Seismic characterization of fractured reservoirs by focusing Gaussian beams

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**ABSTRACT**

Naturally fractured reservoirs occur worldwide, and they account for the bulk of global oil production. The most important impact of fractures is their influence on fluid flow. To maximize oil production, the characterization of a fractured reservoir at the scale of an oil field is very important. For fluid transport, the critical parameters are connectivity and transmissivity plus orientation. These can be related to fracture spacing, compliance, and orientation, which are the critical seismic parameters of rock physics models. We discovered a new seismic technique that can invert for the spatially dependent fracture orientation, spacing, and compliance, using surface seismic data. Unlike most seismic methods that rely on using singly scattered/diffracted waves whose signal-to-noise ratios are usually very low, we found that waves multiply scattered by fractures can be energetic. The direction information of the fracture multiply scattered waves contains fracture orientation and spacing information, and the amplitude of these waves gives the compliance. Our algorithm made use of the interference of two true-amplitude Gaussian beams emitted from surface source and receiver arrays that are extrapolated downward and focused on fractured reservoir targets. The double beam interference pattern provides information about the three fracture parameters. We performed a blind test on our methodology. A 3D model with two sets of orthogonal fractures was built, and a 3D staggered finite-difference method using the Schoenberg linear-slip boundary condition for fractures was used to generate the synthetic surface seismic data set. The test results showed that we were able to not only invert for the fracture orientation and spacing, but also the compliance field.

**INTRODUCTION**

Naturally fractured carbonate reservoirs occur worldwide. When the matrix reservoir rock is impermeable (limestones, dolomites, chert, etc.), fractures may dominate the porosity and permeability. In major oil-producing regions, such as in the Middle East and Mexico, the bulk of oil production is from fractured reservoirs. The presence of fractures can promote or impede fluid flow depending on whether or not the fractures are sealed (e.g., Nelson, 2001). They can also cause short-circuit flow during production. Therefore, to enhance oil production in fractured reservoirs, we need to better characterize the fractures. For example, Kang et al. (2011) show that if fracture plane orientation, spacing, and permeability are known, one can use the continuous time random walk method to better capture flow transport behavior in fractured media.

In the past, several direct and indirect methods have been developed to estimate fracture parameters. Direct borehole coring or Formation MicroImager logs provide local information on the scale of centimeters. Extrapolating what has been observed from these near-wellbore measurements outward to larger distances is difficult, and we need to make unjustified assumptions. Seismic waves impinging upon a single fracture that intersects a borehole can squeeze fluid contained in the fracture into the borehole and generate borehole tube waves that can be observed with vertical seismic profiling and studied to estimate the compliance of the single fracture (e.g., Bakku et al., 2013). Fractures are mechanical discontinuities, and they scatter seismic waves. A fractured reservoir, if the fractures are closely spaced, can be treated as an equivalent anisotropic medium. Common methods based on seismic anisotropy include the amplitude-versus-azimuth analysis of reflected P-waves (Ruger and Tsvankin, 1997) or shear-wave splitting analysis (e.g., Vetri et al., 2003). On the other hand, when the fractures are not closely spaced, seismic wave scattering can, in principle, be used to characterize the fractures. Seismic scattering and diffraction by a fracture may have...
different moveout from that from a specular reflection. By properly treating these diffracted signals (Landa, 2007), one can image fracture tips or edges. However, these singly scattered waves usually have low signal-to-noise ratios (S/Ns). For surface seismic, Willis et al. (2006) show that scattering characteristics are different for source-receiver lines oriented normal and parallel to the fracture planes. Based on the azimuthal variation of the common-midpoint gather (CMP) stacks, they propose a seismic scattering index (SI) method. In our numerical experiment using models with multiple sets of fractures with different fracture compliances, the SI method can at best resolve only the stronger fracture set and has limited resolving power for fracture spacing and compliance.

