Using co-propagating P waves to extract nonlinear elastic characteristics of rock

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Summary

We propose a dynamic method to detect the nonlinear elastic hysteretic characteristics of rock. The method is easily implemented and requires injecting two co-propagating longitudinal waves into the rock and receiving them. One of the waves is a high energy low-frequency longitudinal wave that loads strain in the rock, and a other is the low energy high-frequency longitudinal wave that is used to detect the elastic modulus variation induced by the loaded strain. Analyzing the relationship between elastic modulus variation and the loaded strain, we quantify the nonlinear elastic characteristics of the rock, including the nonlinear coefficients and slow dynamics, and evaluate hysteretic loops. A typical sandstone experiment is displayed and results show the nonlinear coefficients are independent of input amplitude of the longitudinal waves, but slow dynamics are a function of the input amplitude of the low fequency longitudinal waves.

Introduction

The nonlinear elastic response of rocks is known to be caused by the rock's microstructure (Darling et al., 2004). Obtaining information about the nonlinear response of a rock mass may help us to better characterize the reservior and to distinguish it from neighboring regions. Several measurement methods have been developed to allow one to characterize the nonlinear elastic response of rocks. In static acousto-elasticity, acoustic waves propagate through a sample, while different amplitudes of static stresses are applied (Winkler and McGowan, 2004). Dynamic acousto-elasticity (DAE) uses a low-frequency (LF) wave source in resonance instead of a static device to apply stress to a rod shaped sample (Renaud et al., 2011; Riviere et al., 2013). Hysteretic nonlinear elasticity has been observed and nonlinear coefficents have been extracted in sandstone sucessfully using DAE (Renaud et al., 2013).

In the field, it is not easy to either load static stresses or to generate resonance in subsurface rock. In this work, we present measurements obtained using a new approach, the co-propagating P wave detection method, that employs high energy low-frequency longitudinal waves pump, instead of resonance used by DAE, to induce the nonlinearity in the rock, and uses a low energy high-frequency (HF) longitudinal wave probe to detect the nonlinear elastic characteristics of the rock. Using propagating waves makes our method easy to implement for field measurements.

Method

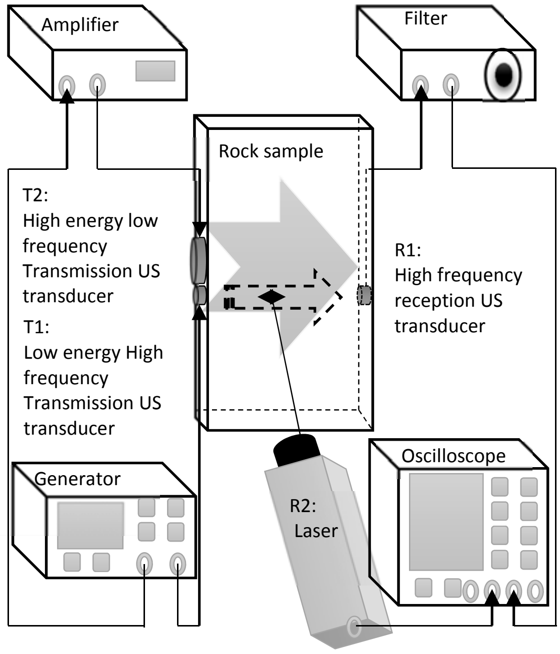

Figure 1: Experimental setup to perform the co-propagating P wave detection method. T1 is a low energy high frequency (HF) Transmission US transducer, the probe, and R1 is a HF reception US transducer. T2 is a high energy low frequency (LF) Transmission US transducer, the pump, and R2 is a laser vibrometer. Gray and dashed arrows represent the pump and the probe signals, respectively.

