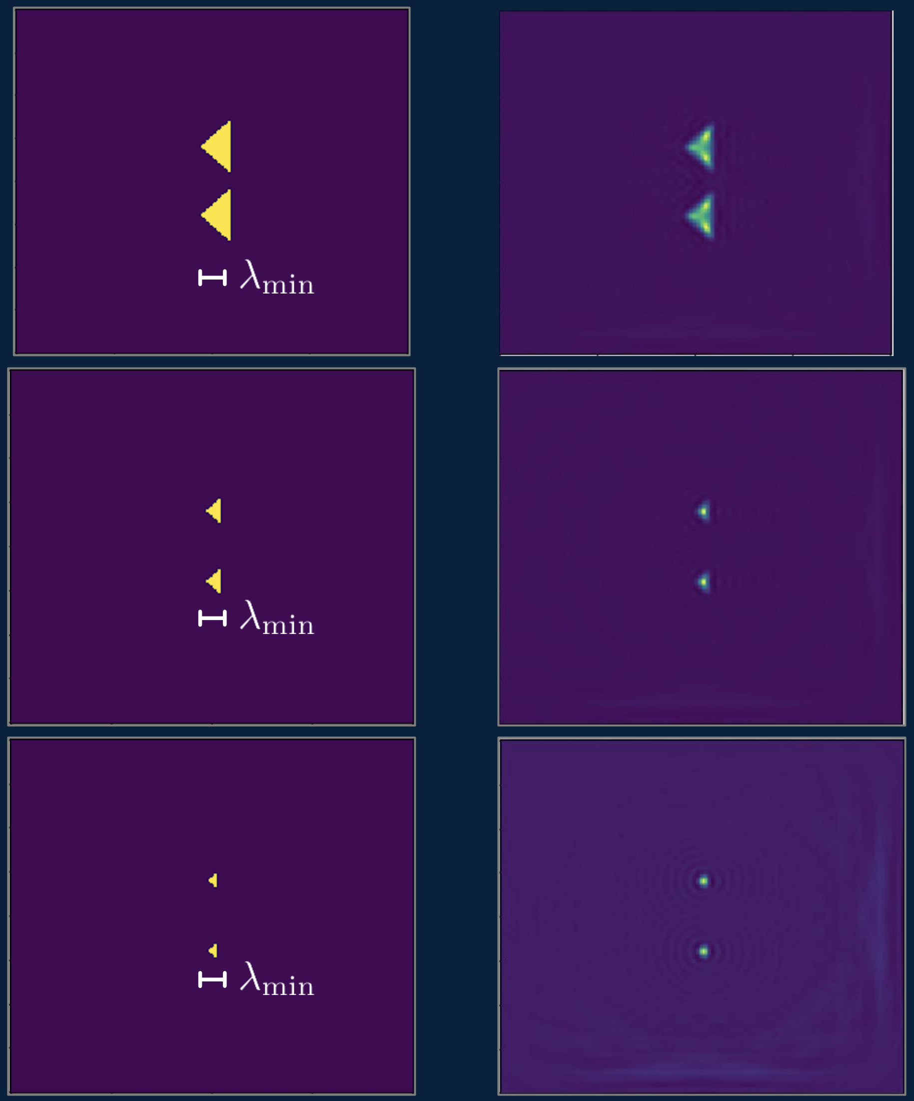


$$\hat{\eta} = \arg\min_{\mu} \left| \sum_{\omega} ||\mathcal{F}^{\omega}[\mu] - \Lambda^{\omega}_{s,r}||^2 \right|$$

nonlinear wave (Helmoltz) equation

Super Resolution

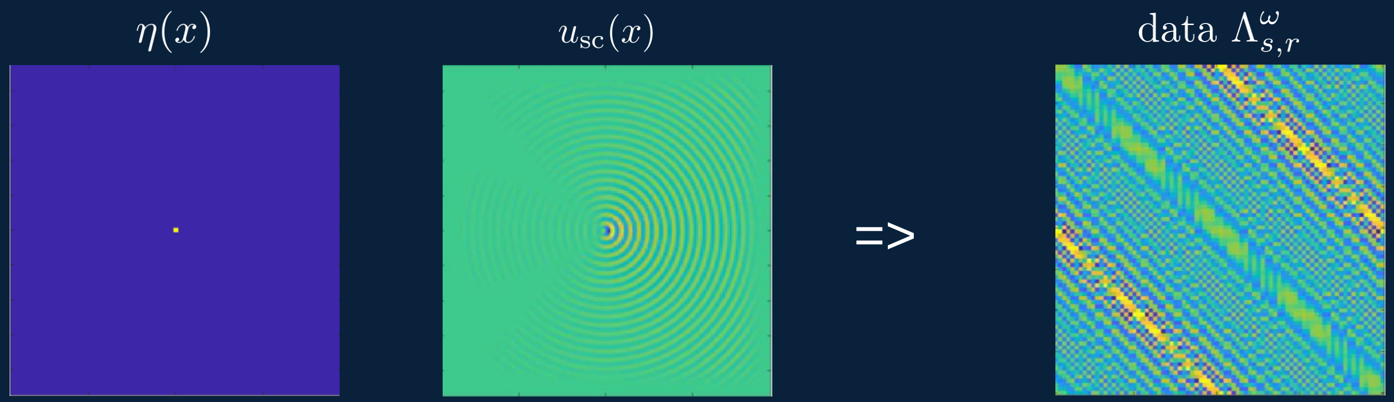
- in the **linearized setup** we are limited to ½ wavelength by the Rayleigh resolution
- in the **nonlinear setup (FWI)** it is less clear what resolution limits are
  - *but sharp images are still possible if you properly regularize*
  - *... consult our resident expert Zhilong Fang to learn more!*
- Donoho (1992) proves **super resolution of point scatterers** is possible:
  - *1. you need multiple frequencies*
  - *2. algorithm must be non-linear*



Architectural Design

QUESTION: Why do we need a custom neural network architecture for this problem?