We seek a method for characterizing a fractured reservoir that has the ability to resolve spatially dependent information about fracture spacing, orientation, and compliance; the ability to distinguish the presence of multiple fracture sets and to extract the fracture parameters of each set; and the ability to overcome the low S/N of the scattered waves. In many fracture characterization studies, a common assumption is that all fractures are vertical. Ideally, the method should also be able to relax this condition because structural beds may be dipping. In this Letter, we briefly delineate a method that is capable of accomplishing all these objectives by yielding spatially dependent fracture parameters for multiple sets using 3D focusing Gaussian beams.

THEORY AND METHOD

In developing our idea, there are several important consecutive steps. We first identify the important role of multiple scattering by fractures because the amplitudes of these waves can be stronger than those of the primary reflections commonly used in seismic migration and imaging. Second, we find that these multiply scattered waves are directional. Third, the multiply scattered waves are back-scattered. Fourth, we relate the incident wavenumber to the scattered wavenumber by fracture parameters (i.e., orientation and spacing). Fifth, we localize the signal in space and time and construct true-amplitude focusing propagators for stacking. We show that the stacking amplitude is proportional to the fracture compliance.

Diagnostic features of fracture-scattered waves

To illustrate the basic idea, let us first consider the simplest fracture systems in which vertical fractures are situated in a homogeneous medium and are parallel to each other (Figure 1). The spacing between neighboring fractures is a, and all fracture planes are within the same vertical interval. Let us consider a plane wave incident upon this periodic medium. If the incident plane wavenumber is $k^i$, the scattered plane wave $k^s$ must be discrete: $k^s = k^0(n)$ (e.g., Rayleigh, 1907; Aki and Larner, 1970; Ishimaru, 1991). The horizontal wavenumbers must satisfy the following relation:

$$
k^s(n) = k^i + n \frac{2\pi}{a} \hat{\phi}, \quad n = 0, \pm1, \pm2, \ldots, \tag{1}
$$

where $k^i = (k^i_x, k^i_y, k^i_z)$, $k^0 = (k^0_x, k^0_y, k^0_z)$, and $\hat{\phi}$ is the fracture orientation defined as a unit vector perpendicular to the fracture plane, $\hat{\phi} = \hat{x}$ in Figure 1. We emphasize that equation 1 is valid not only for singly scattered waves but also for multiply scattered waves in the vicinity of the fractured reservoir. The important information in equation 1 is that the scattered wavenumber $k^s(n)$ is directional and is related to the fracture spacing $a$ and orientation $\hat{\phi}$. For $n = 0$, we have $k^s = k^0$, which represents the case of CMP stacking, and in this case there is no role for the fracture parameters, $a$ and $\hat{\phi}$. It is also worthwhile to point out that in the elastic case, equation 1 is not only valid for P-to-P scattering, but it is also valid for P-to-S, S-to-P, and S-to-S scattering. Although in equation 1 there are infinite possibilities for the scattered wavenumber $k^s(n)$ for a given incident $k^i$, a large $n$ is not important. If $n$ is too large, the vertical wavenumber will be an imaginary number because the horizontal and the vertical wavenumbers must satisfy the dispersion relation. Consequently, the scattered plane wave will be evanescent and decay exponentially away from the fractures. Therefore, $n$ has to be small. In this Letter, we focus on the case of $n = -1$ because we have observed in numerical simulations that the first-order back-scattered wave has strong energy; in this case, equation 1 reduces to

$$
k^s_T = k^i_T - \text{sgn}(k^i_T \cdot \hat{\phi}) \frac{2\pi}{a} \hat{\phi}, \tag{2}
$$

where sgn takes the sign of $k^i_T \cdot \hat{\phi}$. In view of the relation between the wavenumbers and the slowness vectors via the angular frequency $\omega$,

$$
k^0 = \omega p^0; \quad k^i = \omega p^i, \tag{3}
$$

equation 2 can be rewritten as

$$
p^s_T = p^i_T - \text{sgn}(p^i_T \cdot \hat{\phi}) \frac{1}{\alpha f} \hat{\phi}, \tag{4}
$$