To test and demostrate the method, we show a laboratory experiment setup using unltrasonic longitudinal waves. Figure 1 shows the laboratory experiment setup of the co-propagating P waves detection method. A room-dry block of sandstone was chosen for this experiment. 0.1MHz LF (T2) and 1MHz HF (T1) diameter compressional US source transducers are mounted on one side of the rock sample to generate the pump and probe signals, respectively. Another 1 MHz HF (R1) matching compressional US transducer is mounted in the opposite side of the rock sample to receive the probe signal. The low and high frequency acoustic beams overlap allowing the signals to interact. A laser vibrometer (R2) measures the particle vibration velocity of the pump signal. A waveform generator excites both a LF signal, which is amplified by a RF power amplifier and sent to T2, and a HF signal, that is sent to T1. The frequencies of the pump and the probe signals were chosen to maximize signal strength in the rock. Probe signals received by R1 are filtered using a high-pass filter to remove contamination by the pump signals. Finally, a digital storage oscilloscope stores both R1 and R2.

We measure two probe signals. One is the probe signal accompanying the pump signal. The other one is the probe signal in the absence of the pump signal. Comparing the time of flight (TOF) of the probe signal with and without the pump signal, we obtain a measure of the TOF variation, which is related to the elastic modulus variation. By varying the amplitude of the pump signal, we can obtain the elastic modulus variation (EMV) as a function of the applied strain, which provides a quantitative measure of the nonlinear elastic behavior of the rock. Based on the TOF variation, we can estimate the EMV, $\Delta M/M$, using (Renaud et al., 2012)：

$$\Delta M/M = -2\,\Delta TOF/TOF_0. \qquad (1)$$

Where TOF_0 is the TOF of the probe in the absence of the pump signal.

Based on the average X-axis particle vibration velocity v, which is measured by the laser vibrometer R2, the X-axis normal strain induced by the pump signals can be calculated:

$$\varepsilon(t) = -v(t)/v_p. \qquad (2)$$

Where v_p is the longitudinal wave velocity.

By analyzing the relationship between the EMV, $\Delta M/M$, and the loaded strain, ε, we can quantify the nonlinear elastic characteristics of the rock, including the nonlinear coefficients and slow dynamics, and evaluate hysteretic loops.

Example using Crab Orchard sandstone

A room-dry CrabOrchard sandstone (COS) (Benson et al., 2005) sample from the Cumberland Plateau, Tennessee, USA, was chosen for this study. Both the width and height of the sandstone sample are 15cm. The thickness is 5 cm. The room temperature was about 21 ^{O}C and the humidity was about 23% when the experiment was performed.

In the experiment, the probe transducer (T1) trigger is delayed relative to the pump transducer (T2) trigger from 0 us to 45 us. Since the amplitude of an pump signal is time dependent, delaying the probe onset relative to the pump onset allows us to estimate nonlinear parameters for different strain amplitudes. The pump induces propagating packets of strain and the probe wavelet rides within a particular pump-induced strain state along its path. Therefore, trigger delay time of probe to pump allows us to vary the strain that the pump sees. Figure 2 (a) shows the X-axis strain induced