ANSWER: ...because the **physics of wave propagation** makes it challenging



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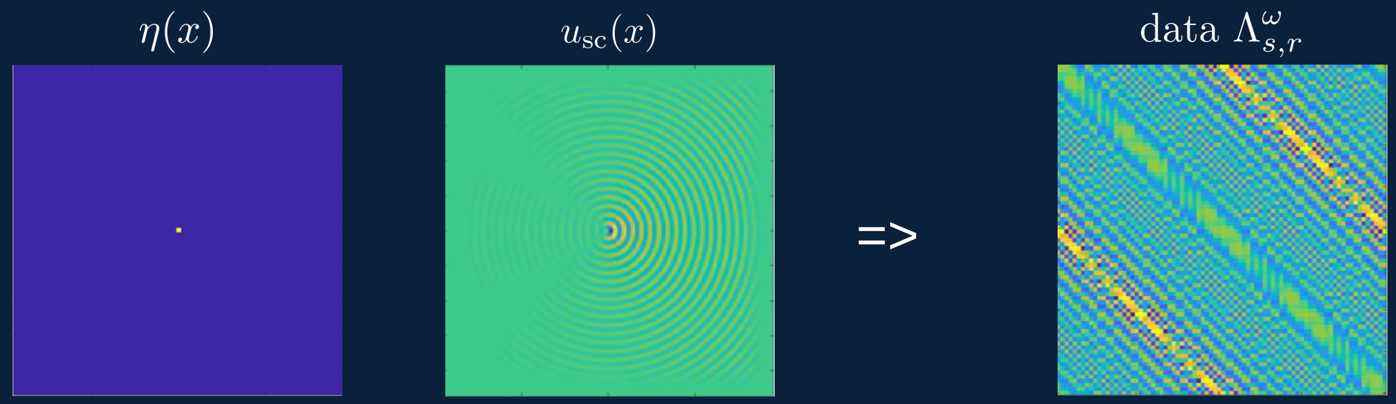


QUESTION: So how should we design our network?

ANSWER: ...to at least perform **as well as linearized Born inversion** (filtered backprojection)

$$\mathcal{F}[\mu] \approx F\mu \qquad \hat{\eta} = (F^*F + \epsilon I)^{-1} F^* \Lambda_{s,r}$$

BORN APPROX

Butterfly Network

- Khoo and Ying (2018) – exploit the **analytical properties** of the operators

$$\hat{\eta} = (F^* F + \epsilon I)^{-1} F^* \Lambda_{s,r}$$

- translation invariant convolution operator
  ⇒ replace with CNN !

- a Fourier Integral Operator (FIO)
  ⇒ well approximated by a BUTTERFLY FACTORIZATION

[Hörmander 1985]

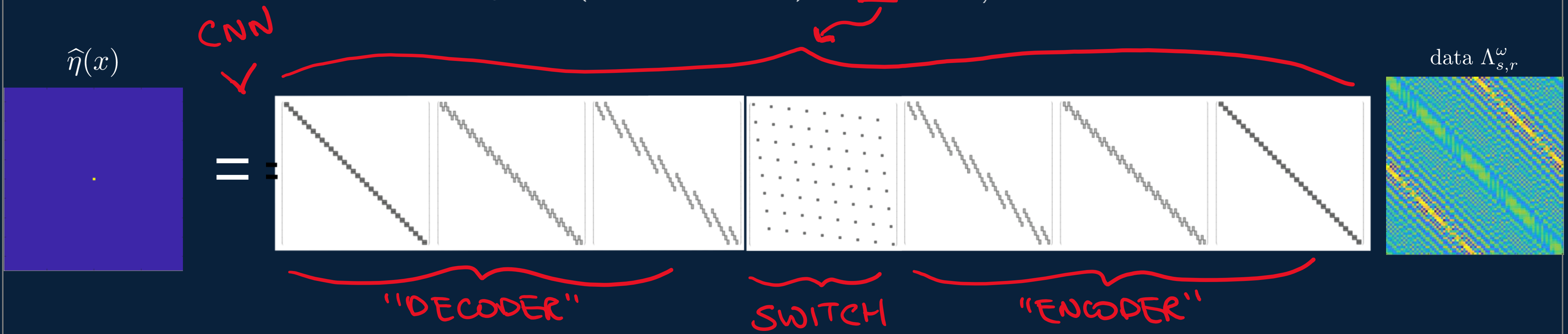
[Michielssen & Boag 1996]