where $p^0 = (p^0_x, p^0_y)$ and $p^i = (p^i_x, p^i_y)$, and $\omega = 2\pi f$, in which $f$ is the frequency of the seismic wave. From equation 4, we can see that the minimum fracture spacing we can resolve is $\lambda/2$ for P-to-P scattering, where $\lambda$ is the P wavelength near the fractured reservoir. Similar analyses can be done for other modes of scattering. For instance, for a typical reflection seismic survey, if the reservoir P-wave velocity is 2500 m/s and the frequency is 50 Hz, then we can resolve fractures spaced at 25 m. However, if 3C seismic data are available, the minimum fracture spacing that can be probed is about 10 m using S waves.

Figure 1. Equally spaced vertical fractures for a plane-wave incidence.
Double focusing Gaussian beam operator

We have shown that fracture-scattered signals are directionally. To exploit this information, the propagator used for stacking the seismic data will be better if it naturally has a dominant propagation direction. Because the fracture parameters may vary with location, the propagator should also be localized in space. A propagator that is simultaneously localized in the space and wavenumber domains is a beam propagator. Hence, Gaussian beams (e.g., Hill, 1990; Gray and Bleistein, 2009) are natural choices. It is well known that a Gaussian beam is a globally regular propagator that preserves the true-amplitude information required for retrieving fracture compliance values. Details concerning the Gaussian beam solution can be found in the above-cited references. To use the scattering property in equation 4, the incident and scattered waves around the target need to be local plane waves. We use $D(x, x, \omega)$ to represent the frequency-domain seismic data recorded at receiver $x$, by a point source $x_s$ at time zero. Our purpose is to synthesize a local incident plane wave at the target using these point sources. Assume that, at the target location $r$, a Gaussian beam with unit amplitude, zero beam-front curvature (i.e., a local plane wave), and beam width $w_r$ (one standard deviation of the Gaussian profile) is shot upward (Figure 2) along the slowness vector $p^s$ and recorded at source location $x_s$:

$$b_s(x_s|p^s, w_s, \omega) = A_s(x_s|p^s) e^{i\omega r(x_s|p^s)}$$

(5)

where $A_s(x_s|p^s)$ is the complex amplitude and $r(x_s|p^s)$ is the complex traveltime. Because the beam propagation is time reversible, if we weight the point source strength and advance the source timing according to $b_s$, we approximately create a synthetic data gather $d_{syn}(x_s|p^s, \omega)$, as if the source were a Gaussian beam of width $w_s$ with unit amplitude and zero curvature (i.e., a local plane wave), incident upon fractures at $r$ at time zero in the direction of $-p^s$. Mathematically, this process can be expressed using the Rayleigh integral:

$$d_{syn}(x_s|p^s, \omega) = 2i\omega p^s \int d^2x_s b_s^*(x_s|p^s, w_s, \omega) D(x_s|p^s, x_r, \omega) d^2x_r$$

(6)

where $p^s$ is the vertical slowness component of $p^s$ at the beam center $x_s$ and $b_s^*$ is the complex conjugate of $b_s$. The integration $d^2x_s$ is over the support of the beam $b_s$ on the source surface. Assuming that we were given fracture spacing $\alpha$ and orientation $\hat{\phi}$, with the slowness vector $p^s$, we can compute the slowness vector $p^s$ for the scattered wave using equation 4. The scattered wave $d_{syn}$ due to this hypothetical beam source can be modeled as a product of the unit incident beam at $r$ at time zero, the scattering coefficient $\alpha$, and the scattered Gaussian beam $b_r$ back to receivers. The Gaussian beam $b_r$ has a unit amplitude, zero beam-front curvature and beam width $w_r$ at $r$. This can be expressed as

$$1 \cdot \alpha \cdot b_r(x_r|p^s, w_r, \omega) \cdot S(\omega) \approx 2i\omega p^s \int d^2x_r A_r(x_r|p^s) e^{-i\omega r(x_s|p^s)} D(x_r|p^s, x_r, \omega) d^2x_r$$

(7)

where

$$b_r(x_r|p^s, w_r, \omega) = A_r(x_r|p^s) e^{i\omega r(x_r|p^s)}$$

(8)

is the receiver beam with complex amplitude $A_r$ and complex traveltime $\tau$.