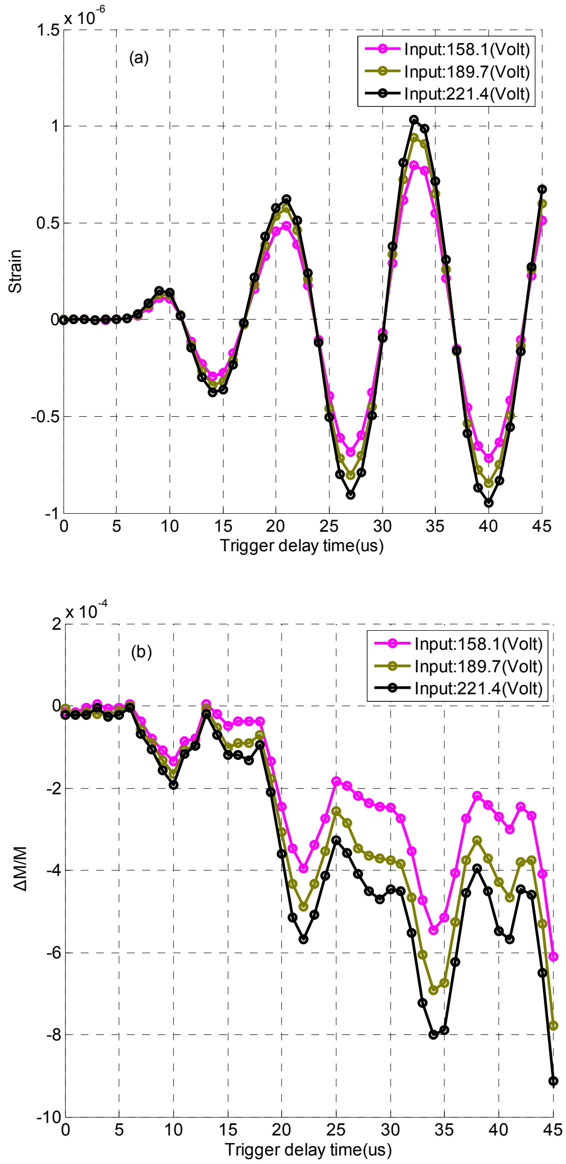

Figure 2: (a) X-axis strain induced by the pump signals as a function of the trigger delay time. (b) Elastic modulus variation, $\Delta M/M$, measured by the probe signals as a function of the trigger delay time. The trigger delay time is the time that the probe trigger is delayed relative to the pump onset. Different solid color lines indicate different input voltage of pump.

along the probe path by the pump signals as a function of trigger delay time. We input 3 different voltages into the pump, and obtained 3 measurements of strains as displayed by solid color lines in the figure. Figure 2 (b) shows the elastic modulus variation measured by the probe signals as a function of trigger delay time. Because 3 different voltages were input into the pump, we obtained 3 curves of elastic modulus variation that are displayed by solid color lines in the figure.

One strain cycle with a maximum value that occurs between 30.1 us and 42.9 us (the trigger delay time) was selected for analysis of the relationship between strain and elastic modulus variation. Figure 3 shows the elastic modulus variation as a function of the X-axis strain in the selected cycle. In the figure, different colors indicate different pump inputs.

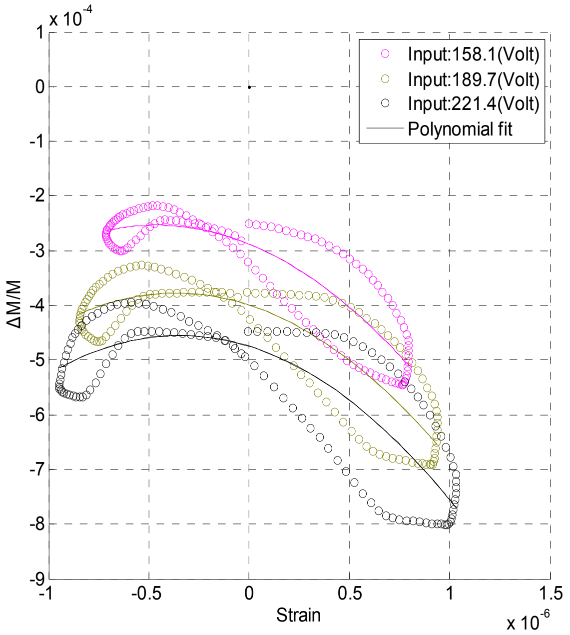

Figure 3: Elastic modulus variation, $\Delta M/M$, as a function of the X-axias strain in the cycle having trigger delay times ranging from 30.1 us to 42.9 us. Solid lines show a second-order polynomial fit to each set of data. Different colors indicate different pump input voltages.

Discussion

Nonlinear Coefficients

Classical nonlinear elastic theory describes,

$$\sigma = \mathrm{M}\varepsilon + \beta M\varepsilon^2 + \delta M\varepsilon^3, \qquad (3)$$

where M is the elastic modulus. β and δ are the nonlinear elastic coefficients for quadratic and cubic elastic nonlinearities, respectively. In this work, we analyze the nonlinear behavior using a practical approach by which the nonlinear coefficients are extracted as a function of the strain by applying a second-order polynomial fit:

$$\Delta M/M = \delta\varepsilon^2 + \beta\varepsilon + \alpha, \qquad (4)$$

Where, α quantifies the offset of the elastic modulus variation. In figure 3, solid lines show the second-order polynomial fit to each set of data.

We subtract α from the elastic modulus variation, and get the term ($\Delta M/M - \alpha$) shown in the figure 4. All the curves overlap very well. Results show that β and δ have similar values among the seven pump inputs studied. The average β is -113.34 and the average δ is -1.82×10^8. Therefore, the nonlinear coefficients are independent of input amplitude of the longitudinal waves in the condition of low strain.