Butterfly Network

THE KEY IDEA

- use the **structure of the matrix factorization** as your architecture

$$\hat{\eta} = (F^*F + \epsilon I)^{-1} F^* \Lambda_{s,r}$$



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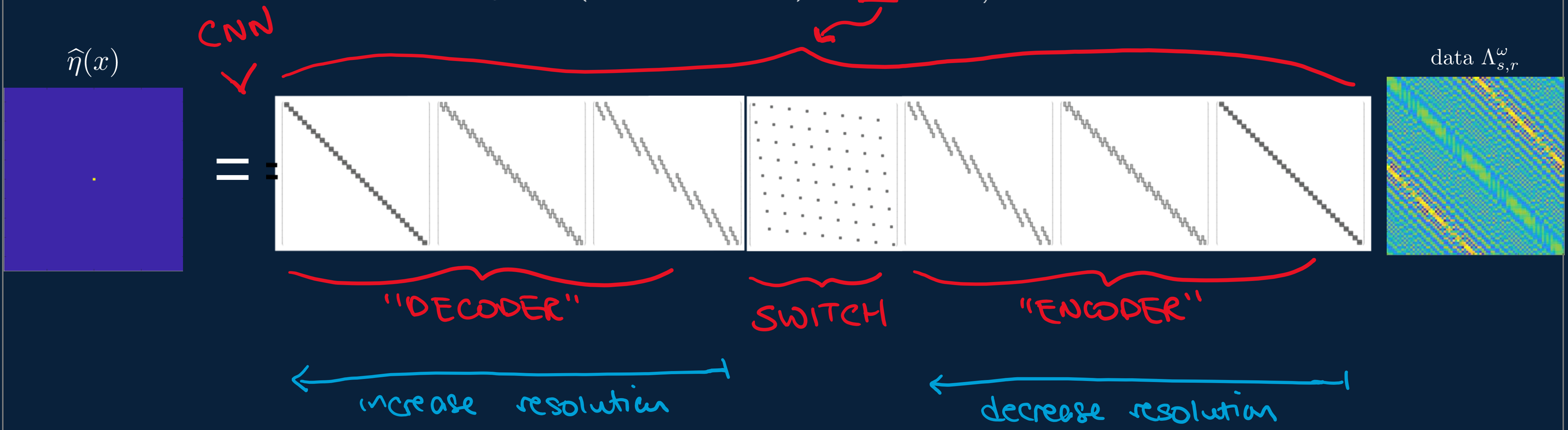
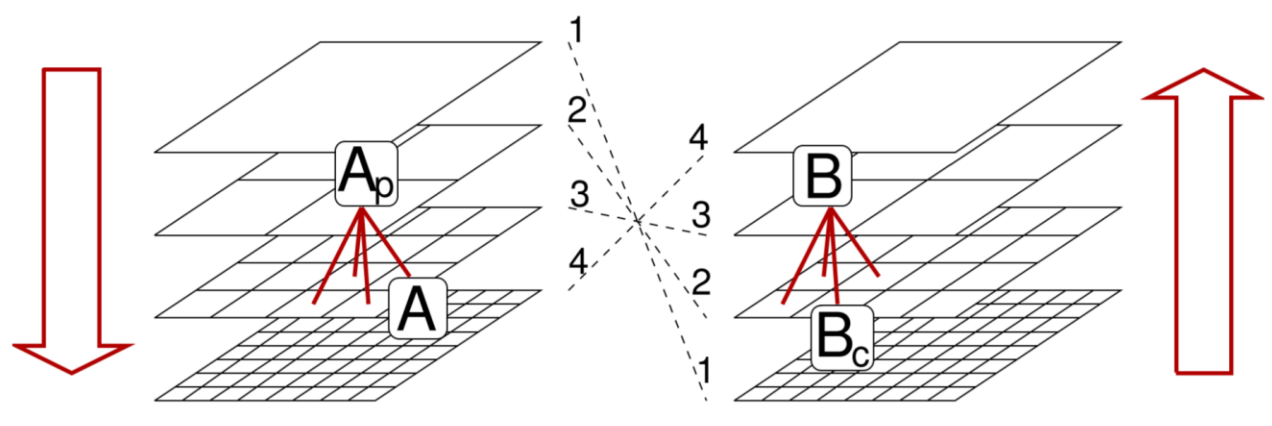


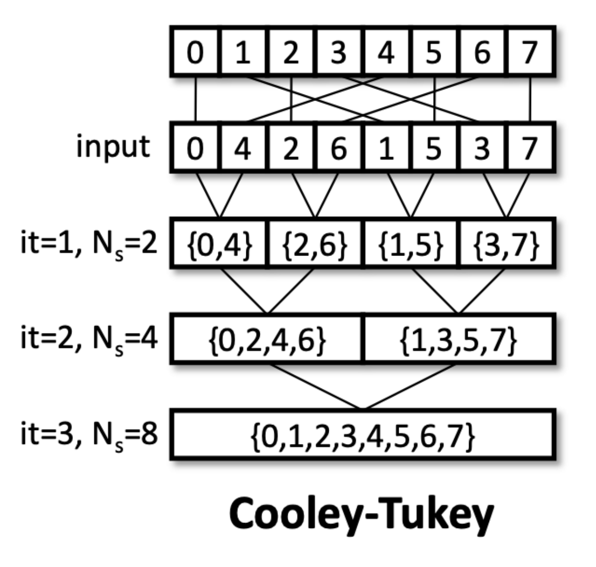
Incorporating Wideband Data

REMEMBER THE REQUIREMENTS FOR SUPER-RESOLUTION

1. need **wideband data to stabilize** inverse problem
2. need **non-linear combination** of frequencies



[Candes, Demanet, and Ying, 2008]



[Cooley & Tukey 1965]