Figure 2. Schematic for double Gaussian beam stacking. Beam centers for the source (stars) beam and the receiver (triangles) beam are represented by $\bar{x}_s$ and $\bar{x}_r$, respectively.
\[ \sigma = \sigma(a, \hat{\phi}, r, w_s, w_g, \omega) \] (9)

is the scattering coefficient of the fractures, and \( S(\omega) \) is the source wavelet spectrum. We can assume \( S(\omega) = 1 \) or absorb it into \( \sigma \).

Combining equations 7 and 8 and summing over all sources and receivers yields

\[
\sigma \cdot \iiint A_g^2(x_g | r) d^2 x_g = 2 \omega p_s \int d^2 x_s \int d^2 x_n F^* (x_s, x_n | r, p_s, p_g, w_s, w_g, \omega) D(x_s, x_n, \omega),
\] (10)

where the double Gaussian beam focusing operator \( F^* \) reads

\[
F^* (x_s, x_n | r, p_s, p_g, w_s, w_g, \omega) = A_s^* (x_s | r) A_g^* (x_n | r) e^{-i \omega \tau (x_s | r) - i \omega \tau (x_n | r)}.
\] (11)

In equation 10, we can window the time-domain seismic data \( D(x_s, x_n, t) \) around \( t = \text{Re} [\tau (x_s) + \tau (x_n)] \) and then transform the windowed data into the frequency domain \( D(x_s, x_n, \omega) \) for stacking. Stacking the windowed data rather than the whole data trace minimizes the effect of interference due to signals that are not scattered from the fractures. A signal that is simultaneously localized in time and frequency is called a wave packet. In this regard, our method becomes a double focusing Gaussian packet stacking algorithm. However, our time windowing in the following example uses a simple edge-tapered window rather than a strict Gaussian taper. Equation 10 can be used for P-to-P and P-to-S scattering.

NUMERICAL EXAMPLE USING A FRACTURE NETWORK

To test our methodology, we used a 3D velocity model that has five layers with a fracture network in the third layer (Figure 3). The fracture network consists of two sets of vertical fractures that cross each other at right angles. We use \( x \) and \( y \) to define a rectangular coordinate system that has an arbitrary orientation relative to geographical coordinates. We call FracX the set of fractures parallel to the \( x \)-axis and FracY the set parallel to the \( y \)-axis. For FracX and FracY, the fracture spacing is uniform at 80 m. However, the fracture normal compliance is spatially dependent (Figure 5) for FracX and FracY. Overall, the FracX compliance values are two times the FracY values. Our compliance values are between \( 10^{-10} \) and \( 10^{-9} \) m/Pa, which are consistent with recent values inferred from borehole tube waves (Bakku et al., 2013). The purpose is to (1) detect the existence of the two crosscutting fracture sets and (2) invert for spatially dependent fracture compliance values for FracX and FracY.

We simulate seismic shot gathers using a 3D staggered grid finite-difference method (Coates and Schoenberg, 1995; Fang et al., 2013). Fractures in the model are treated as linear-slip boundaries (Schoenberg, 1980). An absorbing boundary condition using perfectly matched layers is imposed on all sides of the model. The