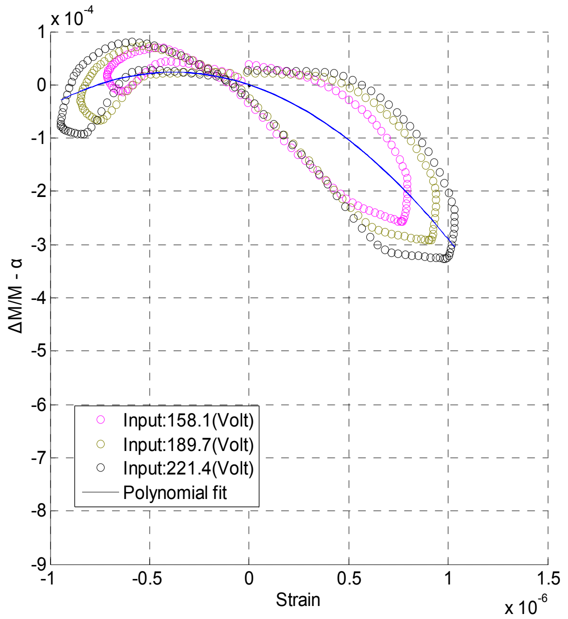

Figure 4: The difference between elastic modulus variation and Alpha ($\Delta M/M - \alpha$) as a function of the X-axis strain in the cycle from 30.1 μs to 43.2 μs in trigger delay time that is the same as that in the figure 3. Solid line shows a second-order polynomial fit to data. Different colors indicate different pump input voltages.

Slow Dynamics

We measured the response of the rock using seven different pump input voltages. We find that the shapes of the hysteretic loops are similar as is shown in figure 4 for three of the measurements. The shapes are controlled by the beta and delta given in equation 4. These parameters give the short-term response of the rock to the applied high-amplitude strain. The offset of one curve relative to the other is given by alpha, which describes the longer-term response of the rock and may be related to slow dynamics (TenCate, 2011). We find that alpha varies linearly with the maximum strain (from Eq. 2) applied during the cycle studied.

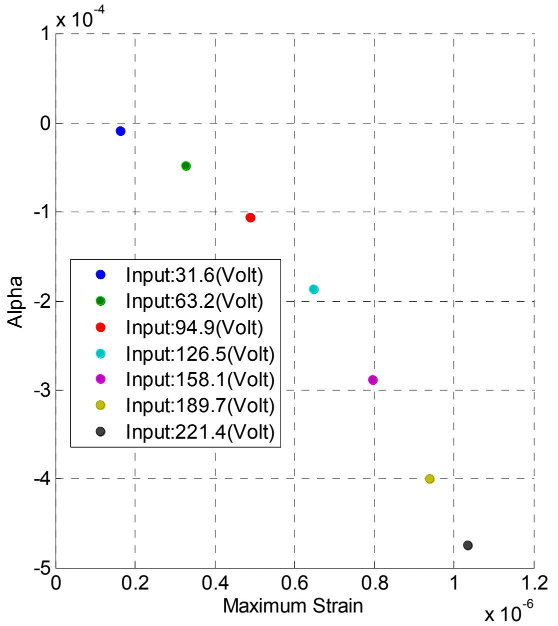

Figure 5: α as a function of the maximum measured strain in the cycle from 30.1 μs to 43.2 μs in trigger delay time that is the same as that in the figure 3. Different colors indicate different pump input voltages.

Hysteretic loops

From the figure 3 and figure 4, we can clearly find the hysteretic loops between strain and elastic modulus variation. The cause is likely the viscoelastic elements in the sandstone, for example clay.

Conclusions

We propose a dynamic method to detect the nonlinear elastic and hysteretic characteristics of rock. The method uses propagating waves to replace the static stresses and resonance used by others. Our method could be easily implemented to make field measurements.

The quadratic and cubic nonlinear coefficients, β and δ, are independent of input energy. Memory/ slow dynamics parameter, α, is a function of the input strain. Also hysteretic loops are clearly observed in Crab Orchard sandstone.

Acknowledgments

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