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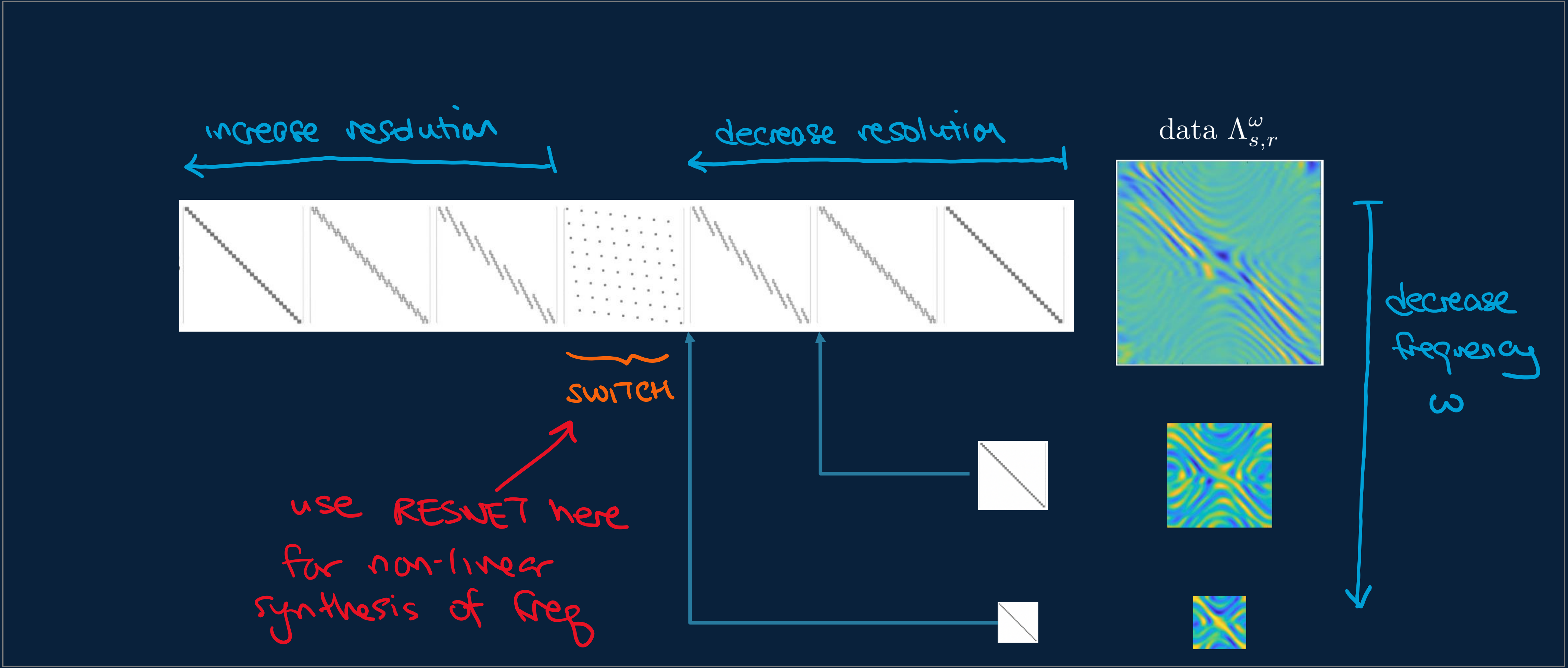
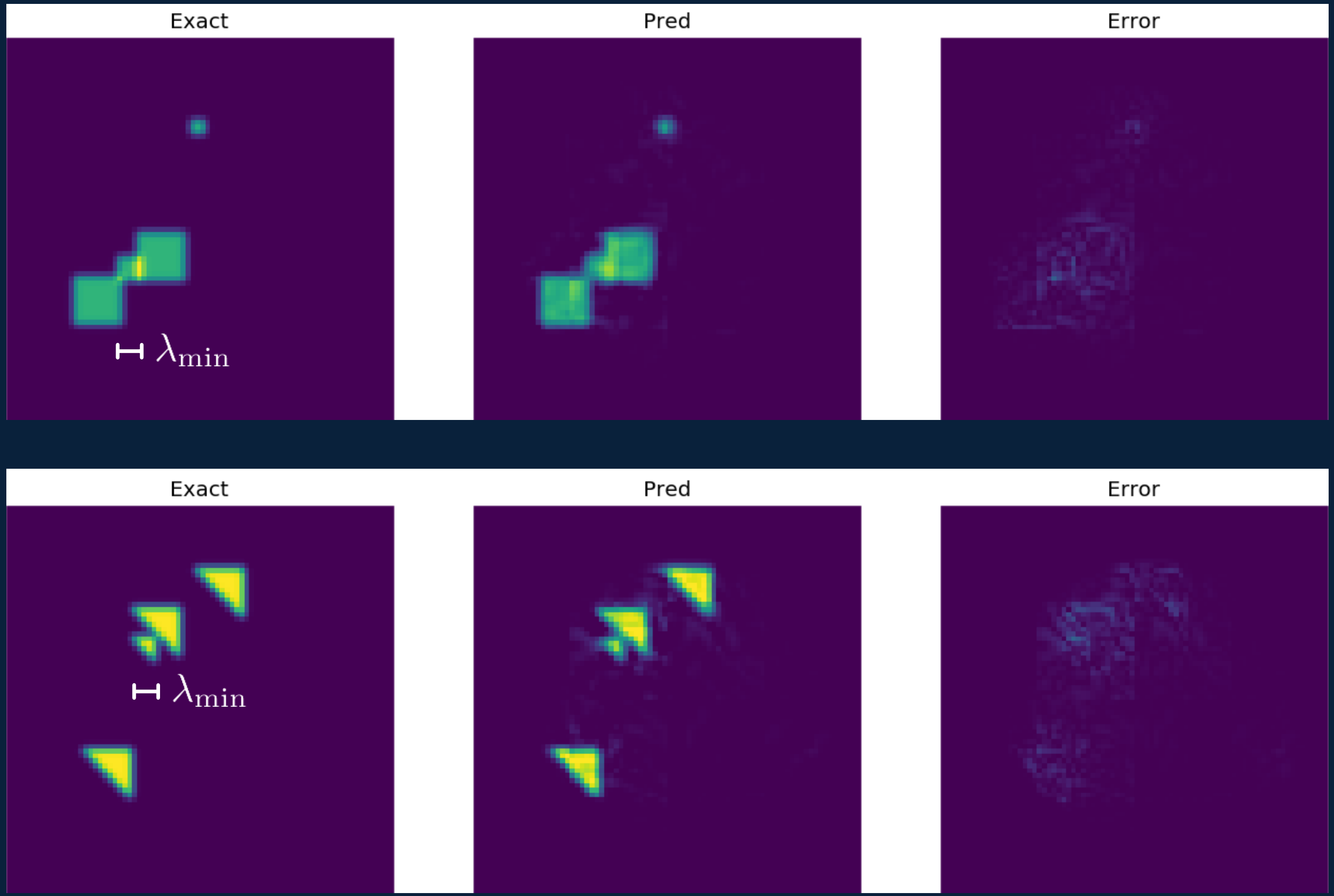