Figure 4. Double beam stacking amplitude |\( \sigma | \) for several targets. Results are displayed in polar plots with radius being the fracture spacing from 40 to 120 m and azimuth being the fracture orientation \( \hat{\phi} \) from 0° to 180°. There is a 180° ambiguity for \( \hat{\phi} \), so the 0° fractures and the 180° fractures are the same. Azimuth 90° corresponds to FracX, and 0° (or 180°) corresponds to FracY.
source time function is a Ricker with a central frequency of 40 Hz. In our test, we simulated full elastic wave propagation in the model with 4C acquisition (i.e., pressure and 3C particle velocities). However, in the double beam stacking, we used only the pressure component recorded by the receivers. So in this case, only the normal compliance field is determined. Figure 3b) shows a sample shot gather along the x-direction for a shot located at (1350, 1350 m). To implement the double beam stacking procedure for fractured reservoir characterization, we use a frequency of 45 Hz. Other frequencies (e.g., 50, 60 Hz) were also tried, and they produced similar results. Hence, we only show the results from the 45-Hz data. At 45 Hz, the P-wavelength in the third layer is about 78 m. Therefore, the minimum fracture spacing we can resolve is ∼39 m. We set the beam widths to $w_x = w_y = 100$ m, which is consistent with our best resolving capability for the fracture spacing and is also comparable to the value suggested for Gaussian beams by Hill (1990). In general, in order for a Gaussian beam to maintain its shape and to have good directionality, the width of the beam must contain a couple of wavelengths. Because our method is target oriented, we choose $21 \times 21$ targets at depth 420 m, within a square $[0, 2400] \times [0, 2400]$ m. The targets are regularly distributed with spacing 120 m between targets in the $x$- and $y$-directions. We divide all surface shots into $24 \times 24$ source beam centers. The source beam centers are regularly distributed in the square $[0, 2400] \times [0, 2400]$ m with the same spacing 104.3 m in the $x$- and $y$-directions. For windowing the seismic data, we use a time window length of 0.06 s, about 2.4 times of the source wavelet period.

Given a source beam center and a target, we first compute the source beam $b_g$. Then we scan all possible fracture spacings $a$ from 40 to 120 m (41 $a$’s), and fracture orientations $\phi$ from 0° to 180° (61 $\phi$’s). For each $(a, \phi)$ pair, we use equation 4 to find the scattered beam direction and construct $b_g$. Note that on the surface, the central ray of $b_g$ may not coincide with any of the source beam centers. We then perform double beam stacking using equation 10 to obtain $\sigma(a, \phi)$ for this source beam. Finally, $\sigma(a, \phi)$ are stacked for all source beams. Our double beam stacking results showed that two fracture sets are evident at each target location (Figure 4). The first recovered fracture set (FracX) is parallel to the $x$-direction, with fracture spacing of ∼80 m; the second recovered fracture set (FracY) is along the $y$-direction, with fracture spacing of ∼80 m. It is clear that fractures in the reservoir form a fracture network. To simulate fluid flow in such a fractured reservoir, it is critical to know the transmittivity and connectivity, which can be related to compliance values. The scattering amplitude is proportional to the compliance (Fang et al., 2013). Because our double focusing Gaussian beam operator conserves the energy flux for the transmitted wave (equation 11), we can search for the maximum stacking amplitude to find compliances for FracX and FracY (Figure 5). It is remarkable that our method provides reliable estimates for the fracture compliance fields for FracX and FracY. Fracture compliance fields together with the fracture network information are essential to build the permeability field for fractured reservoirs (Brown and Fang, 2012) for enhanced oil recovery.

**CONCLUSIONS**

Characterizing the geometry of the fracture networks within fractured reservoirs and their spatially dependent fracture compliance fields is essential for relating seismic scattering to fluid flow within the reservoir. We have presented a new target-oriented technique, which involves interfering two focusing Gaussian beams: one from the sources and the other from the receivers at the fracture reservoir location. The method promises to distinguish multiple sets of fractures in the reservoir and, therefore, provides information about connectivity of the fracture network and their compliance values in a spatially dependent fashion. Our method may be used for land and marine seismic acquisitions given adequate spatial sampling of sources and receivers. It can also be used to characterize a fractured reservoir whose fracture planes are not vertical. With proper rock physics models, our method can help bridge the gap between surface seismic data and fluid flow in the reservoir.

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Queries

1. In the following phrase, is “its” meant instead of “their”: “provides information about connectivity of the fracture network and their compliance values”?