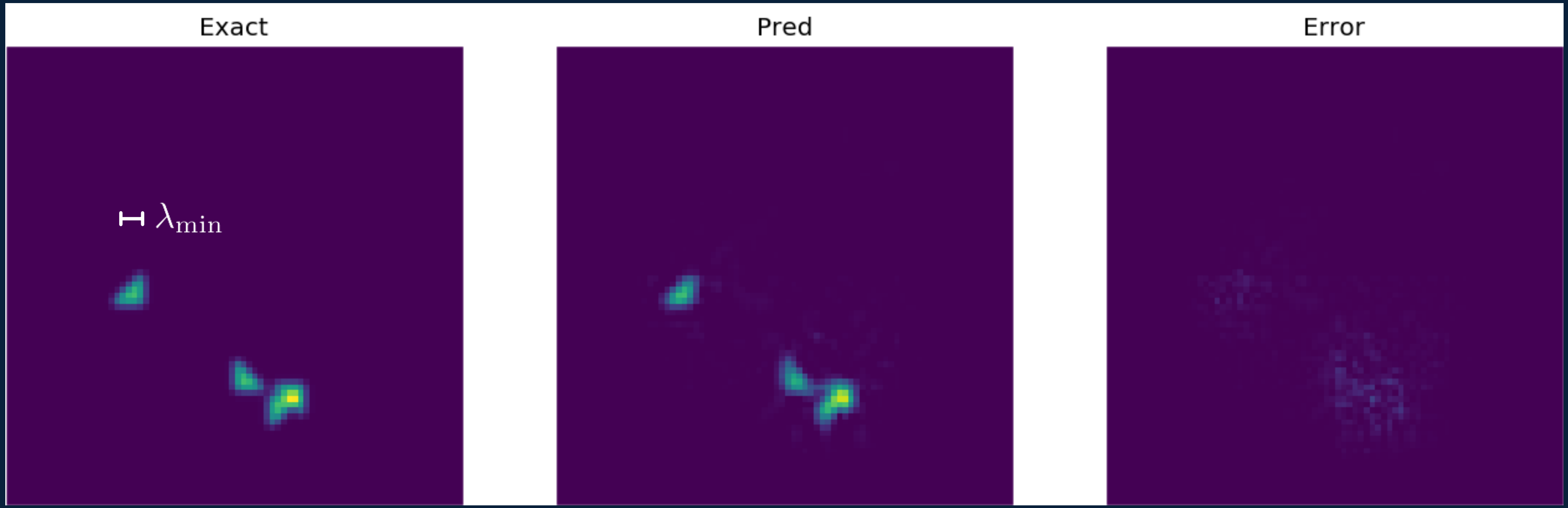
Numerical Results

MIXED RESOLUTION



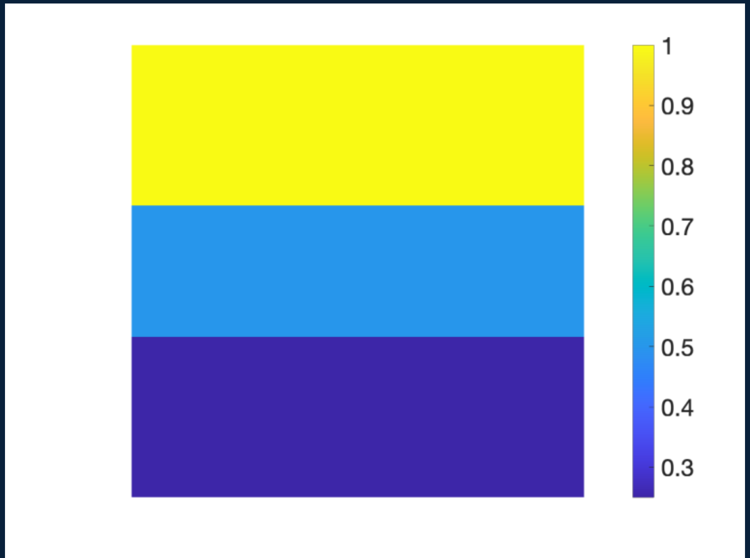
Numerical Results

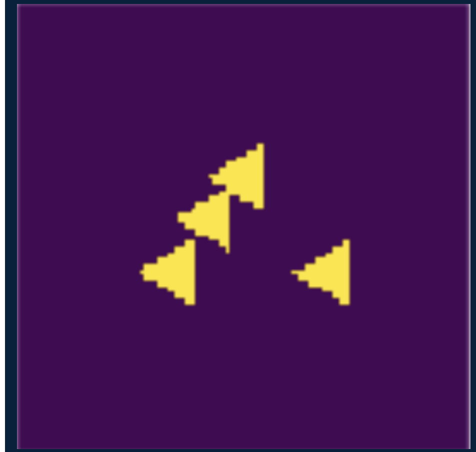
ROTATIONS

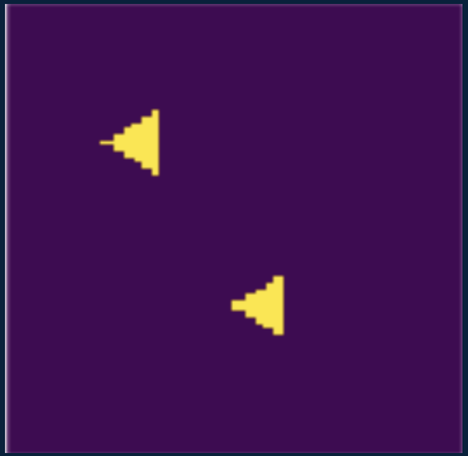


Numerical Results

INHOMOGENEOUS BACKGROUND







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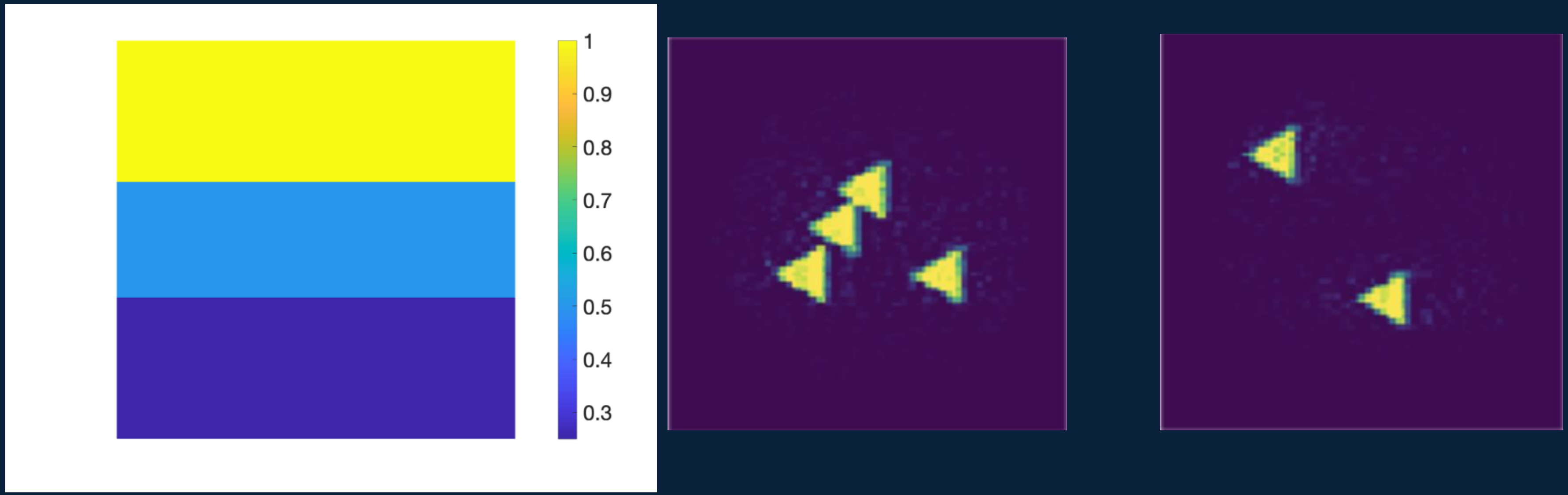

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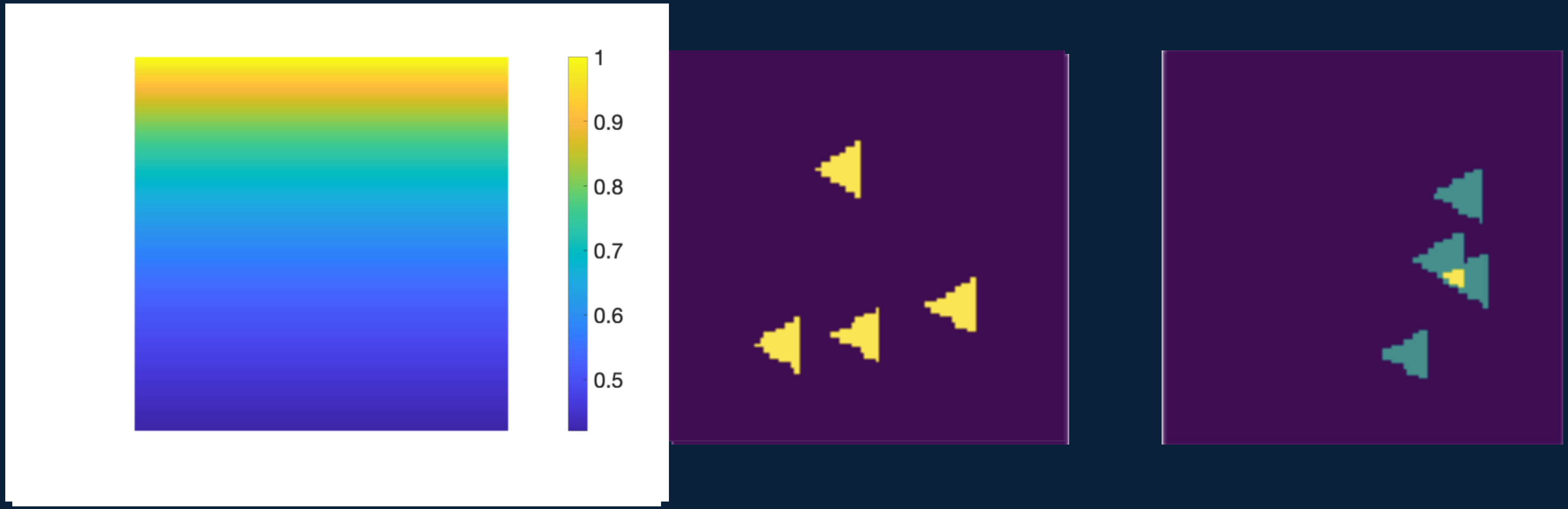

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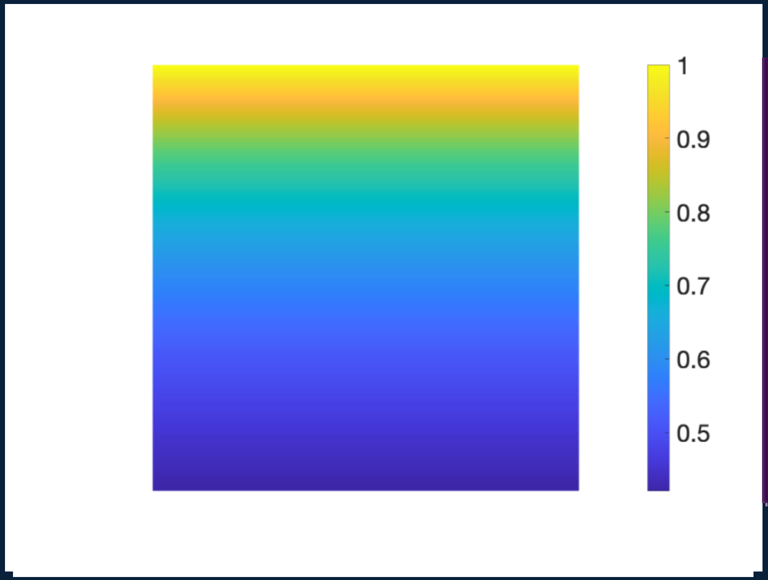

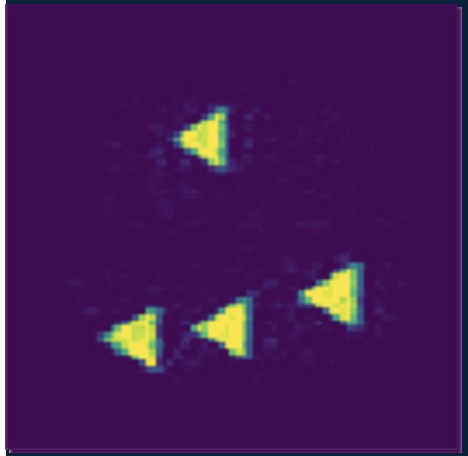

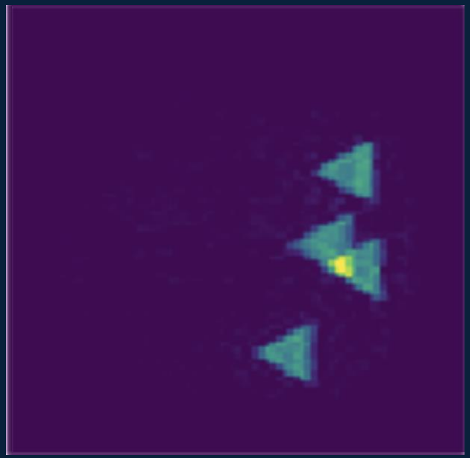

Thank You! Questions?

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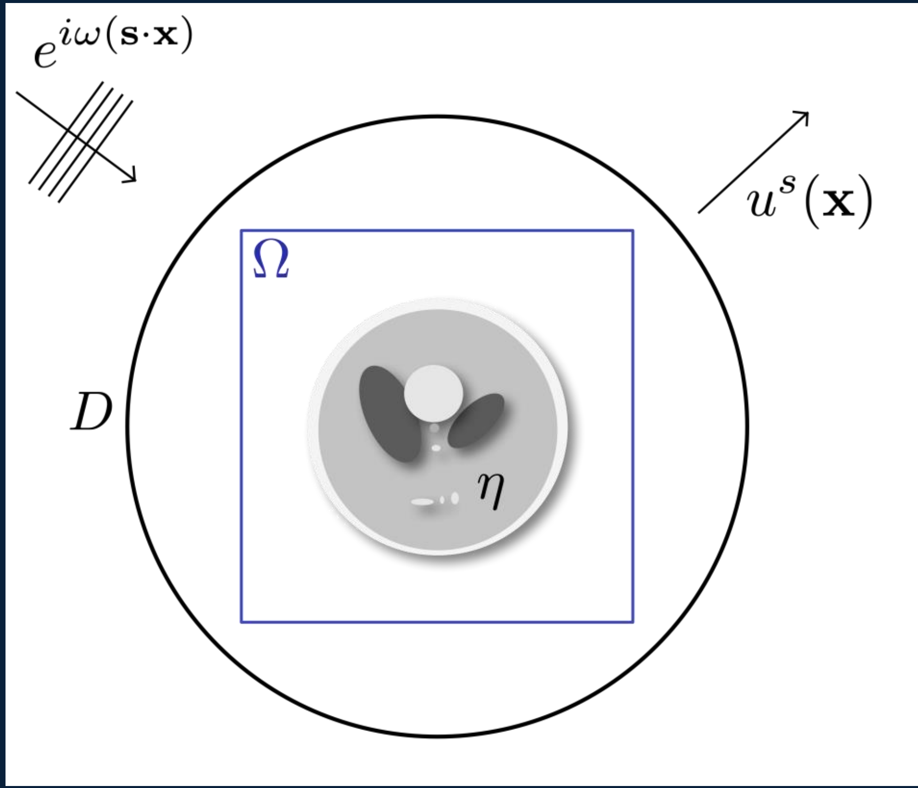
Bonus Slides

Mathematical Setup



$$(\Delta + \omega^2 m(x))u_{\text{tot}}(x) = 0$$

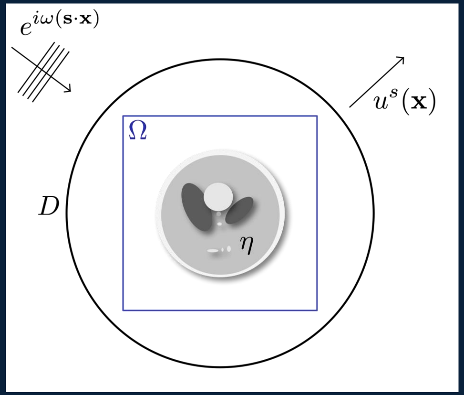
$$m(x) = 1 + \eta(x)$$

$$u_{\text{tot}}(x) = e^{i\omega\mathbf{x}\cdot\mathbf{s}} + u_{\text{sc}}(x)$$

$$(\Delta + \omega^2 m)u_{\text{sc}} = -\omega^2 \eta e^{i\omega\mathbf{x}\cdot\mathbf{s}}$$

$$(\frac{\partial}{\partial|x|} - ik)u_{\text{sc}} = o(|x|^{-1})$$

Imaging Setup



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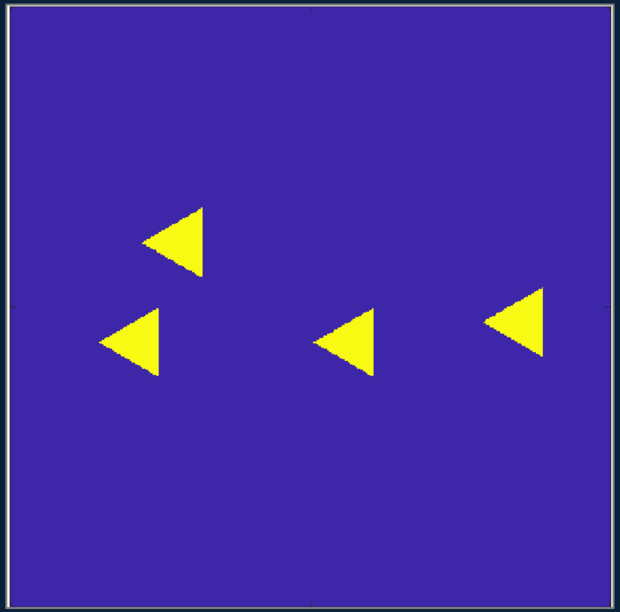
$\eta(x)$



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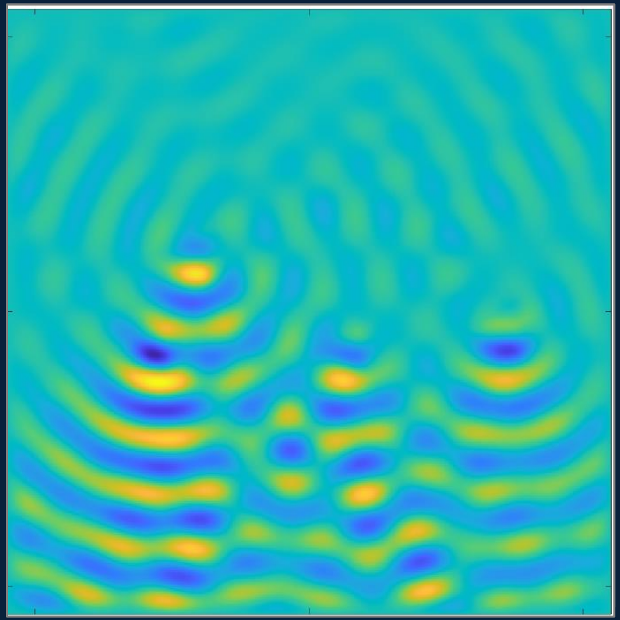

Geophysical Relevance

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Diffraction imaging by focusing-defocusing: An outlook on seismic superresolution

reconstruct the geometry of strong and smooth reflectors—most often with success. At the same time, however, correct identification of geological discontinuities, such as faults, pinch-outs, and small-size scattering objects, is an important problem in interpretation of seismic data. Local structural and litho-logical elements in the subsurface of a size comparable to the wavelength are usually ignored during processing and identi-fied only during interpretation. Unfortunately, the reliability of such identification is generally low. It is precisely the seismic

dynamical behavior can be analyzed and exploited in more re-fined algorithms. The phase content of the diffraction image can be analyzed to extract more information on fault edges, their orientation, and their relation to reflectors. In three dimen-sions, diffractions differentiate between edges and tips (Klem-Musatov, 1994). In 4D (time-lapse) studies, diffractions can help to monitor the displacement of the oil/gas–water contact during production.

In any case, we have shown that diffraction imaging is feasi-